

# A New Approach to Round-Optimal Secure Multiparty Computation

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**Computational** security.

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In the CRS model: 2 rounds [Garg-Gentry-Halevi-Raykova14, Mukherjee-Wichs16, Dodis-Halevi-Rothblum-Wichs16]

# Brief history



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Constant round protocols:

[Beaver-Micali-Rogaway90, Katz-Ostrovsky-Smith03, Pass04, Pass-Wee10, Wee10, Goyal11]

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# Our results

Result 1:

Assuming **DDH**, there exists a **5 round** MPC protocol.

Result 2:

Assuming **OWP + sub-exponentially secure DDH**, there exists a **4 round** MPC protocol.

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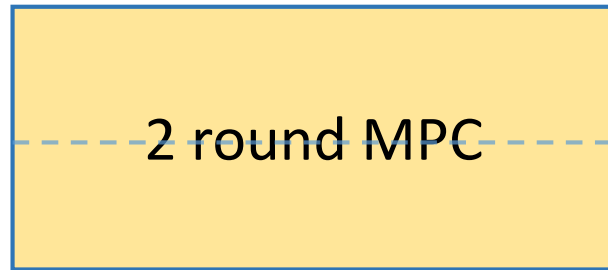
Concurrent work [Brakerski-Halevi-Polychroniadou17]:

4 round MPC protocol assuming **adaptive commitments + sub-exponential LWE**.

[Garg-Mukherjee-Pandey-Polychroniadou 16] template

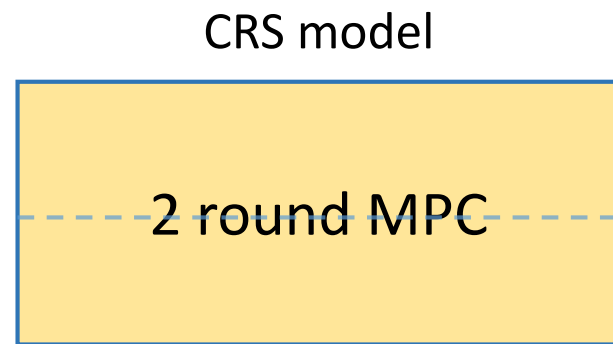
# [Garg-Mukherjee-Pandey-Polychroniadou 16] template

CRS model



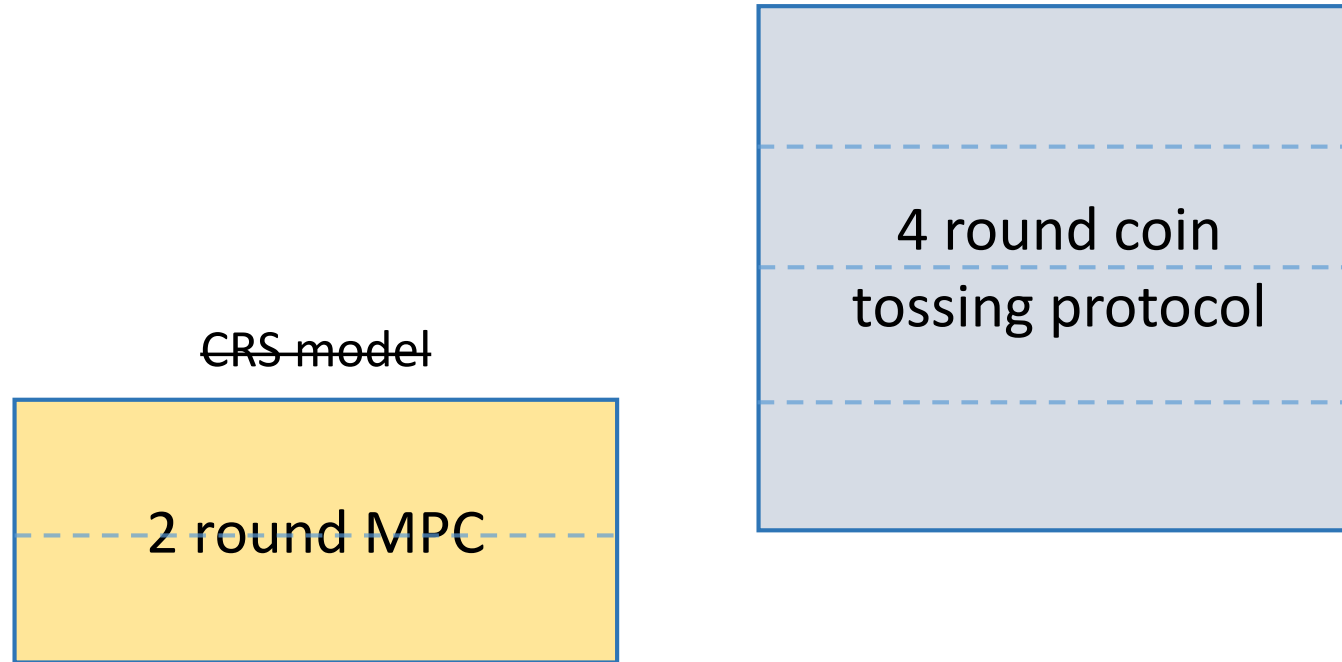
- from **iO** [GGHR14]
- from **LWE** [MW16]

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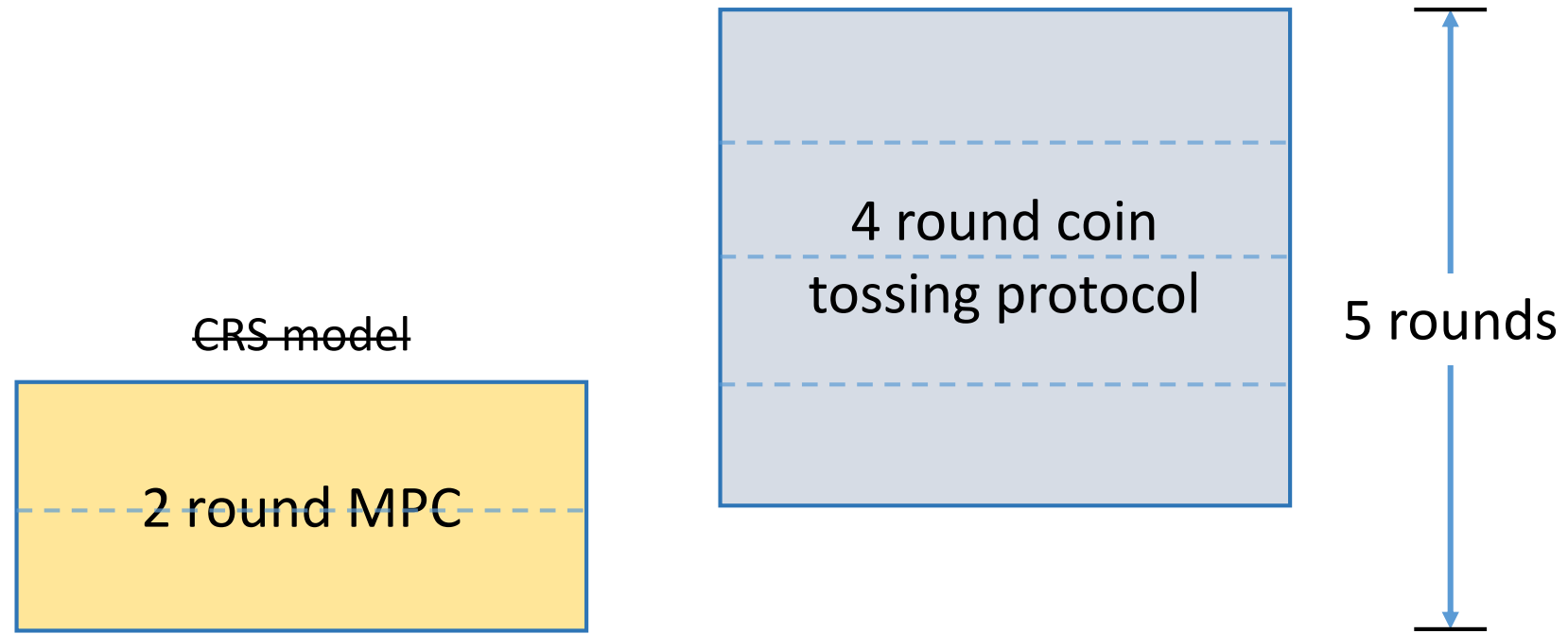
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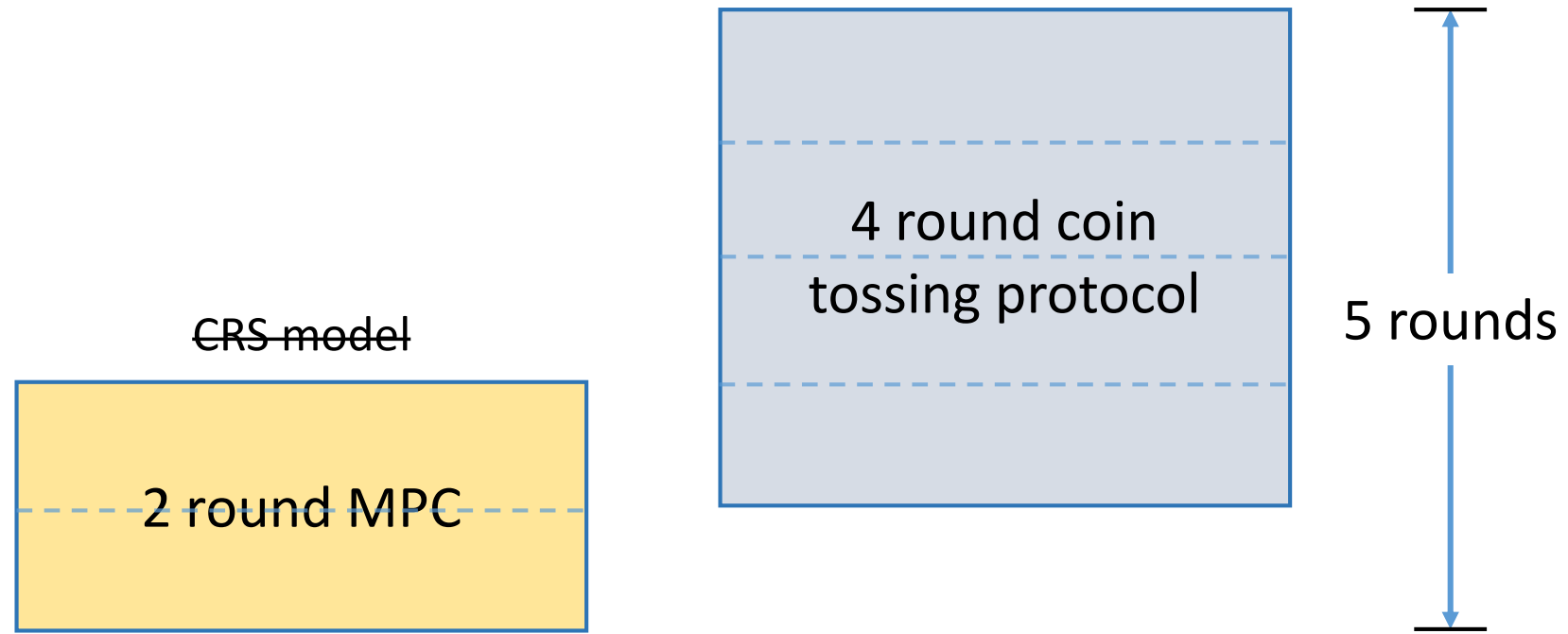




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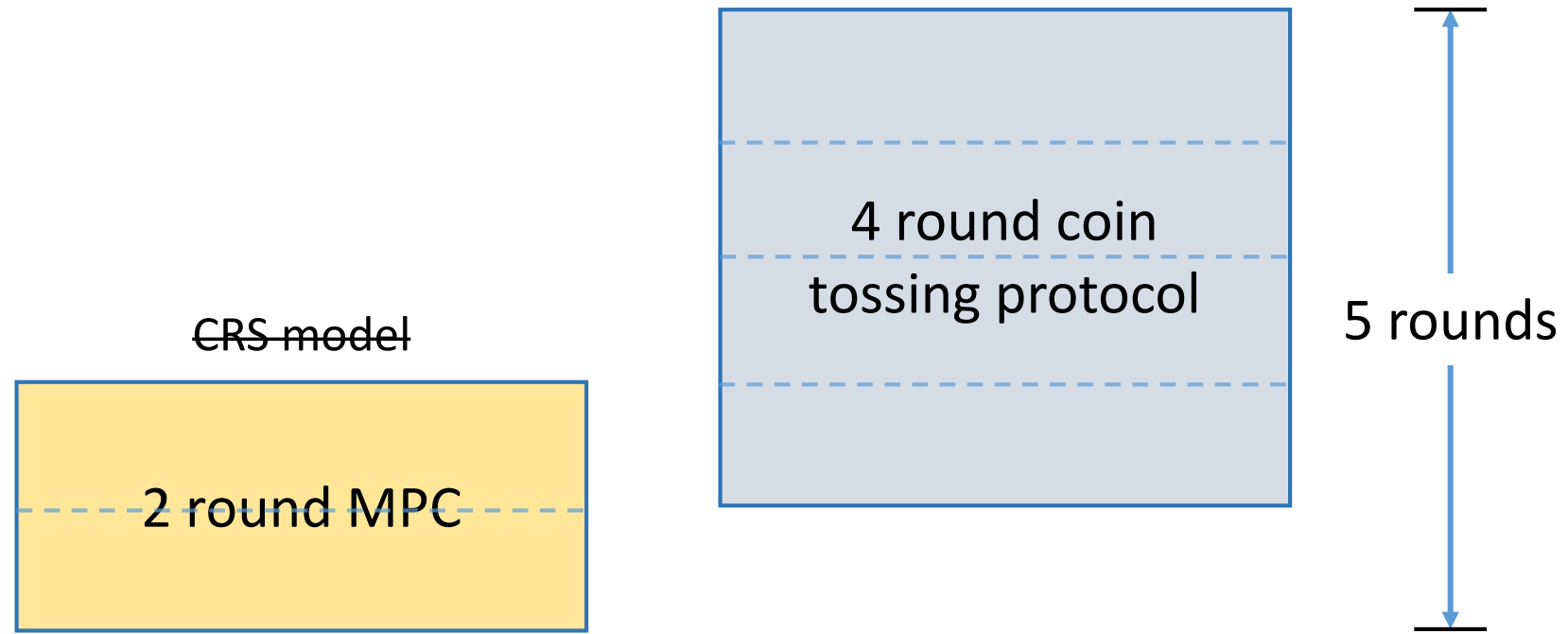


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Limitations of this approach:

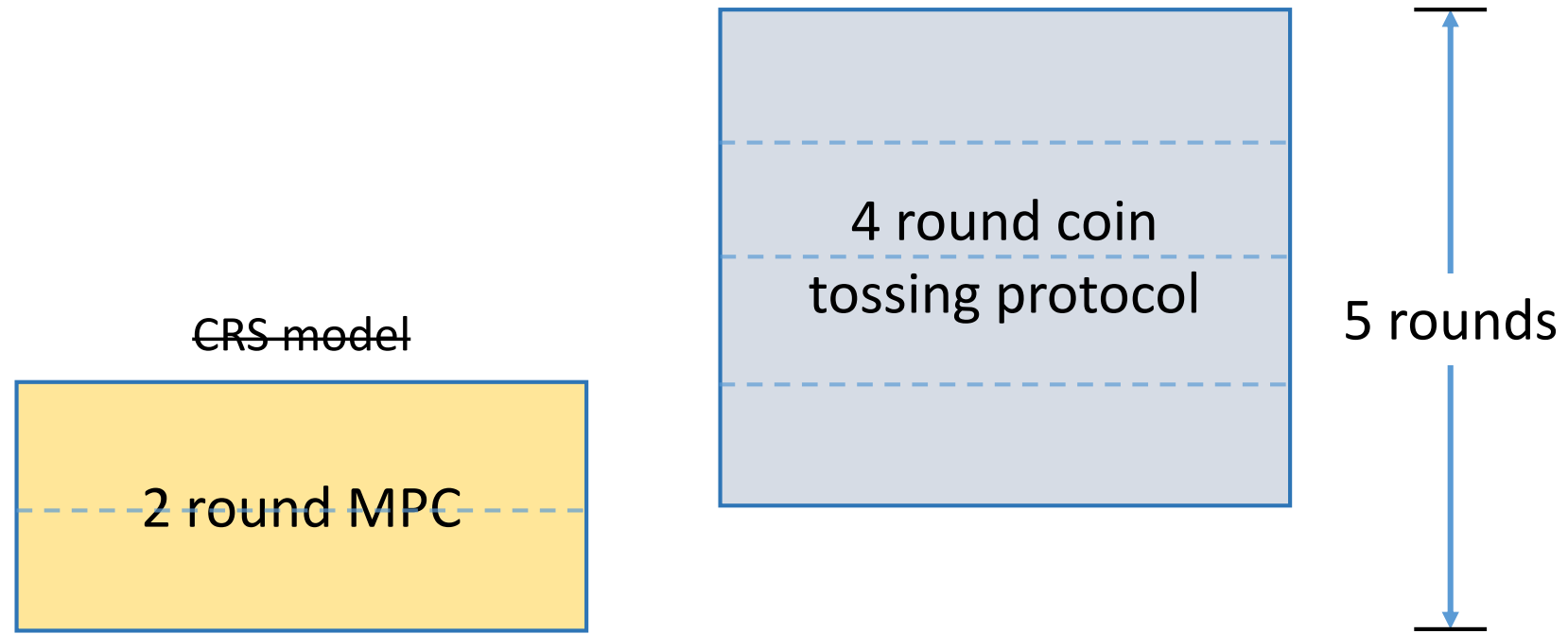
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Limitations of this approach:

Unclear how to parallelize both rounds.

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## Limitations of this approach:

- Unclear how to parallelize both rounds.
- Limits to the 2 round MPC assumptions.

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GMW compiler.

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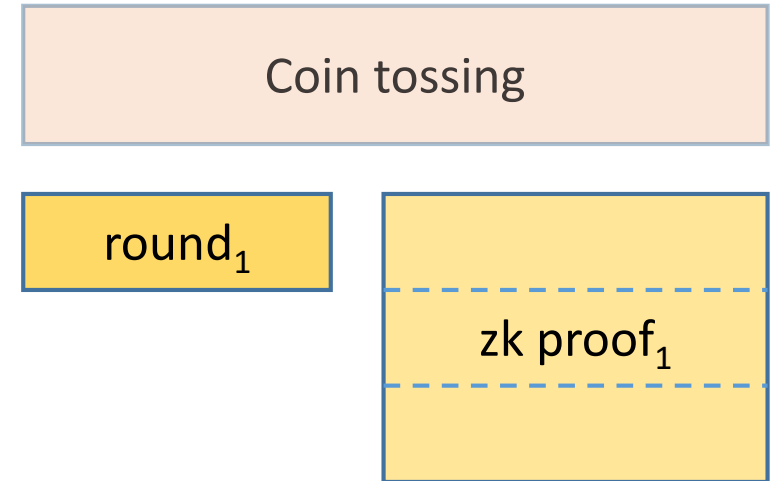
round<sub>1</sub>

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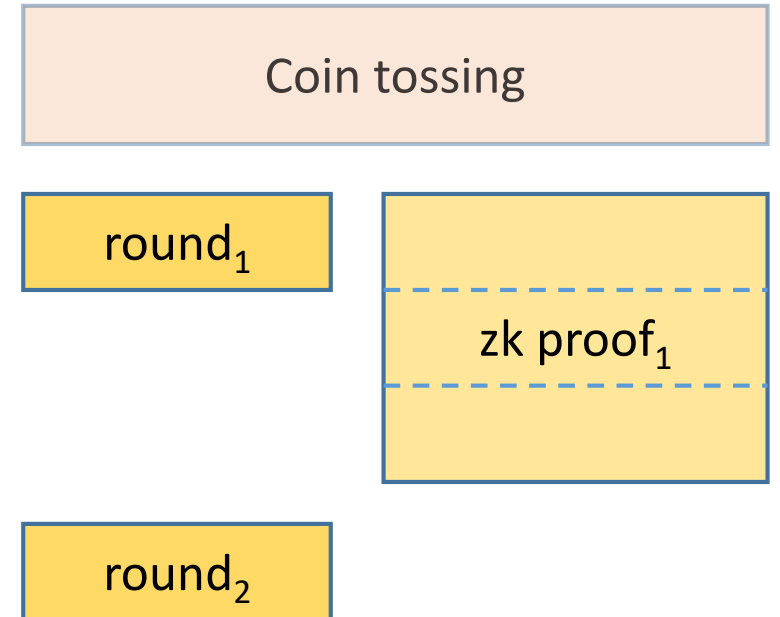


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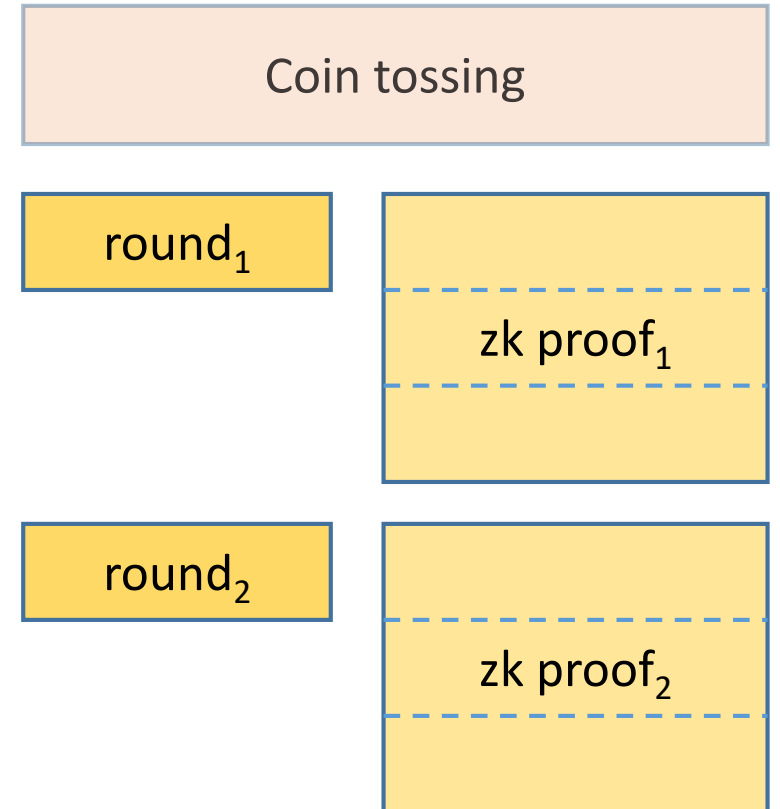


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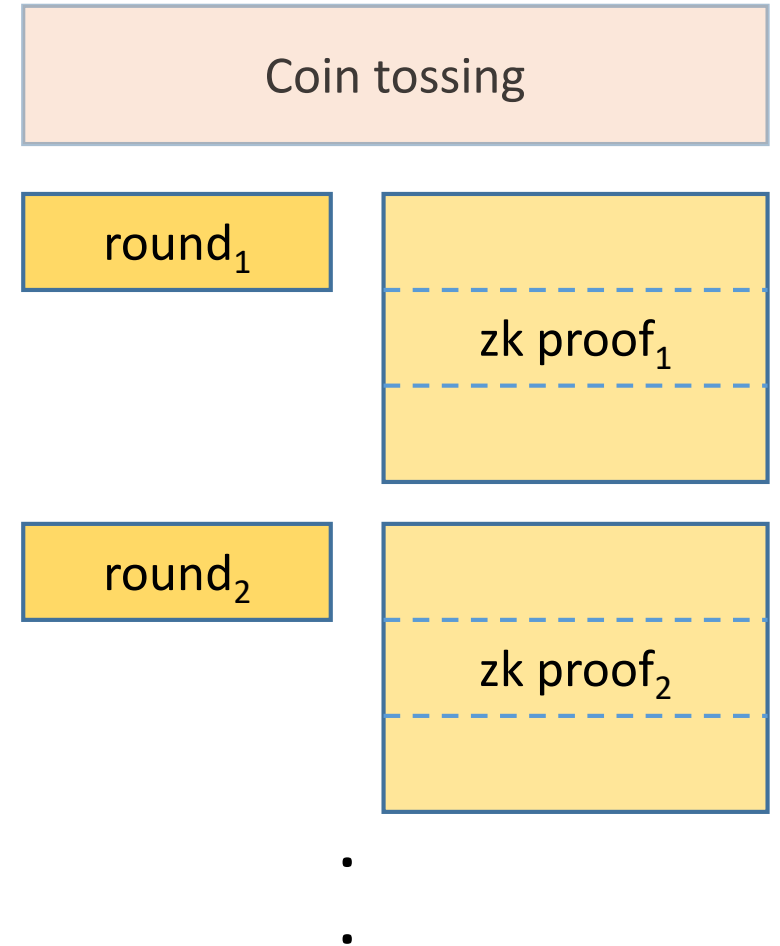


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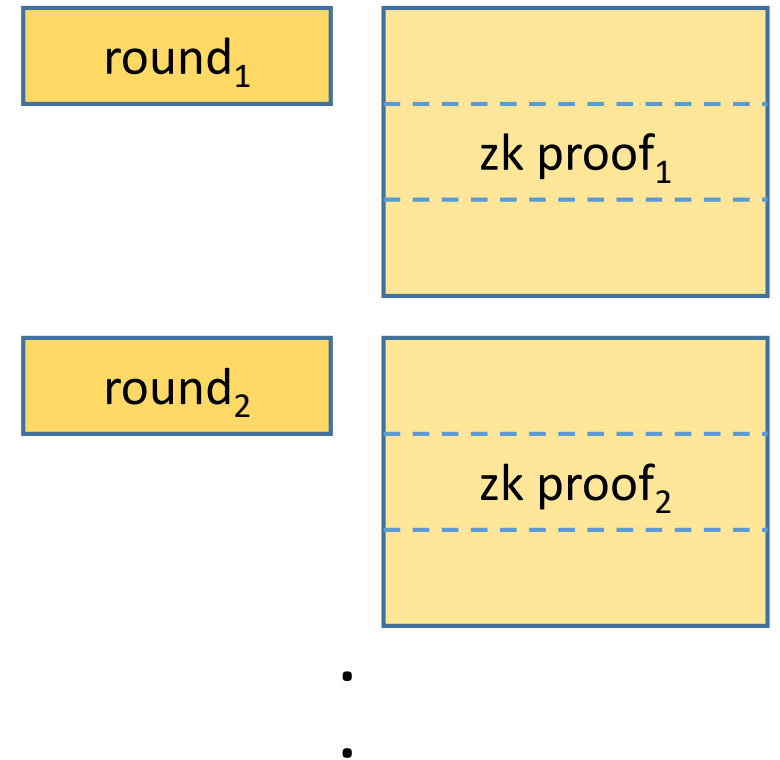


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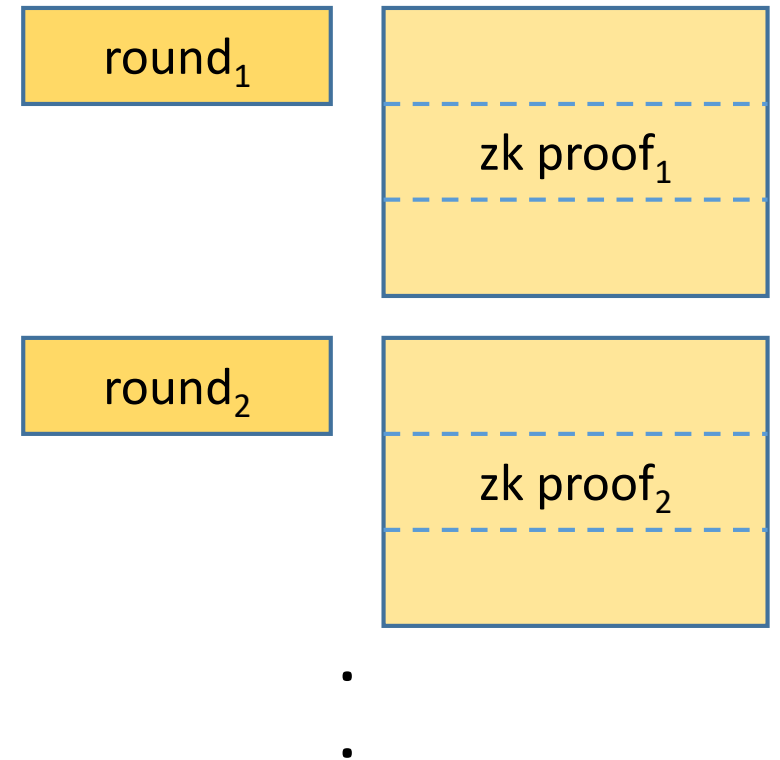
# [Goldreich-Micali-Wigderson 87]

GMW compiler.

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Proof of **honest behavior** with each round.

Main challenge is to reduce the number of proofs.



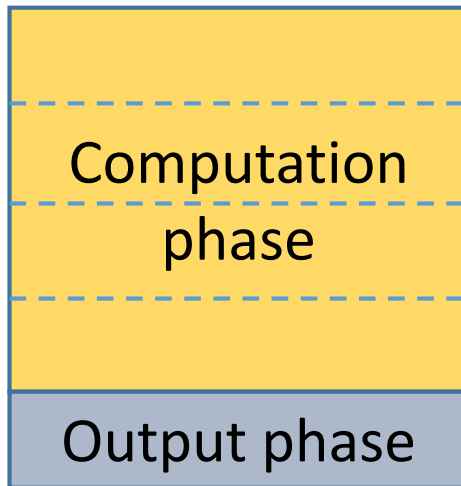
# Our strategy

Semi-honest protocol

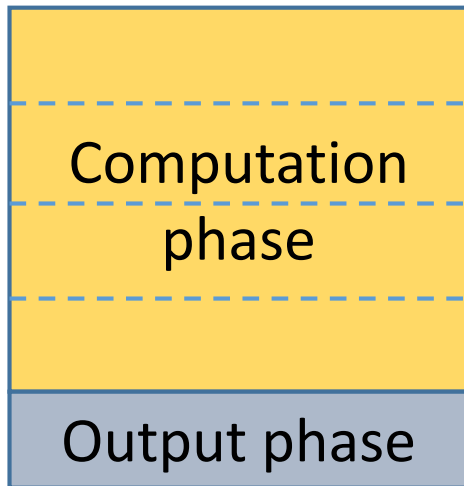


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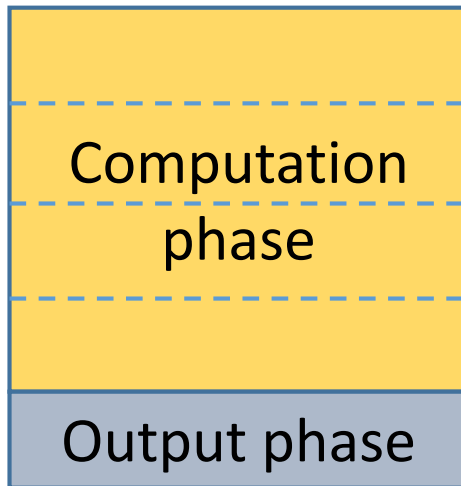


# Our strategy



Semi-honest protocol whose structure is satisfied by most MPC protocols.

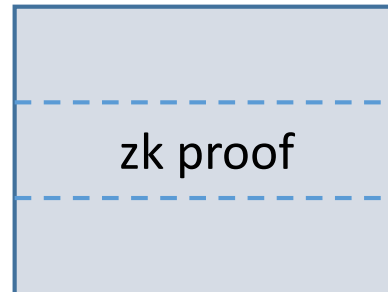
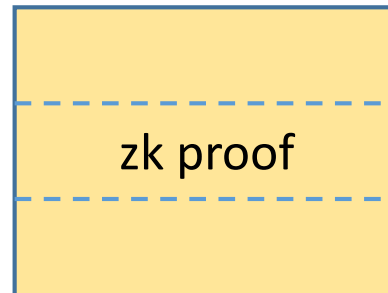
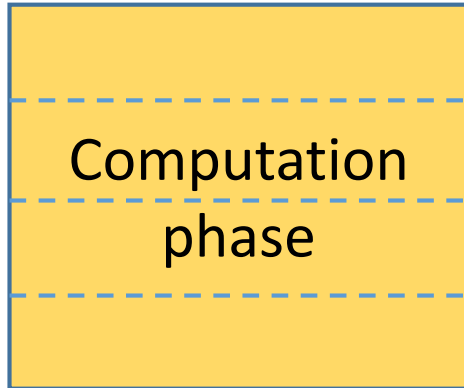
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Prove **honest behavior once** for the computation phase?

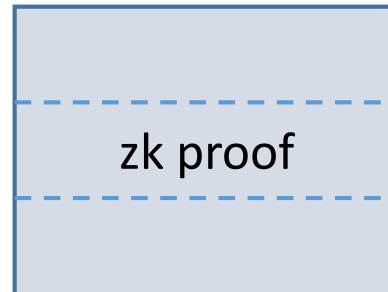
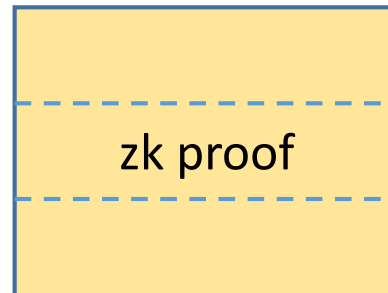
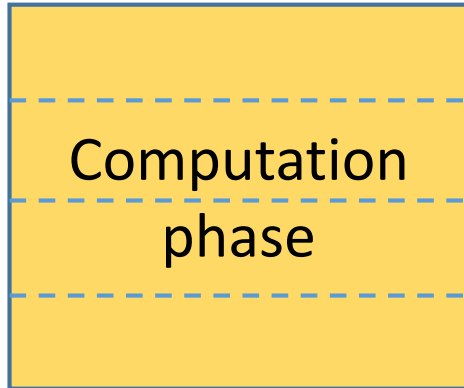
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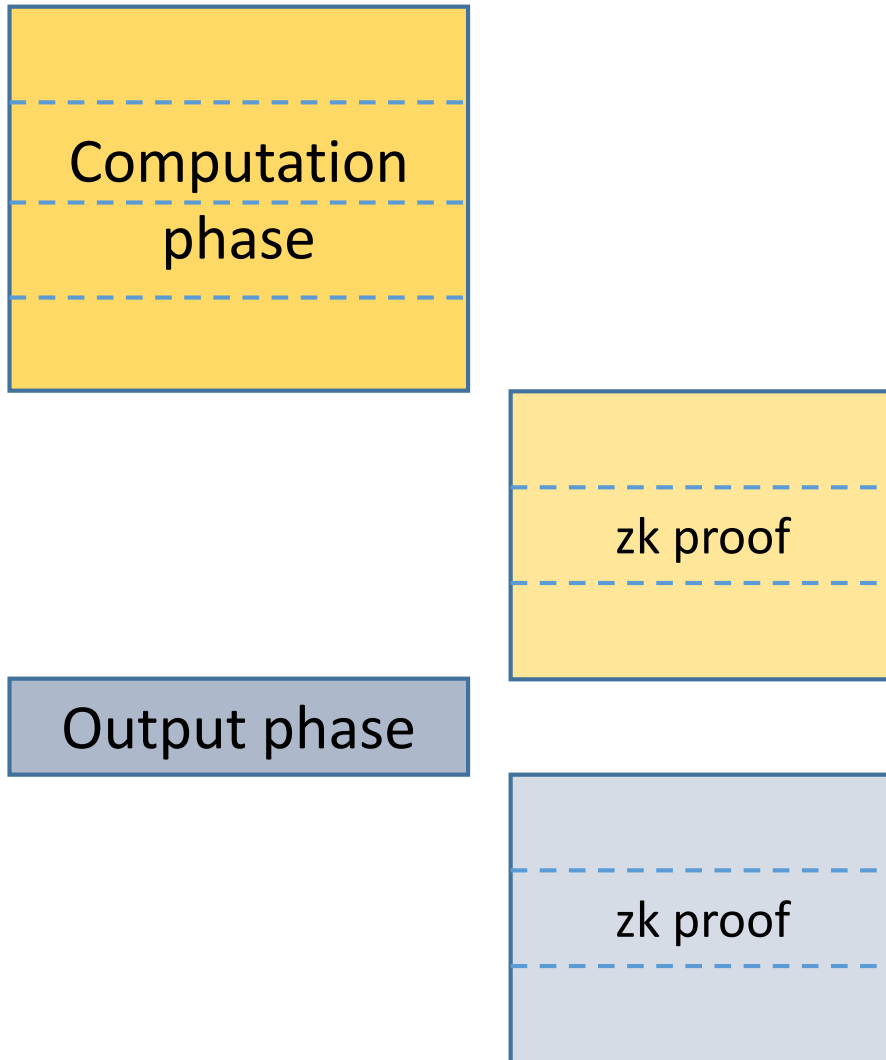


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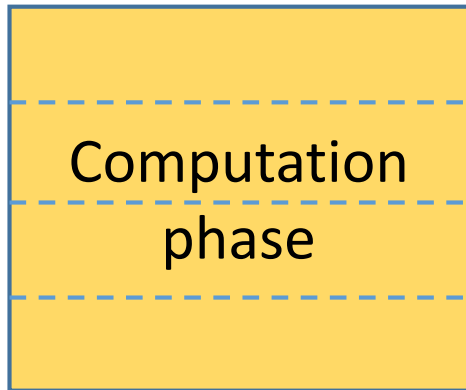
Prove **honest behavior once** for the computation phase?

Might be too late.

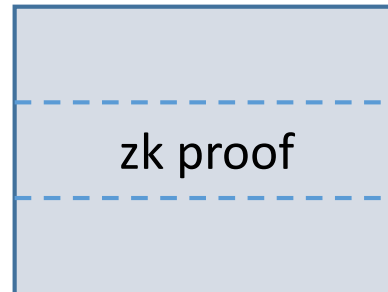
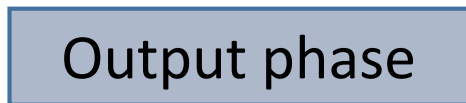
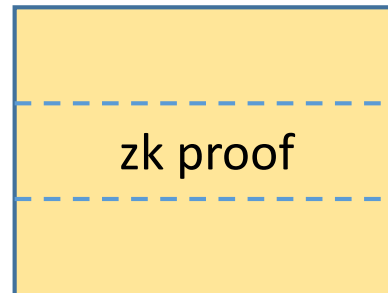
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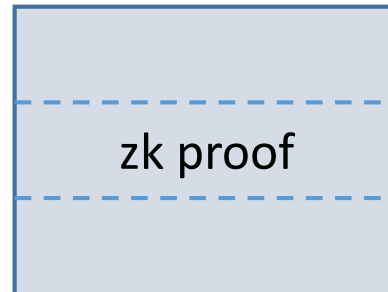
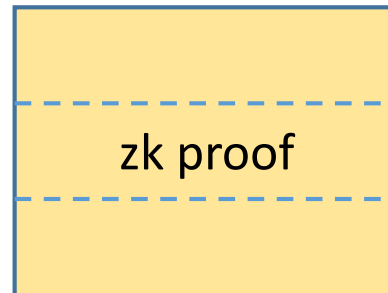
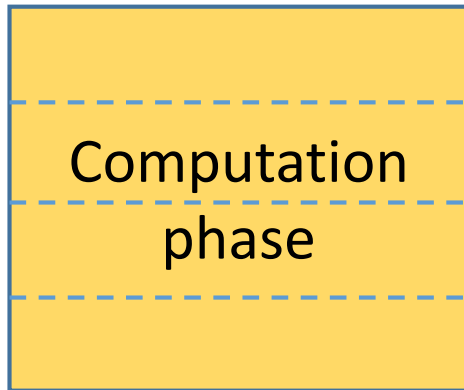
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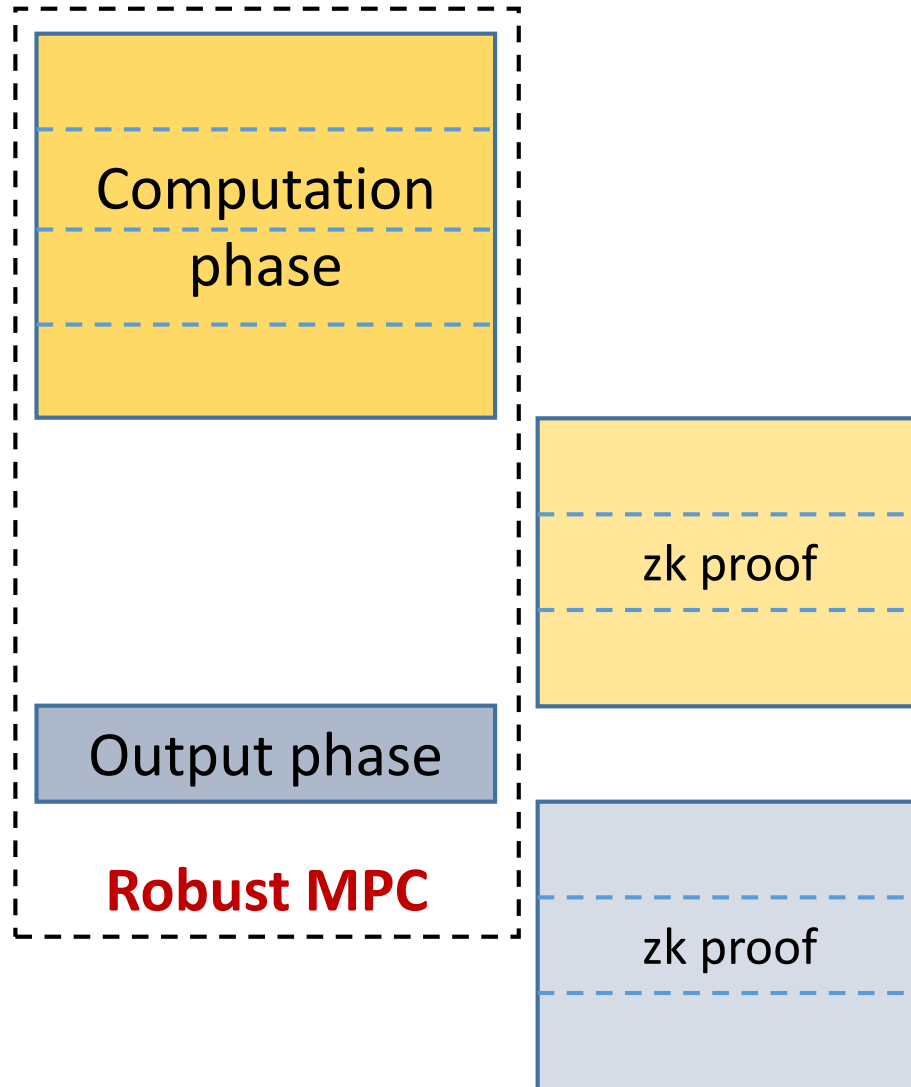


Require additional property: **Robustness**

Computation phase:  
maintains privacy against malicious  
adversaries till end of phase, however the  
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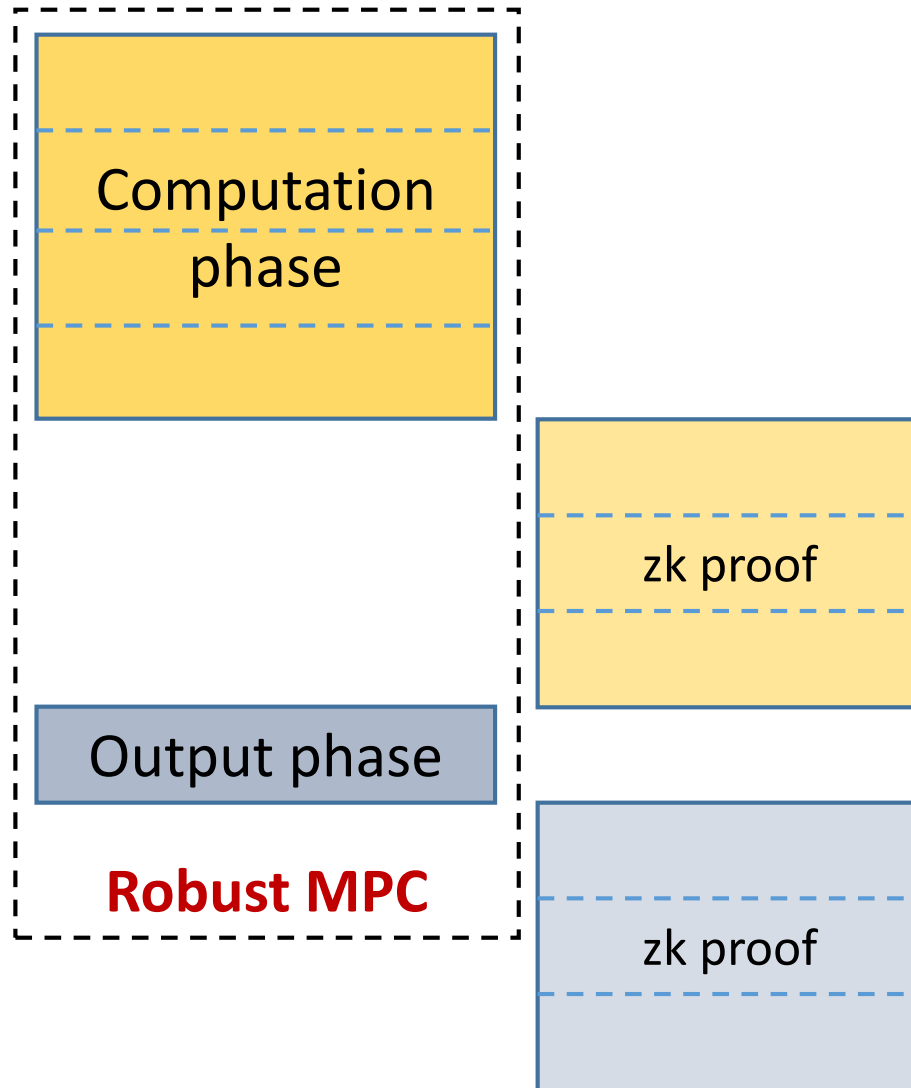
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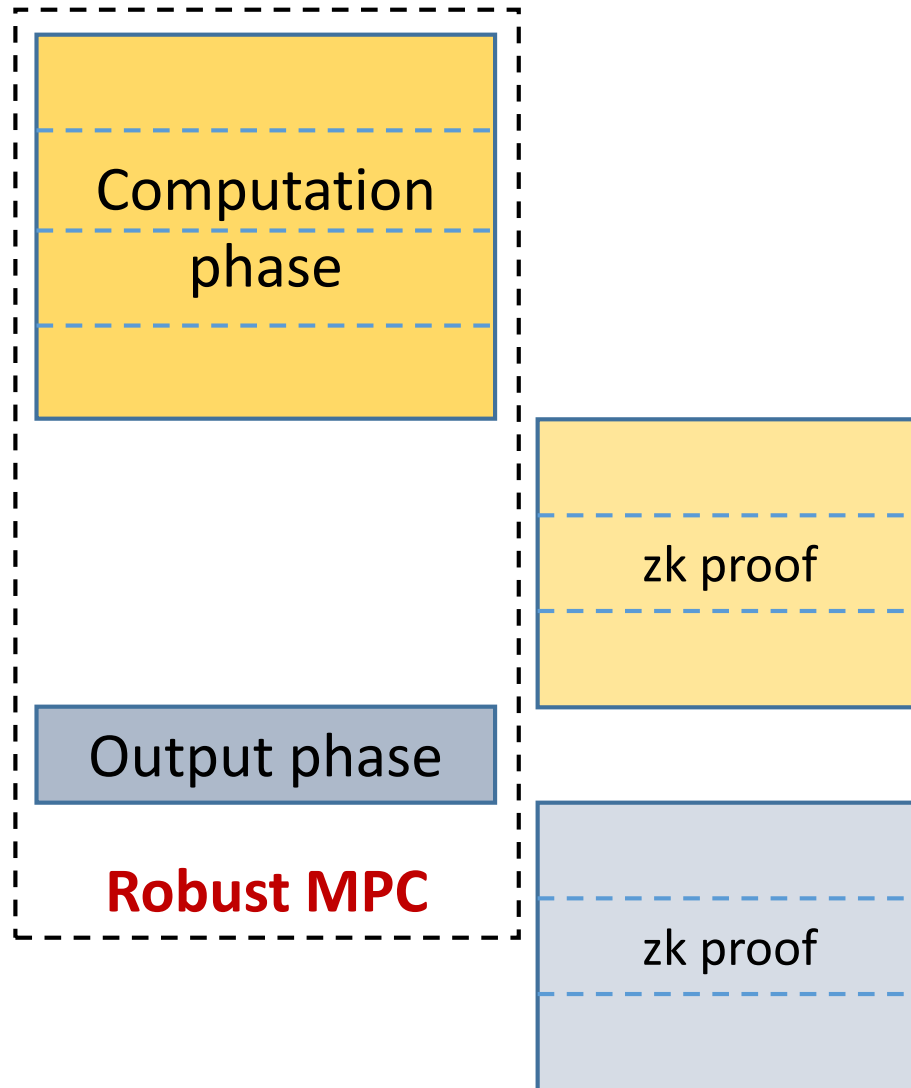


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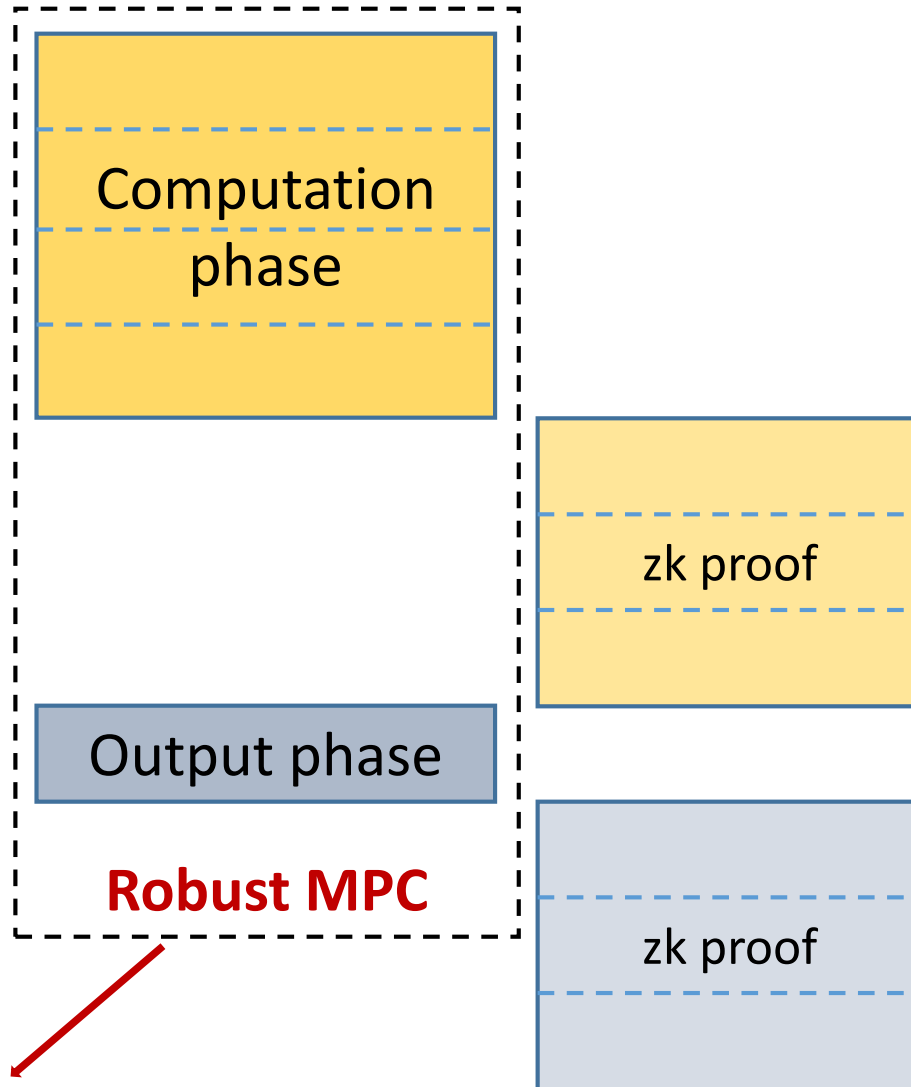
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Developed by [Chandran-Goyal-Ostrovsky-Sahai07] in a different context.

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Developed by [Chandran-Goyal-Ostrovsky-  
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Already secure against bad randomness.

# Rest of the talk

## Compiler from

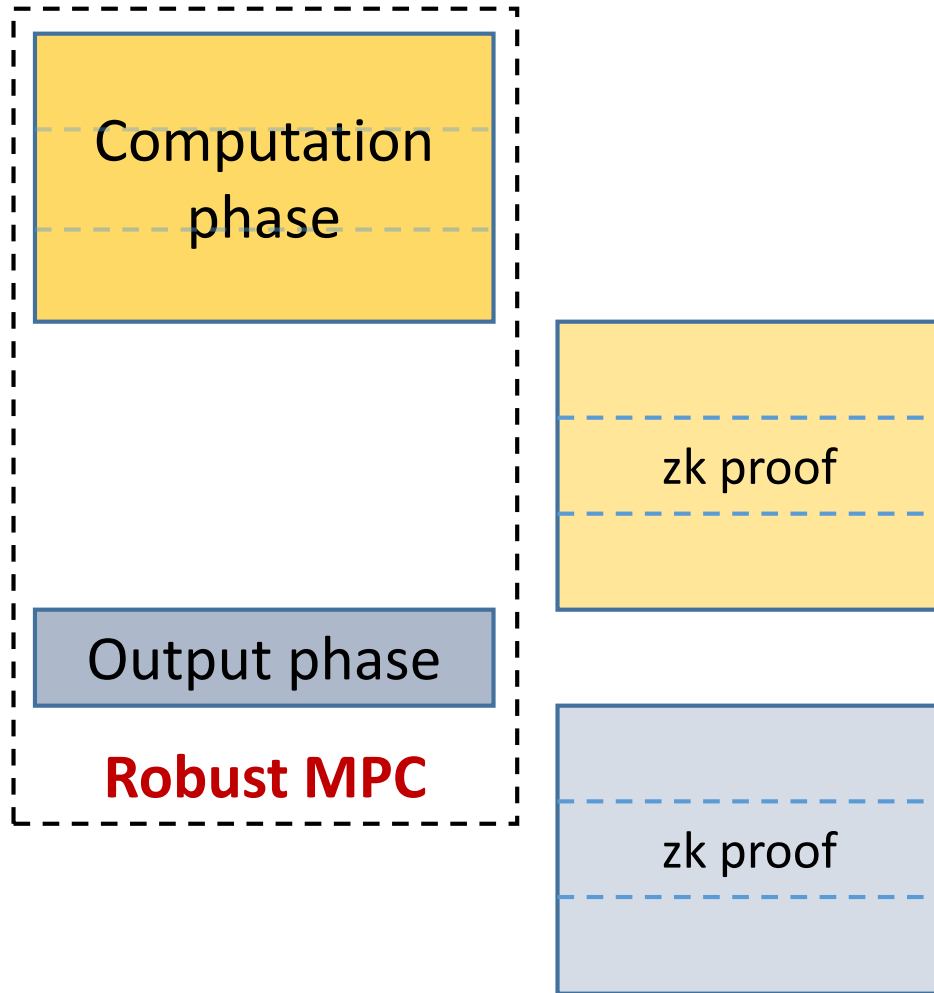
- 4 round robust MPC to 5 round protocol

- 4 round robust MPC to 4 round protocol

## Construction of 4 round robust MPC

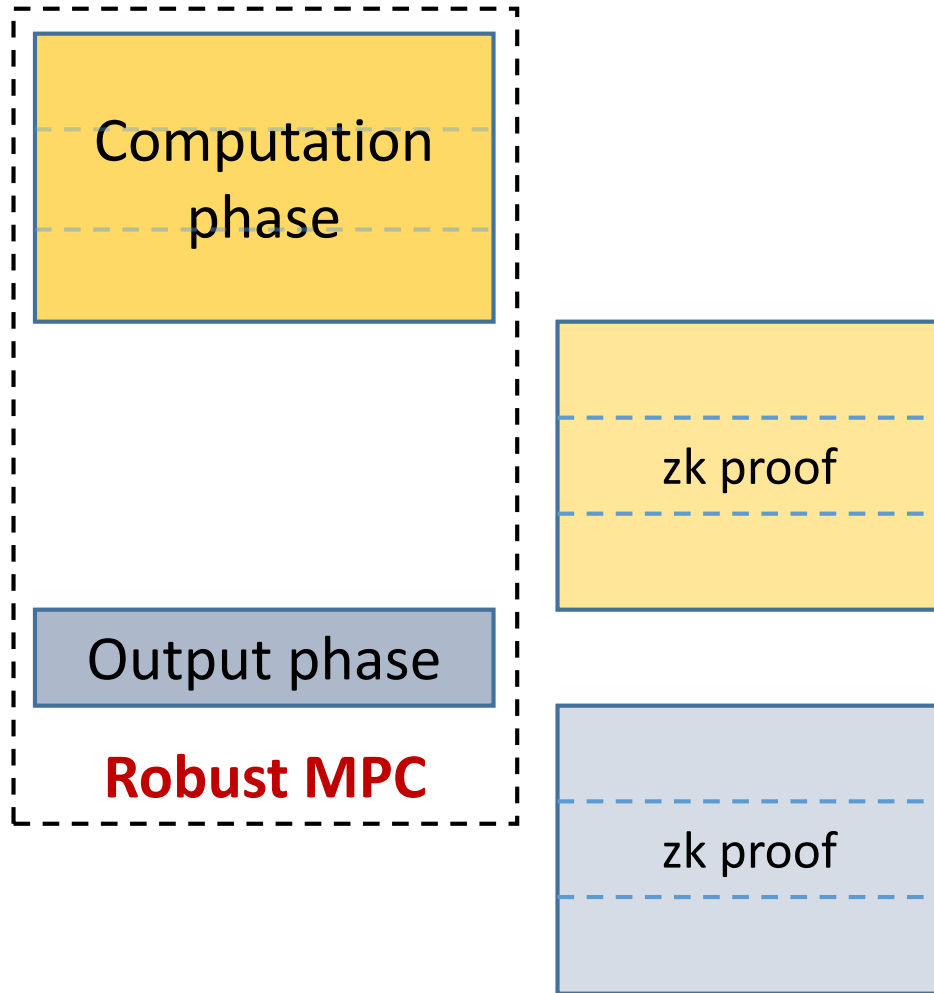
# 5 Round Protocol

# Blueprint of 5 round protocol



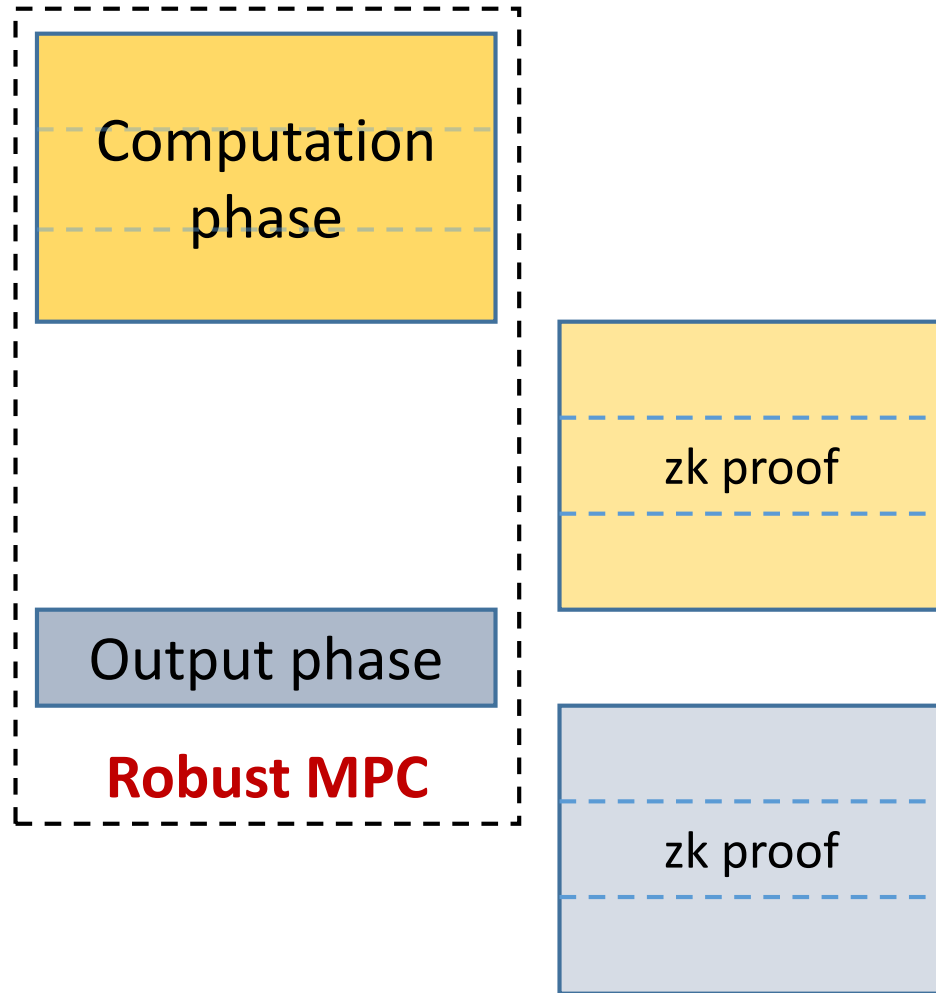
# Blueprint of 5 round protocol

Parallelize proofs.





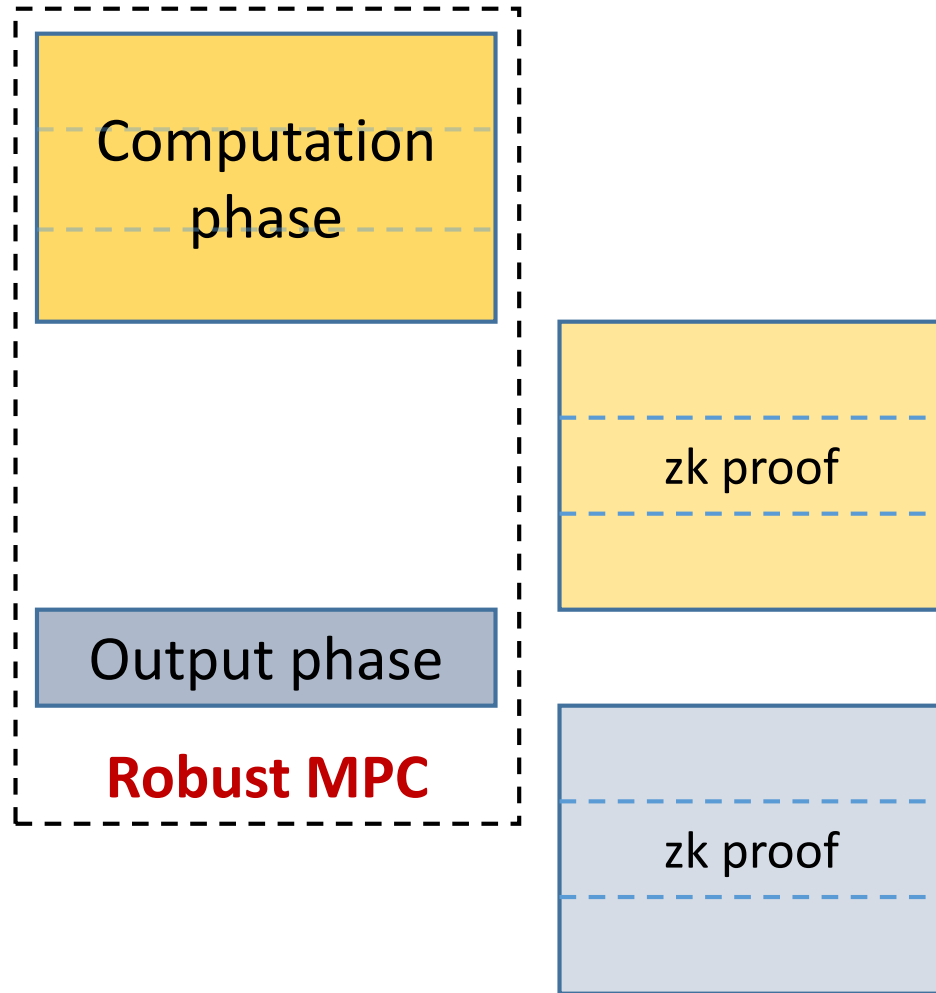
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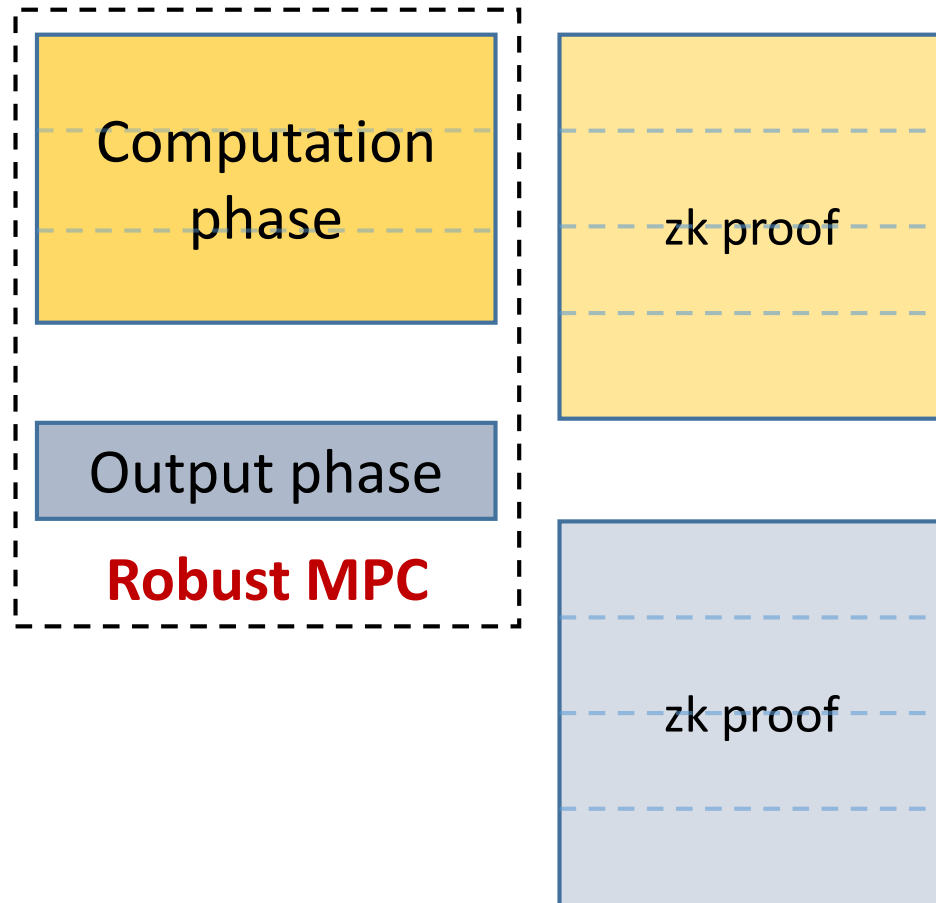


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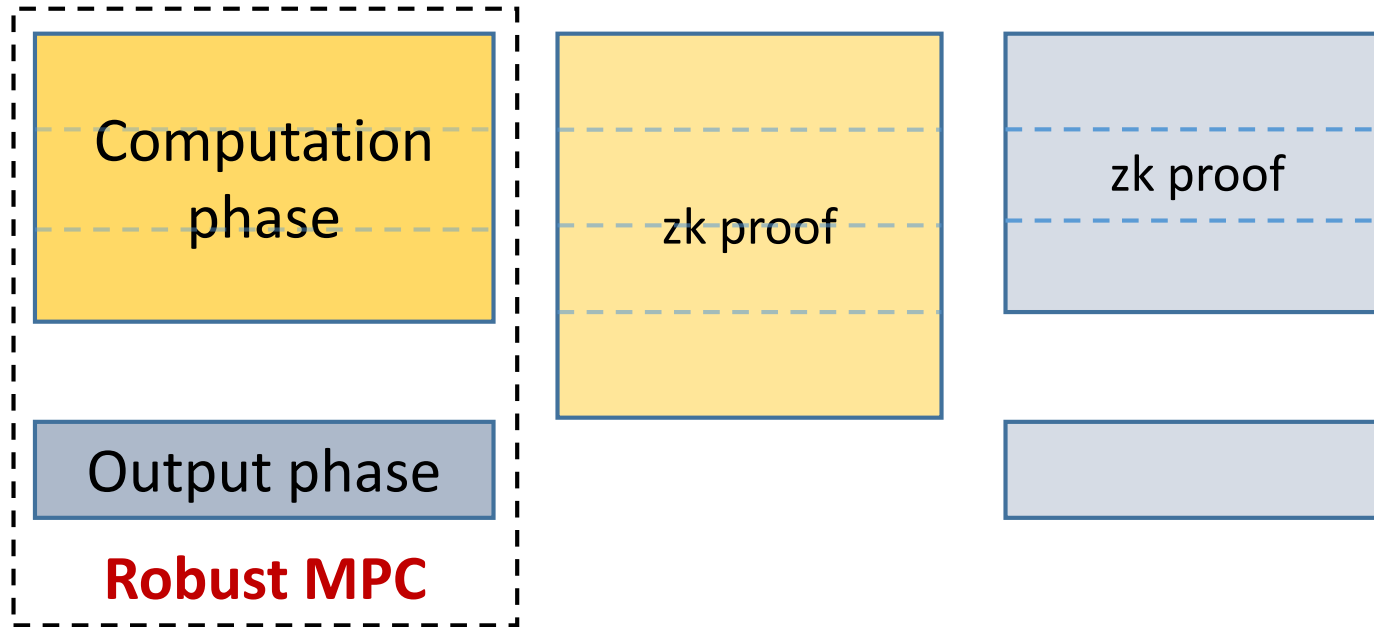


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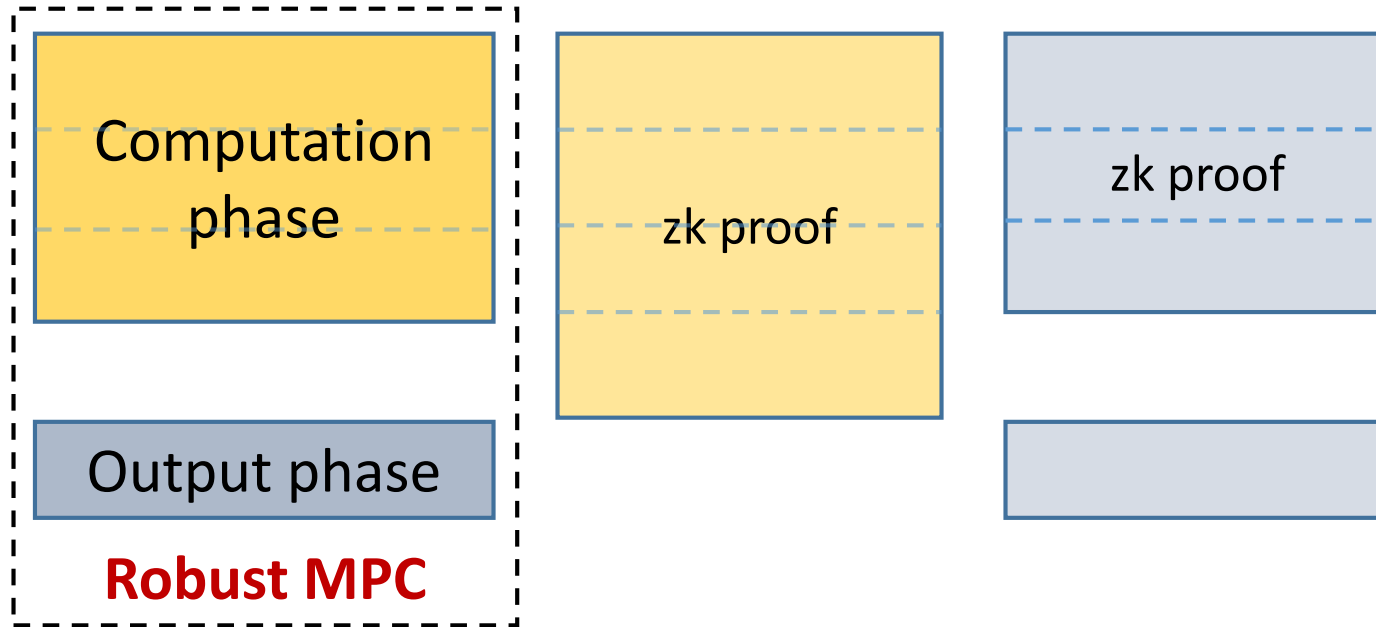


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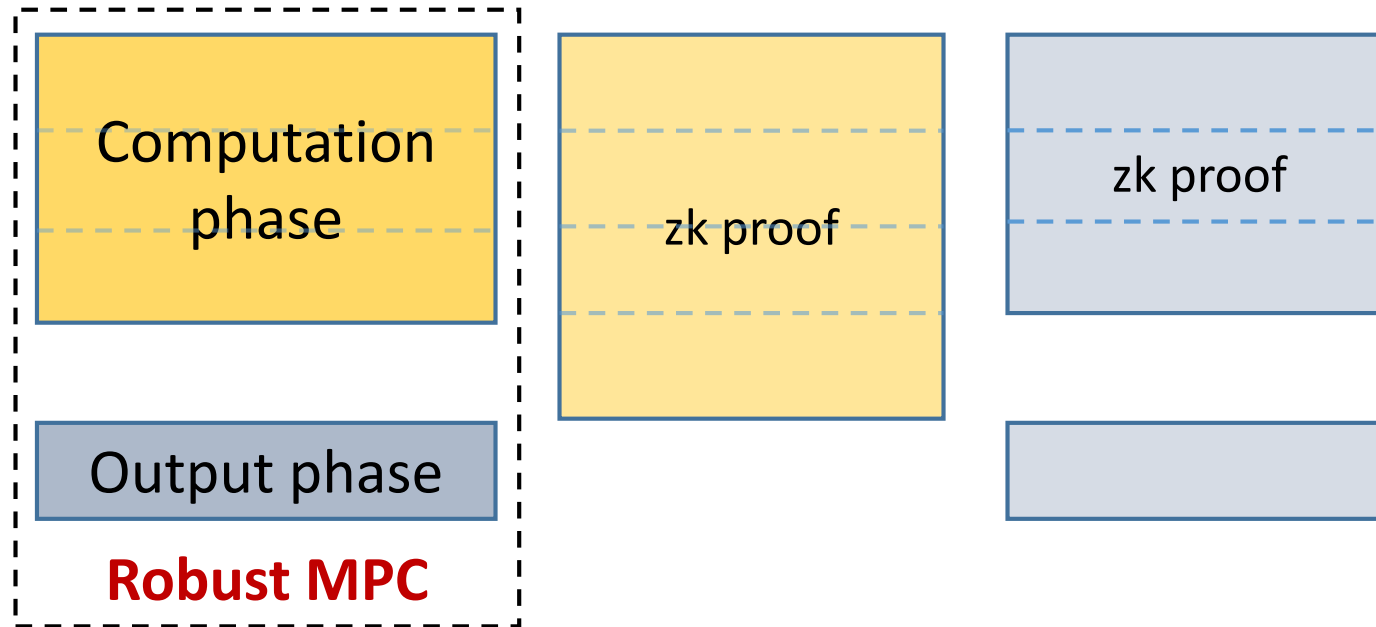
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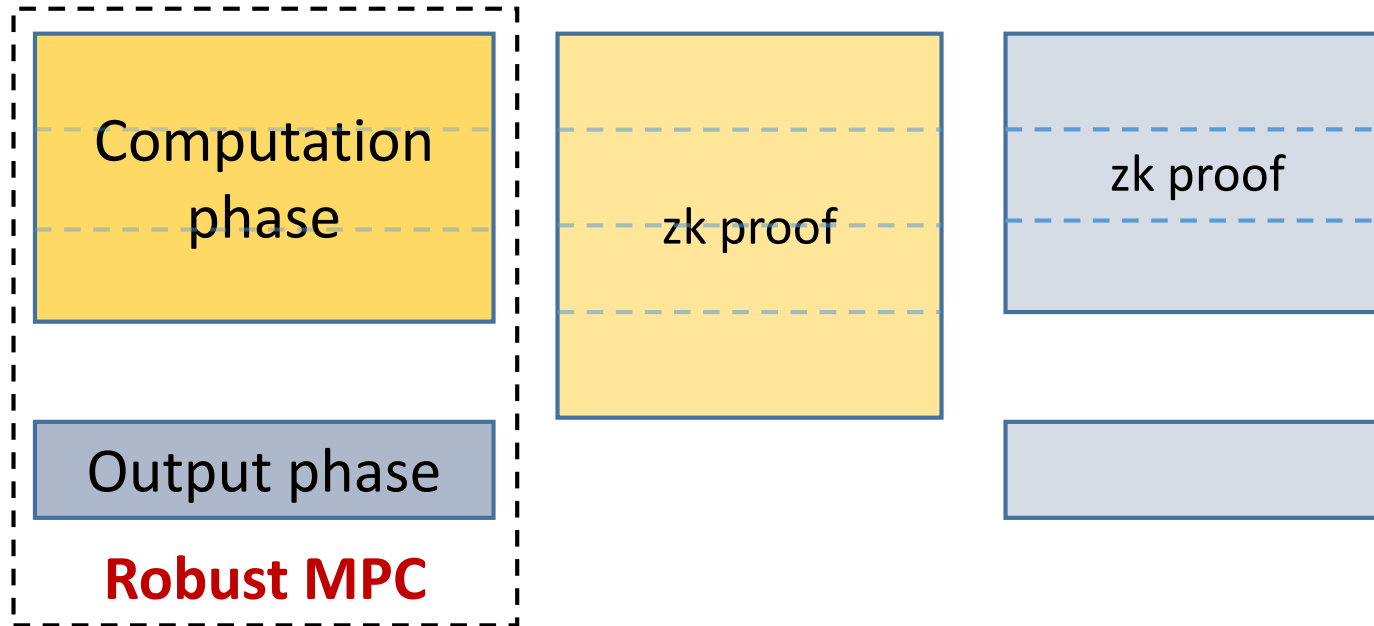


# Blueprint of 5 round protocol



**Non-malleability** is a big challenge.

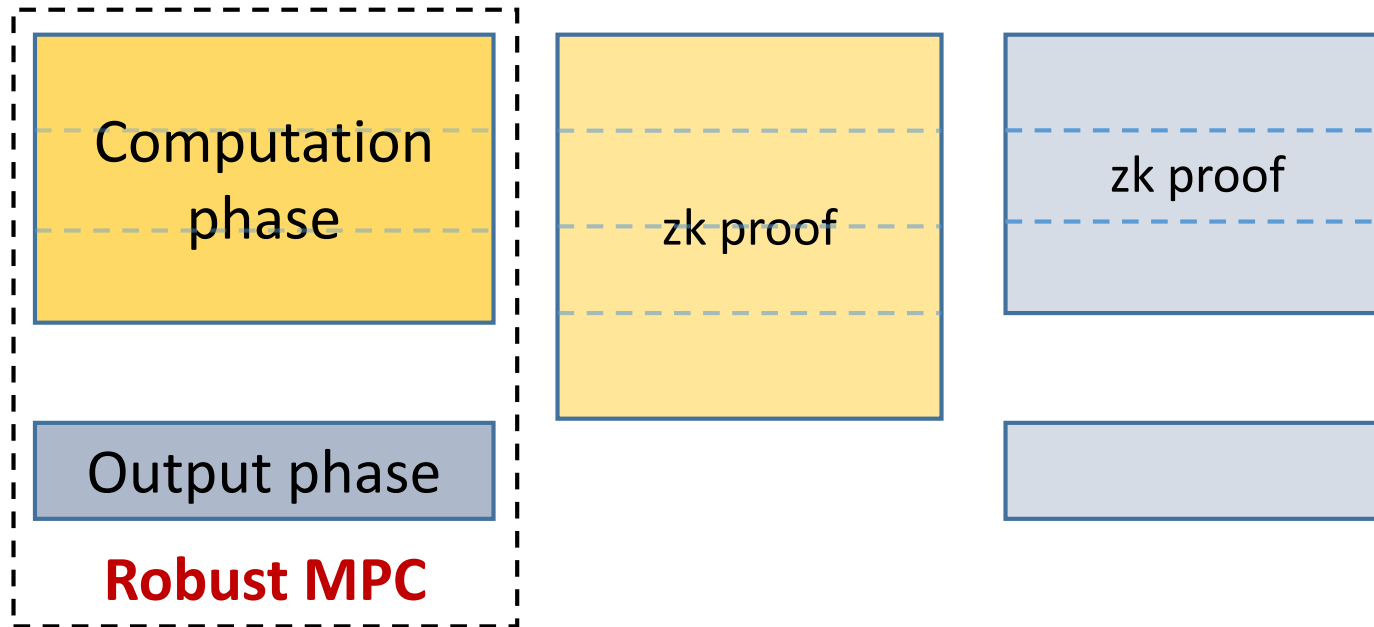
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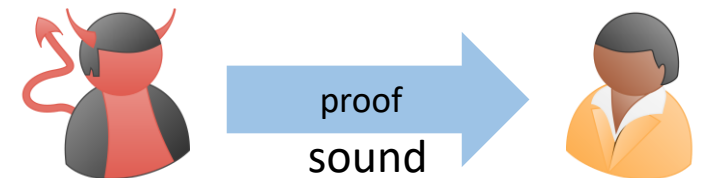
Standard soundness does not suffice.

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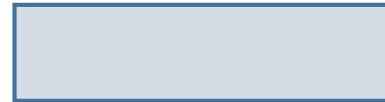
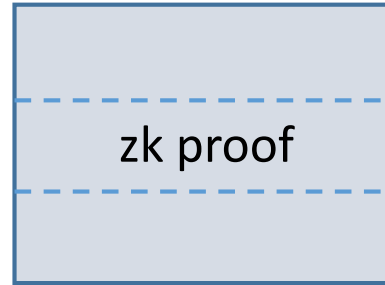
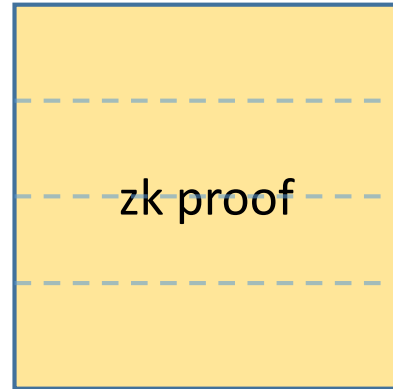
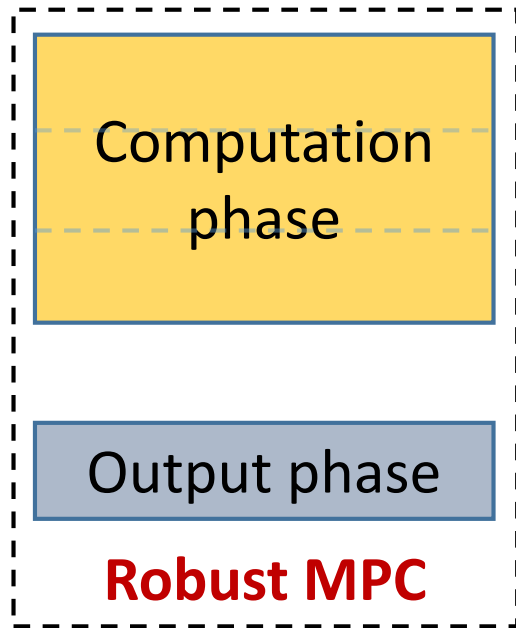
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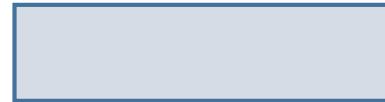
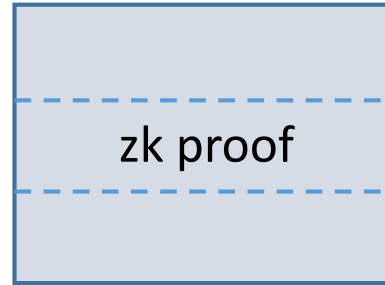
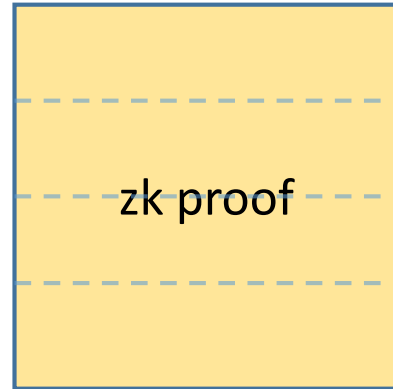
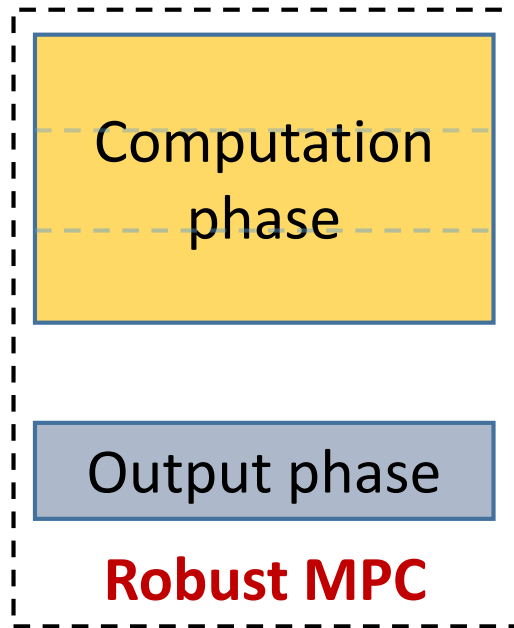


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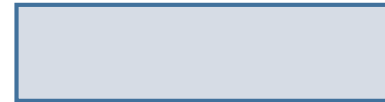
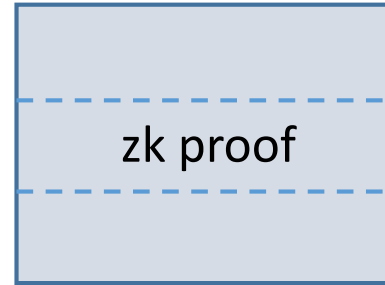
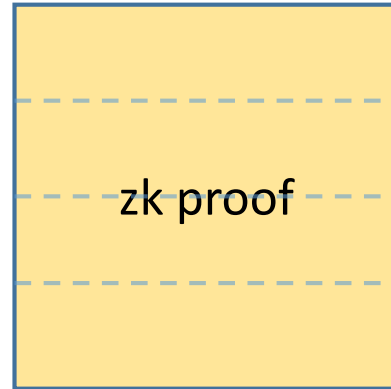
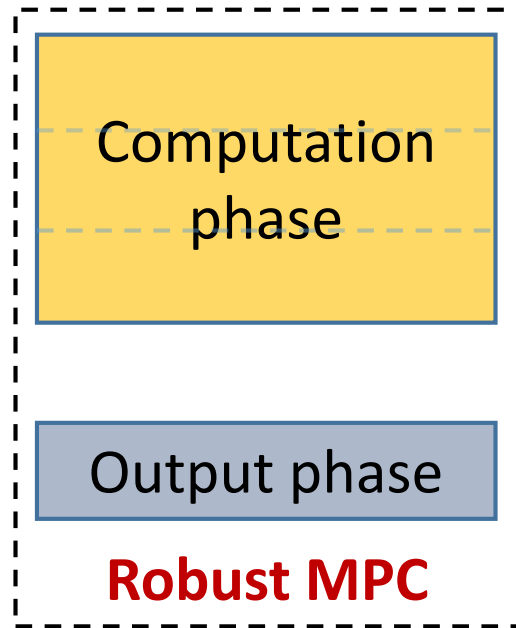


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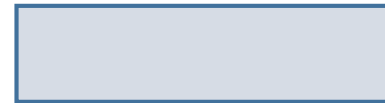
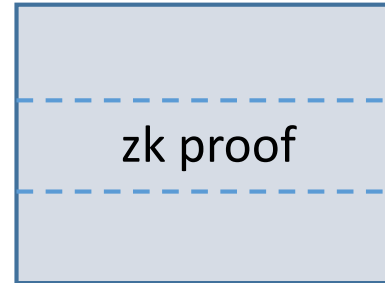
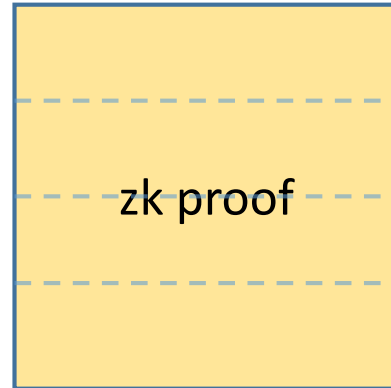
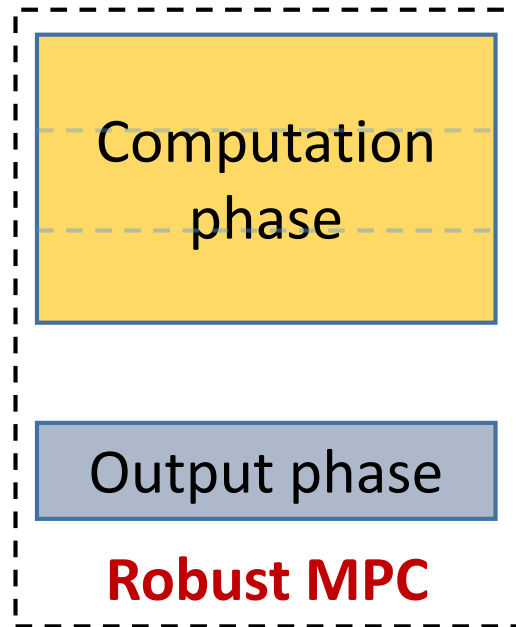


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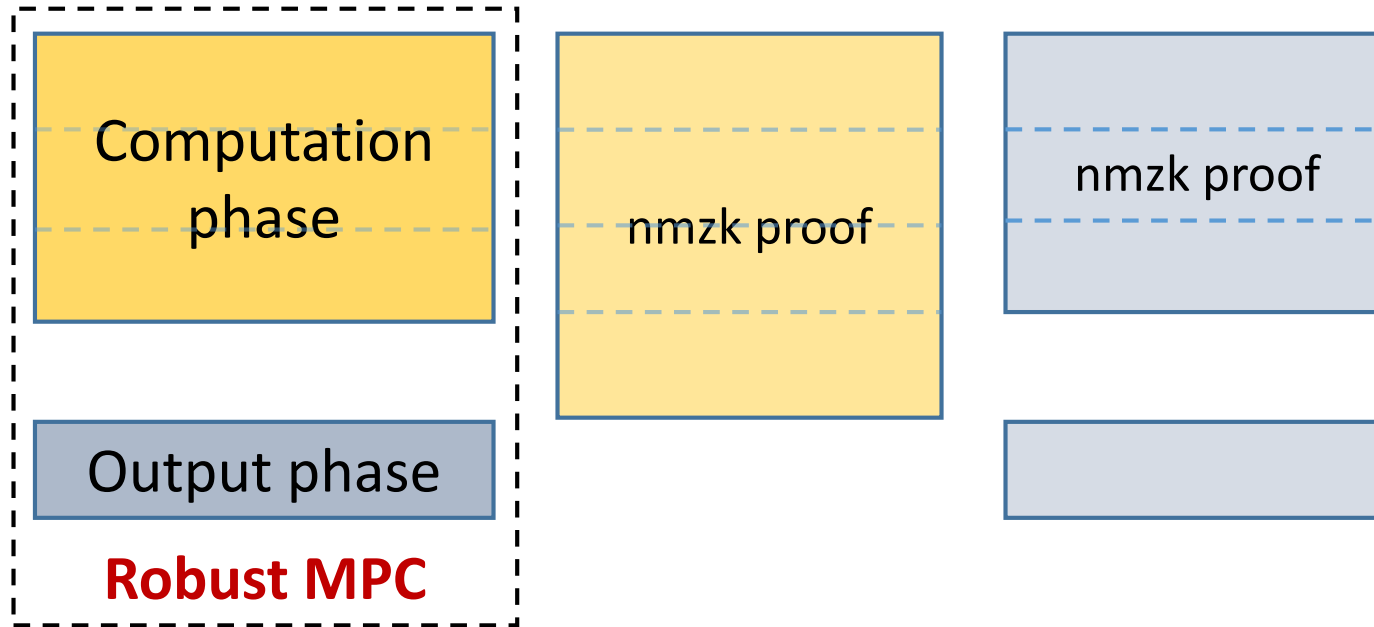
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simulation-soundness [DDN91,Sah99]

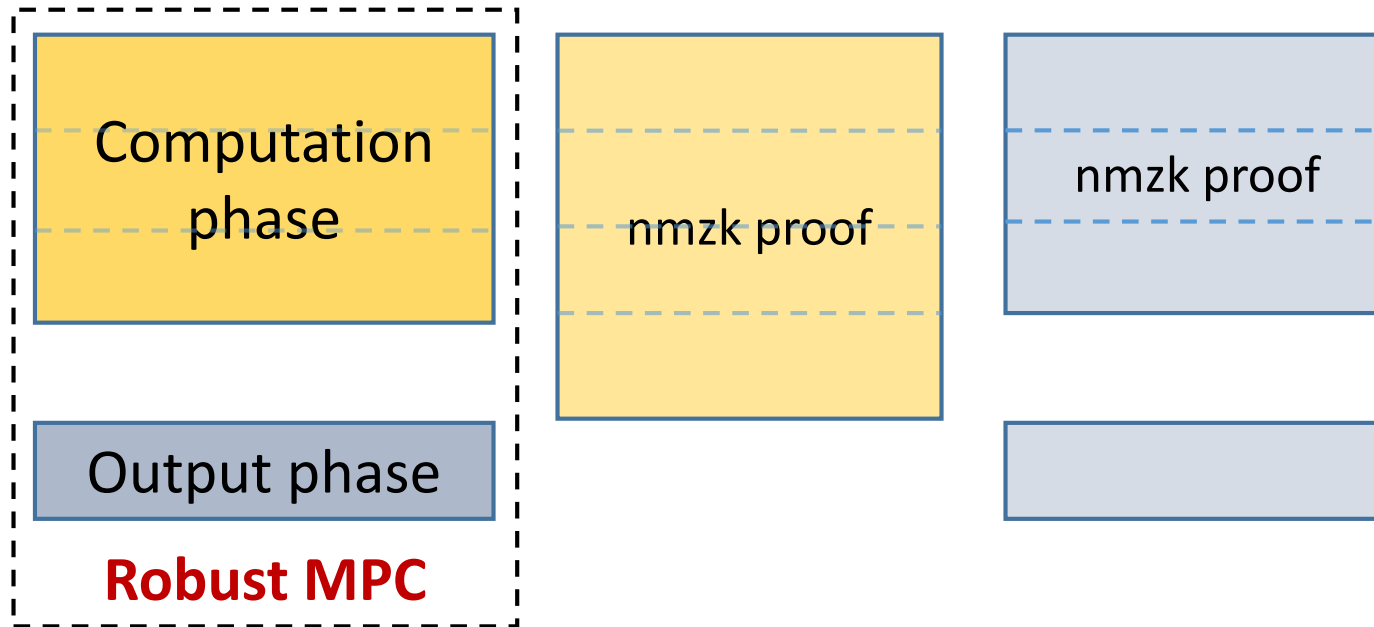
# Blueprint of 5 round protocol



**Non-malleability** is a big challenge.

Standard soundness does not suffice.

# Blueprint of 5 round protocol



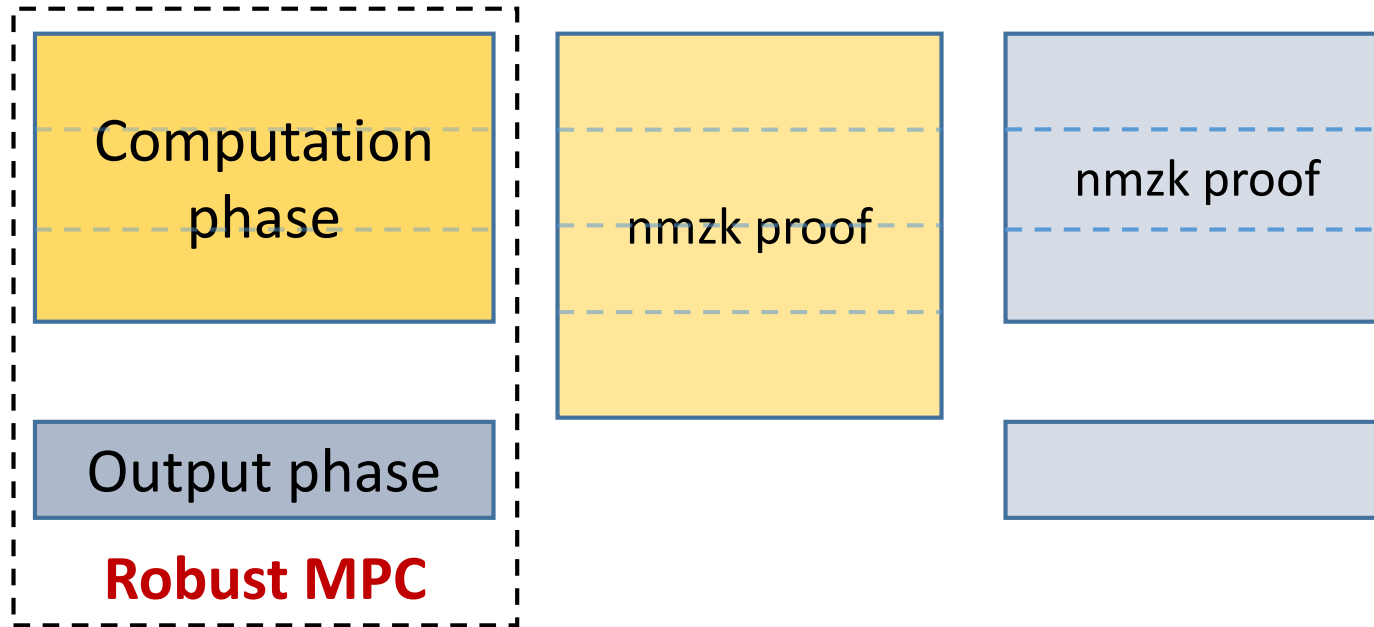
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4 round input delayed NMZK can be constructed from **CRHF** [Ciampi-Ostrovsky-Siniscalchi-Visconti17].

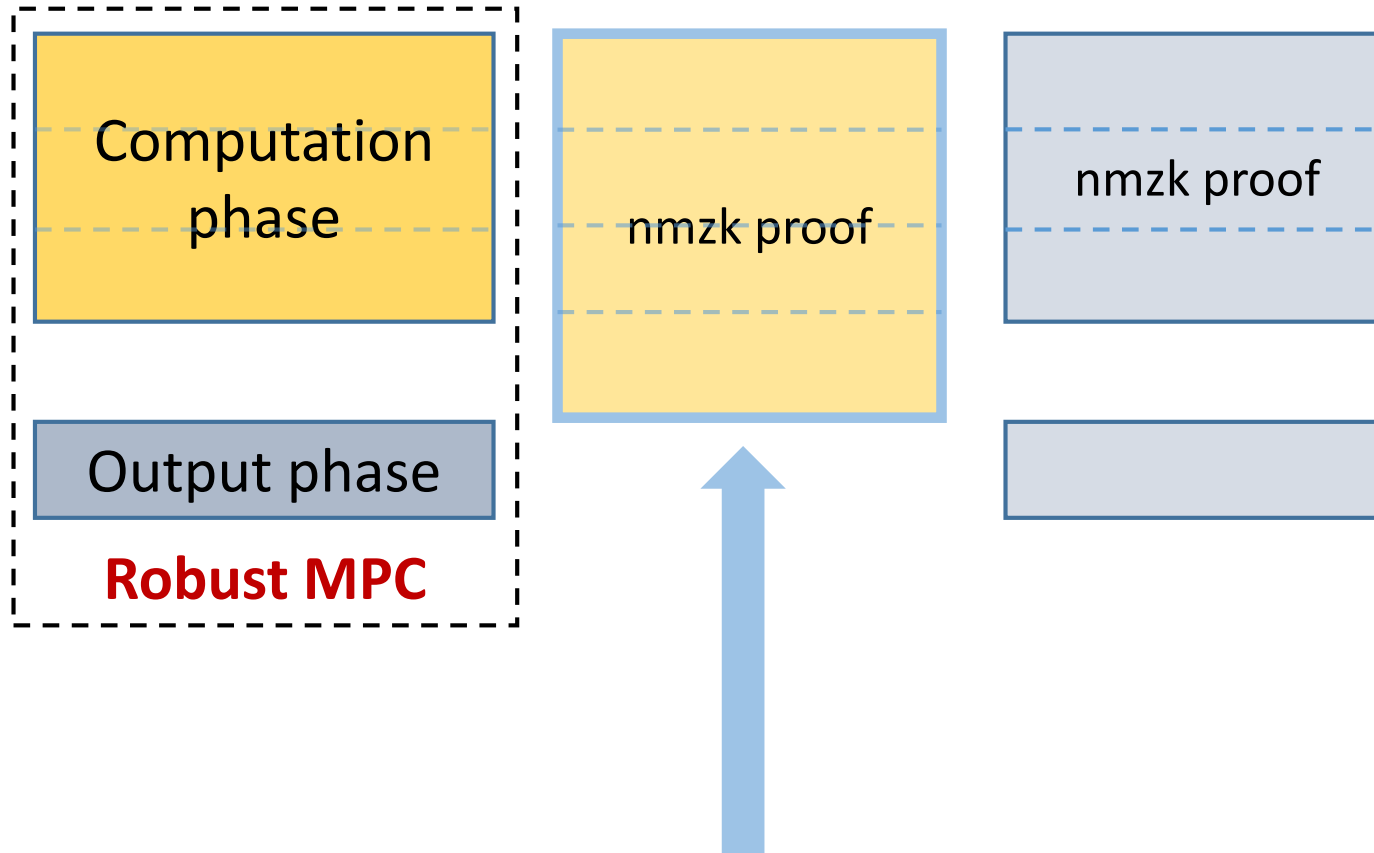
# 4 Round Protocol

# Main challenge

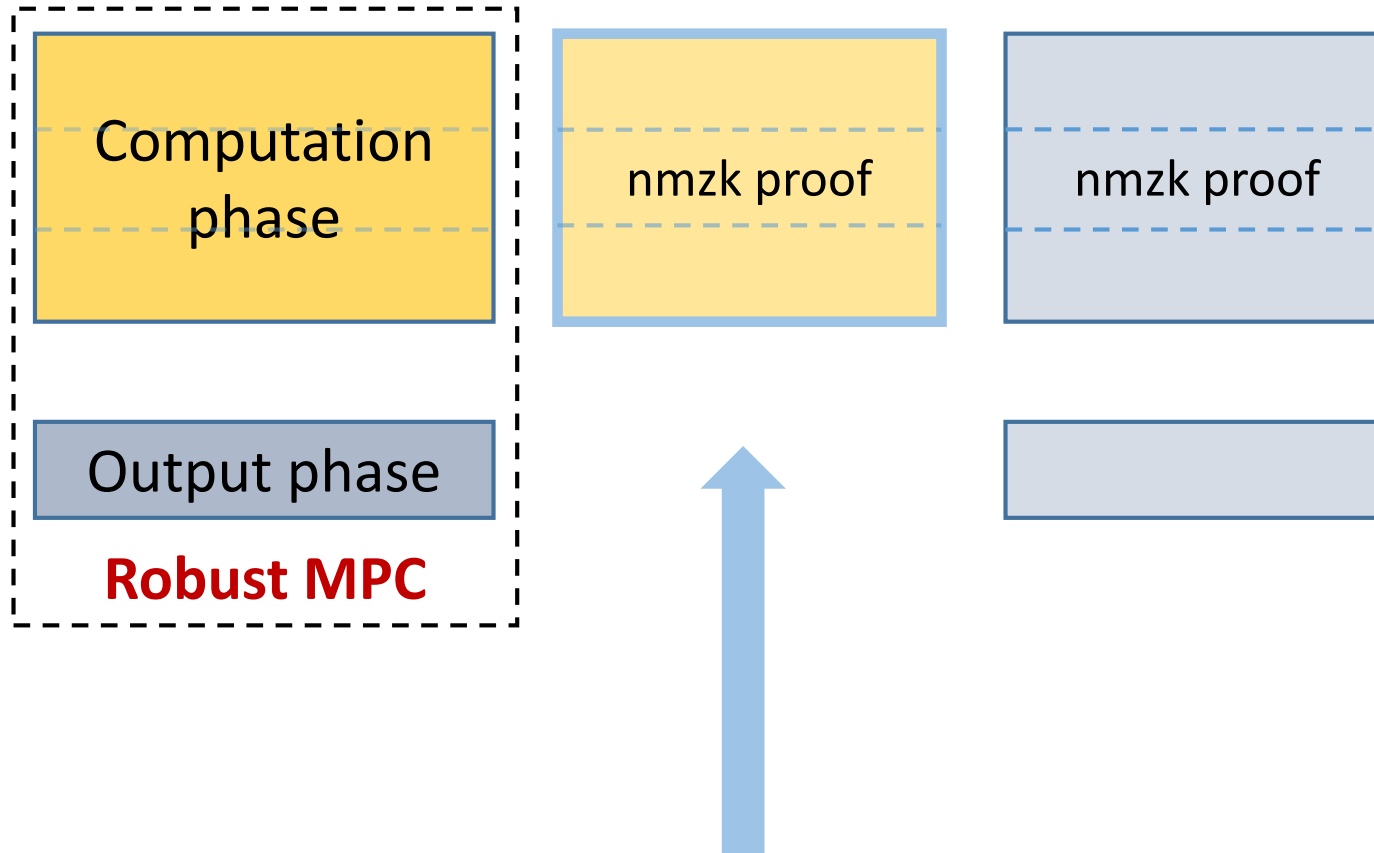




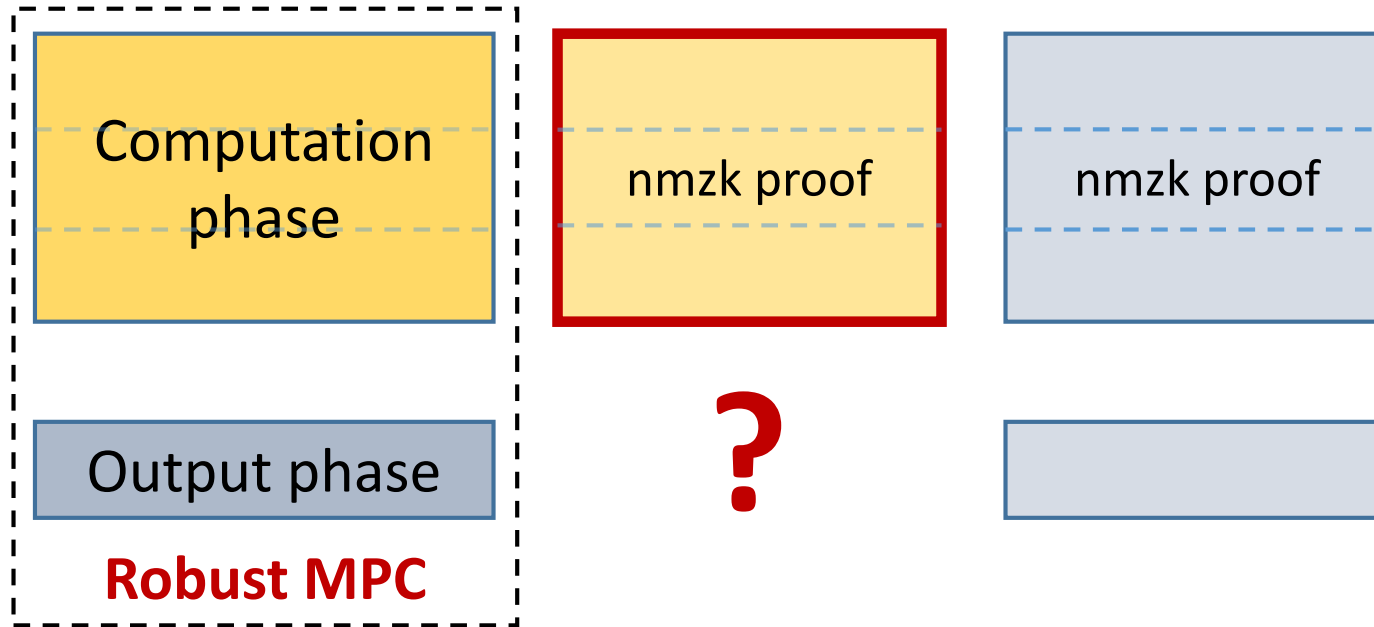
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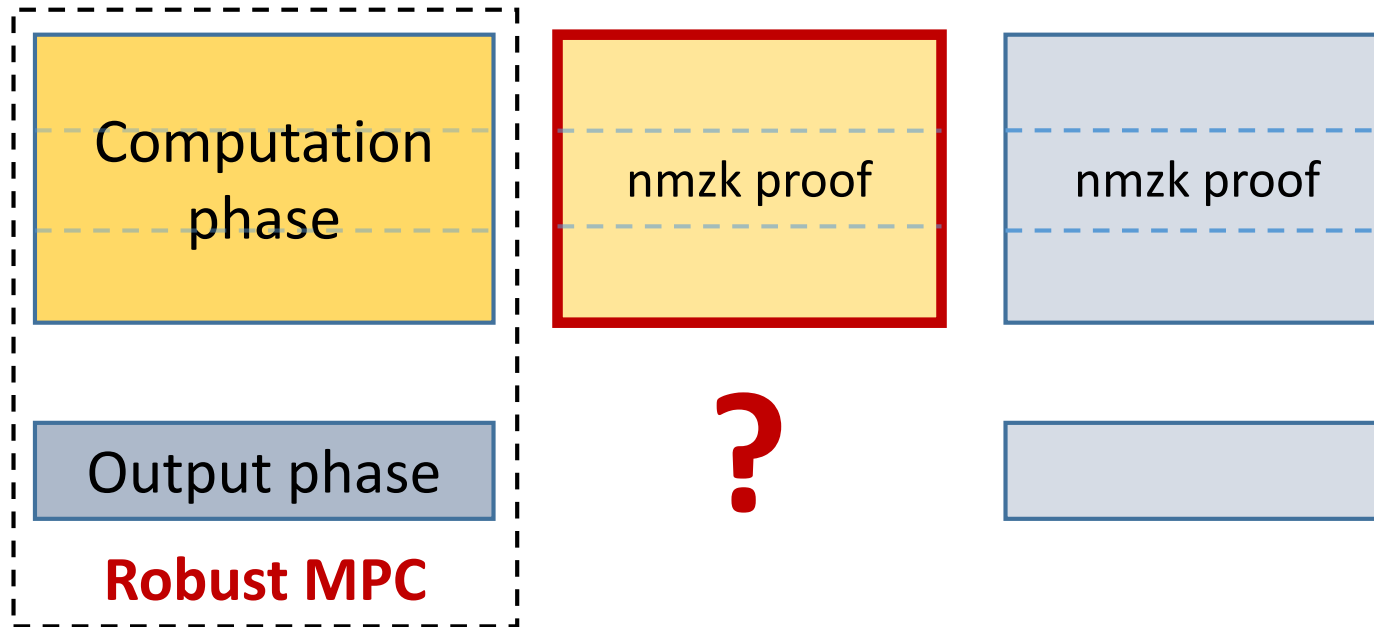
# Main challenge



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# Main challenge



Not clear how to go beyond 5 rounds.

# Key idea

Robust MPC:

Simulator needs to **cheat only in the output phase**.

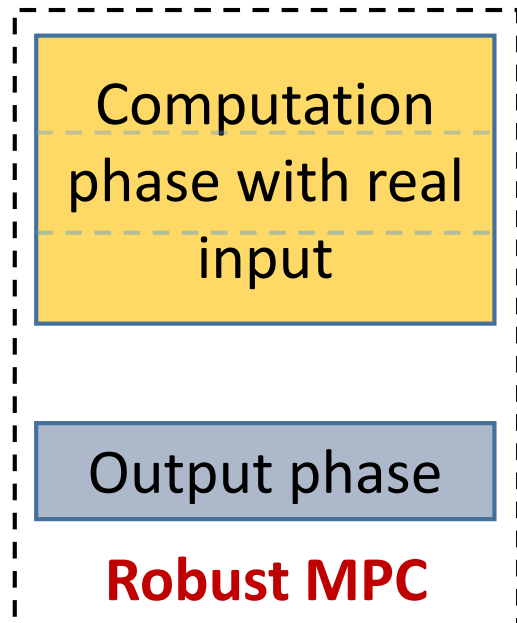
In the computation phase, simulator uses a *random input*.

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Computation  
phase with real  
input

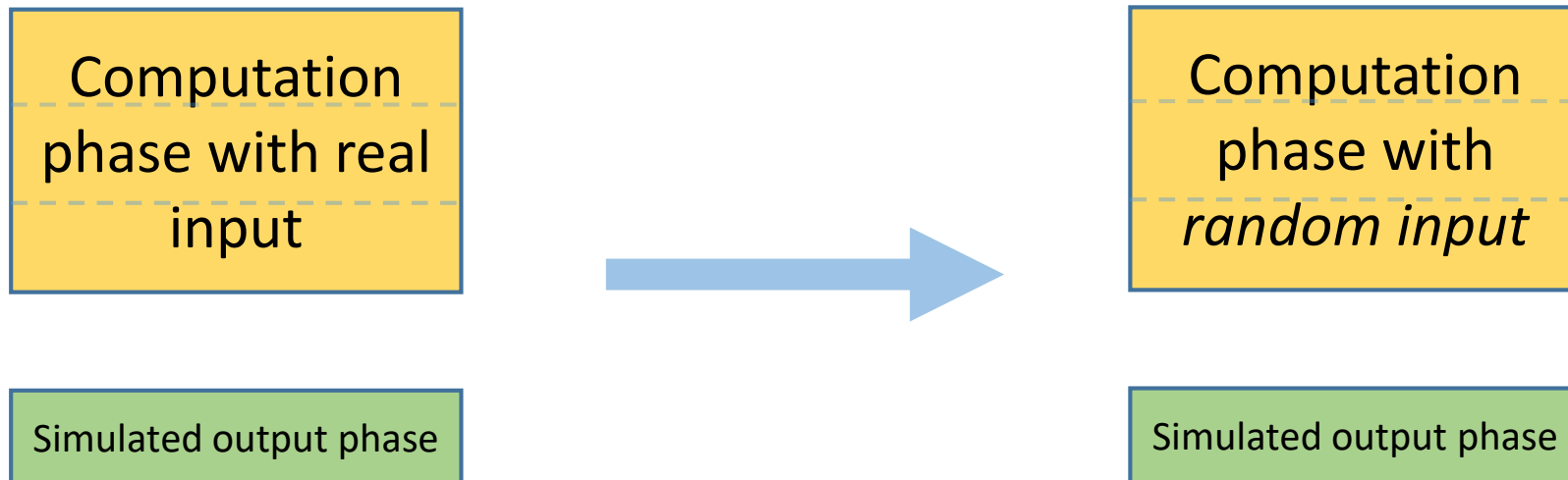
Simulated output phase

# Key idea

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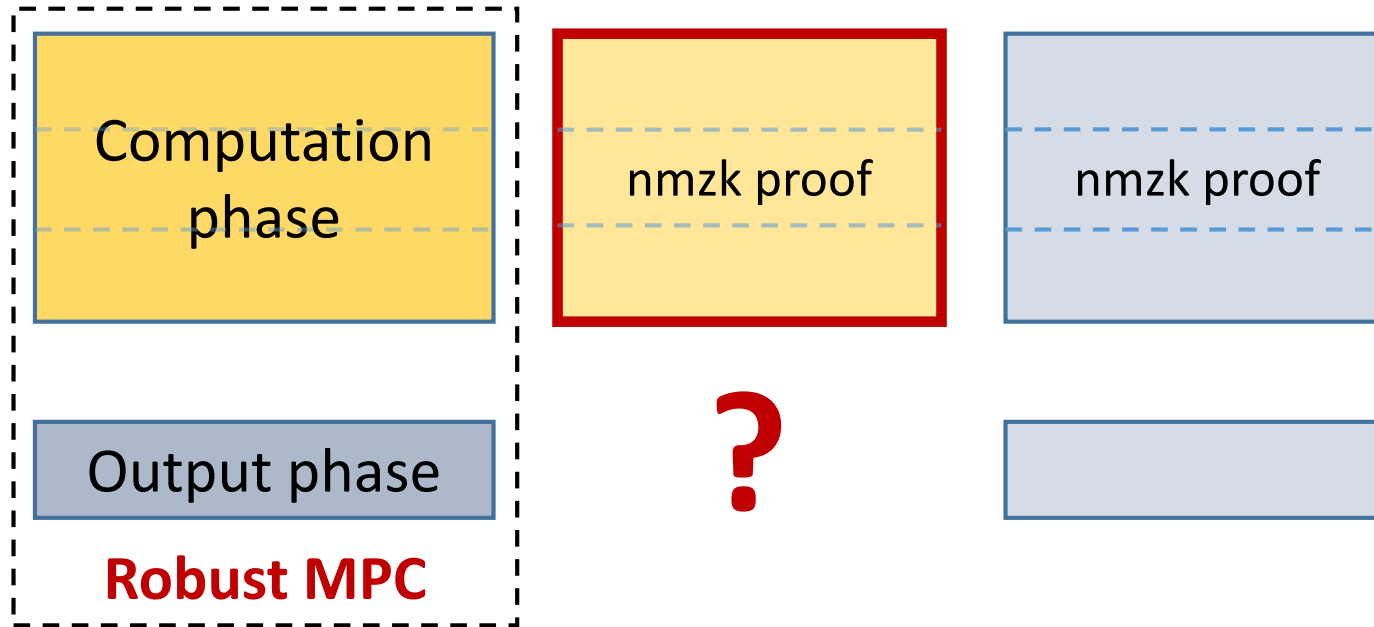
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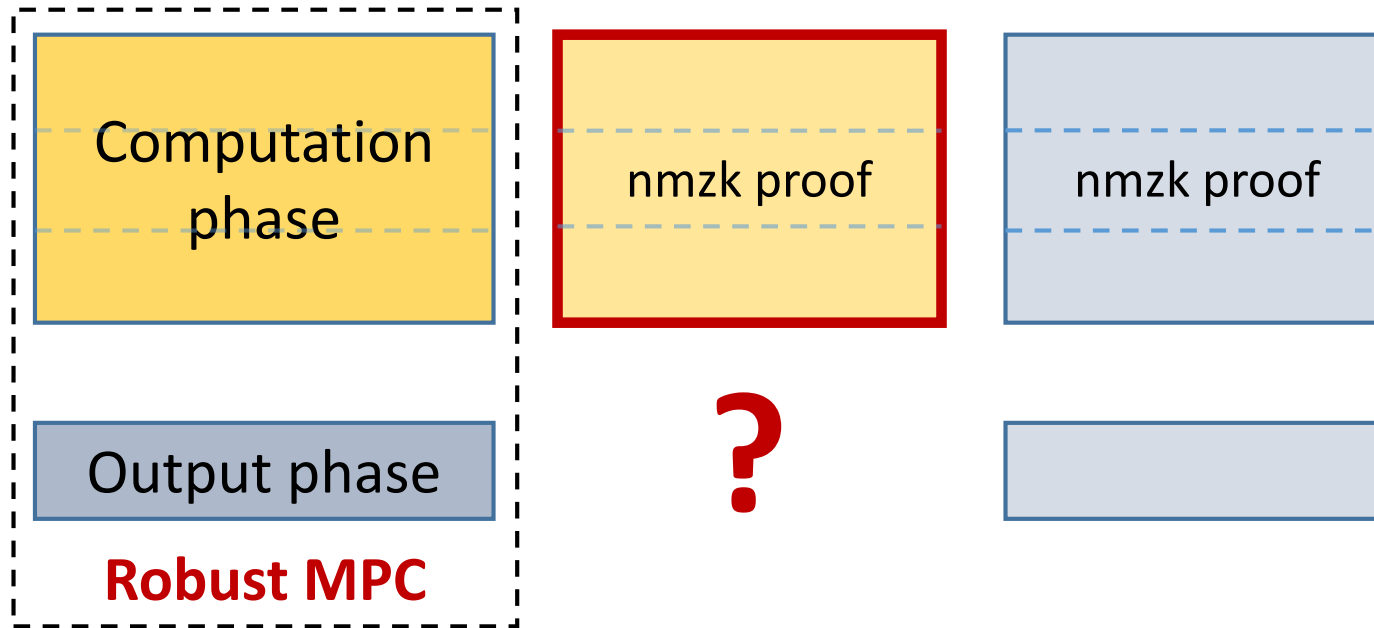




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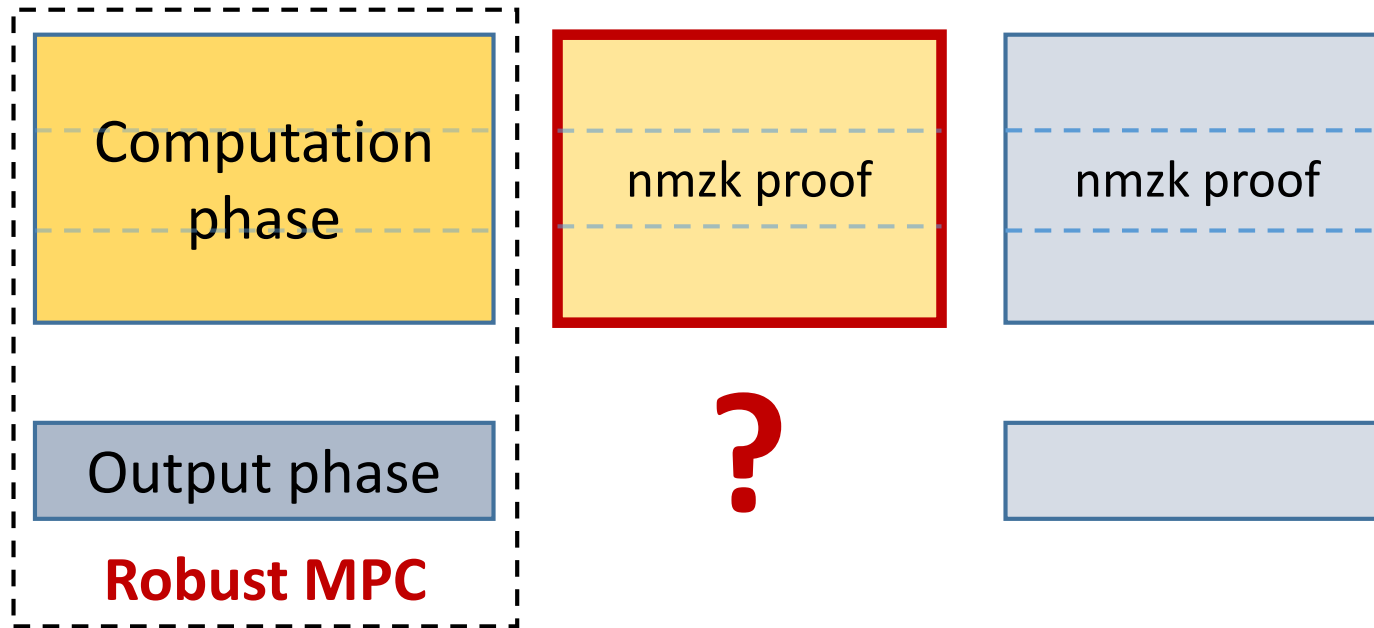


# Key idea



Not necessary to simulate the NMZK.

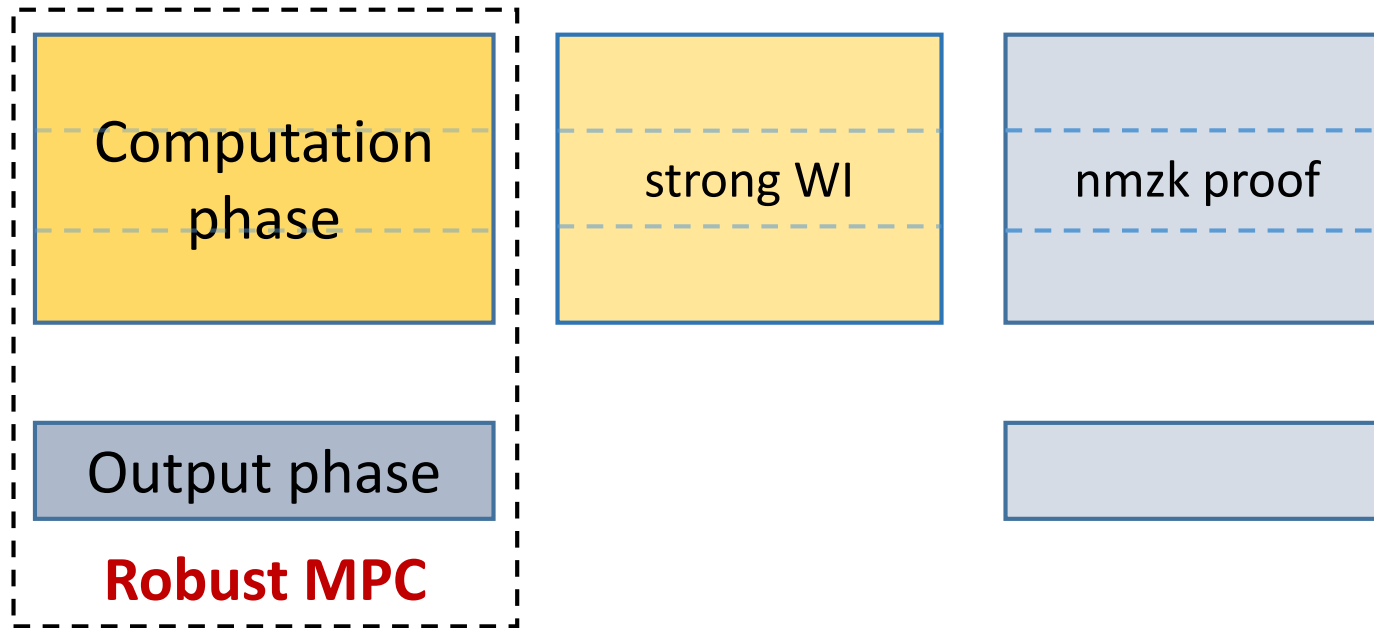
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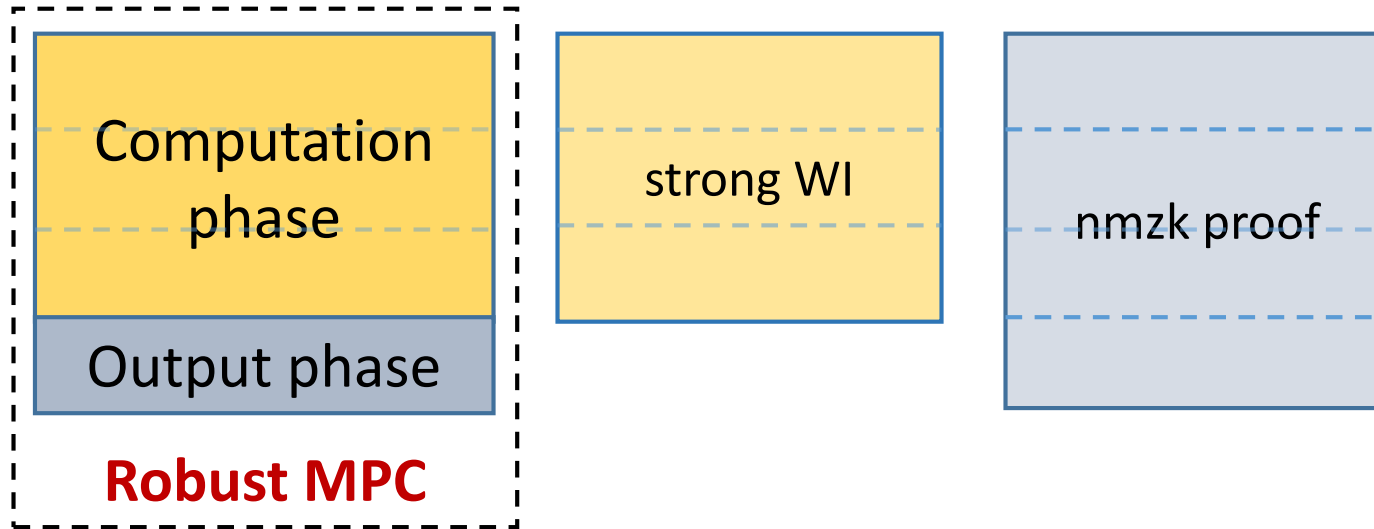
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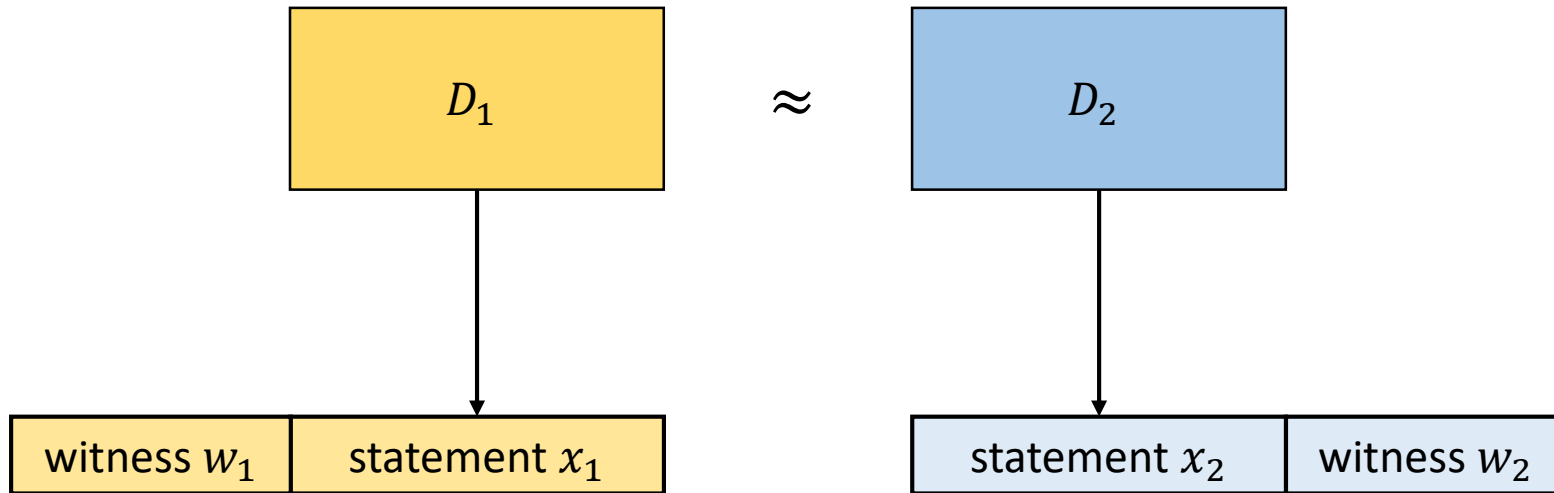
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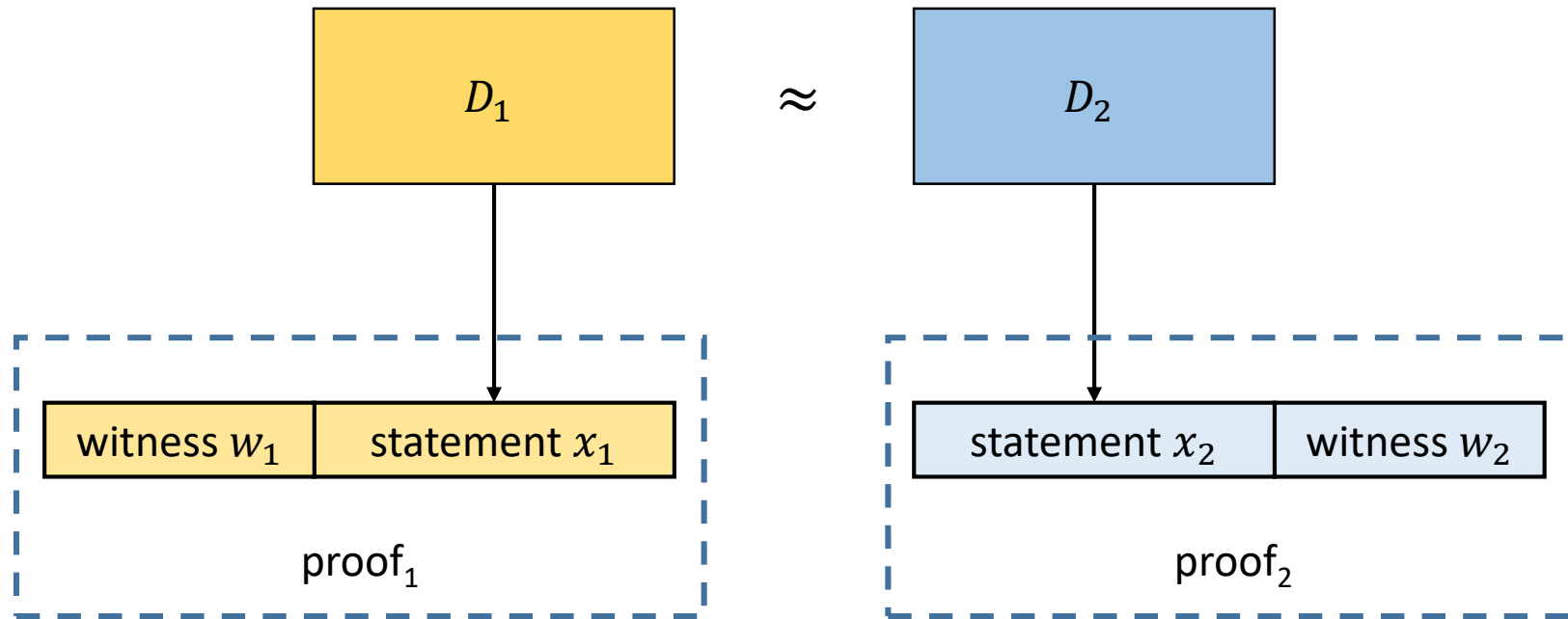
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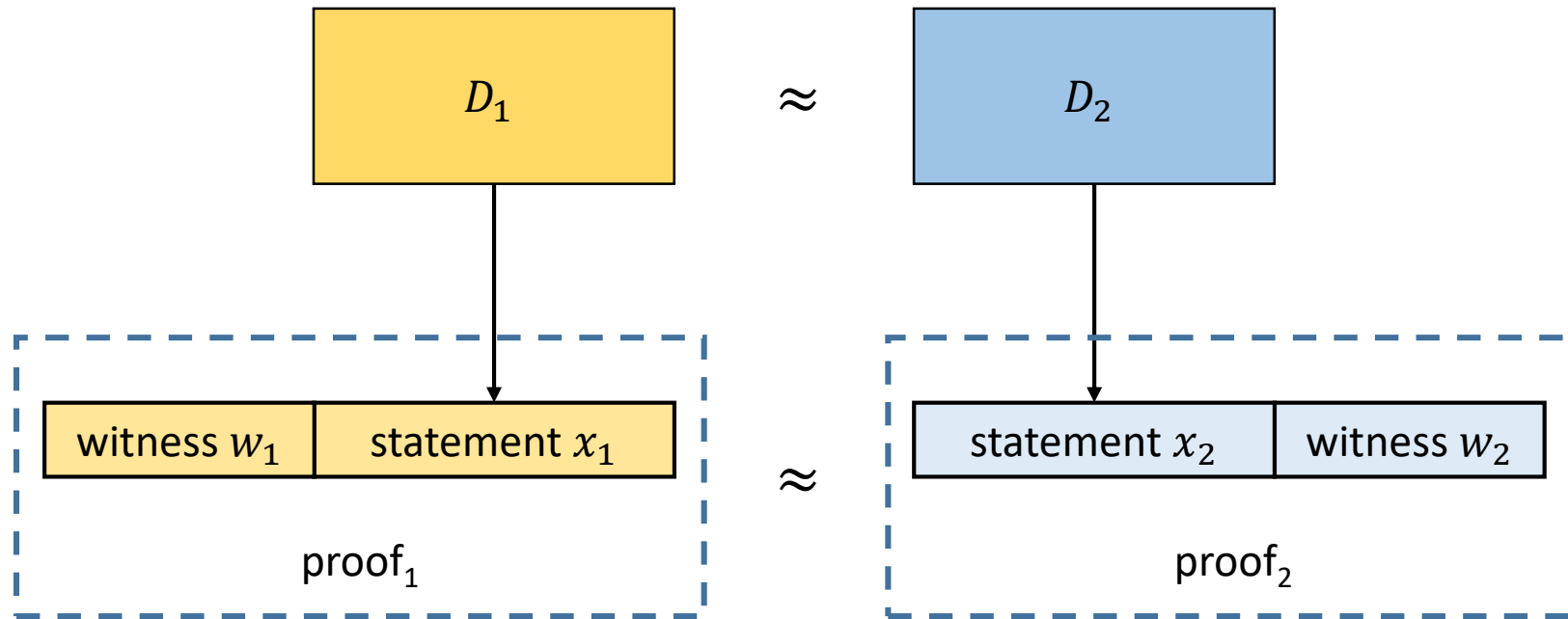


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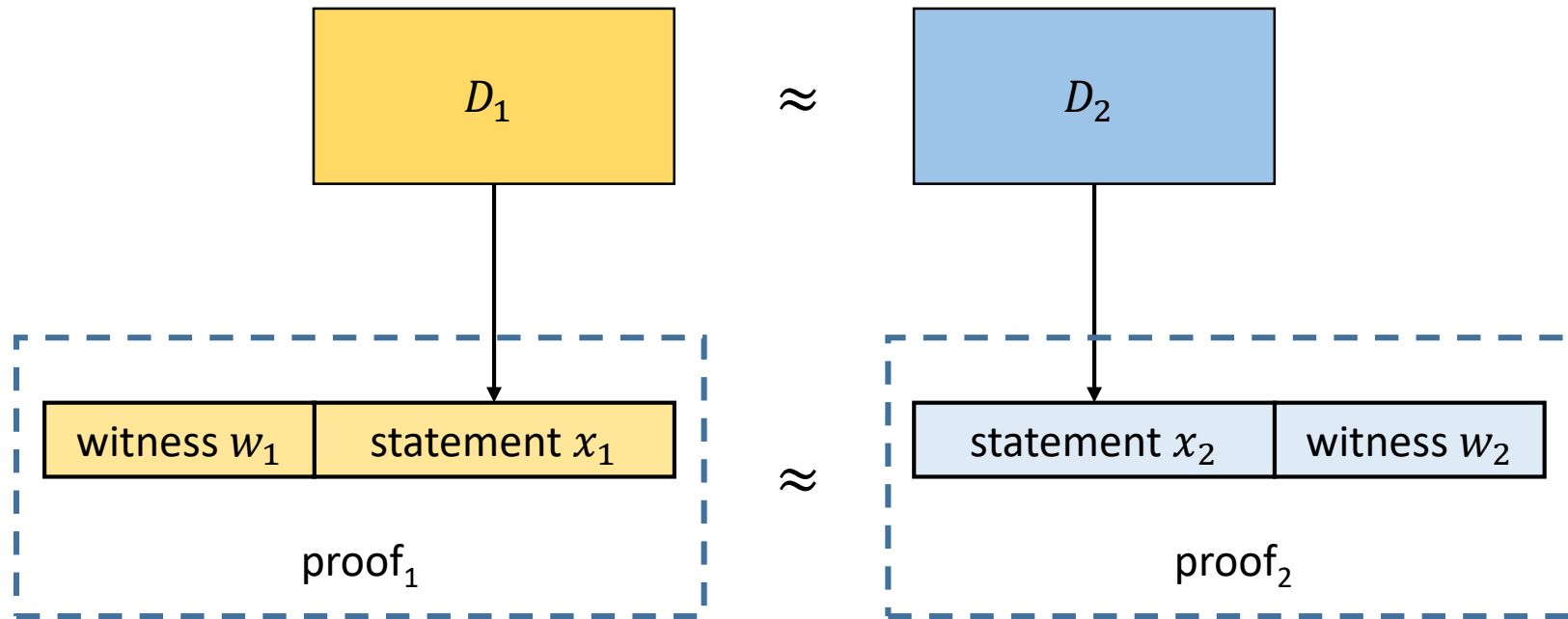




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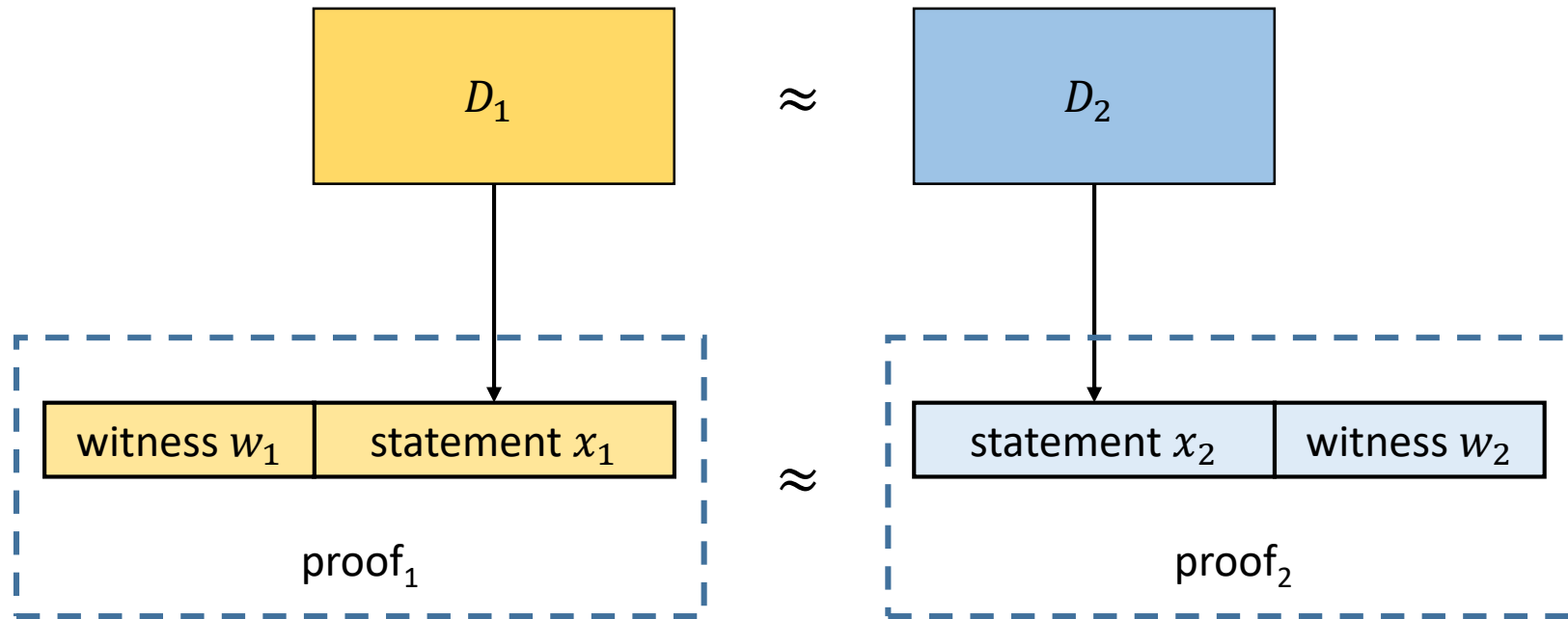


# Strong WI proof system



We change both witness and statement during simulation.

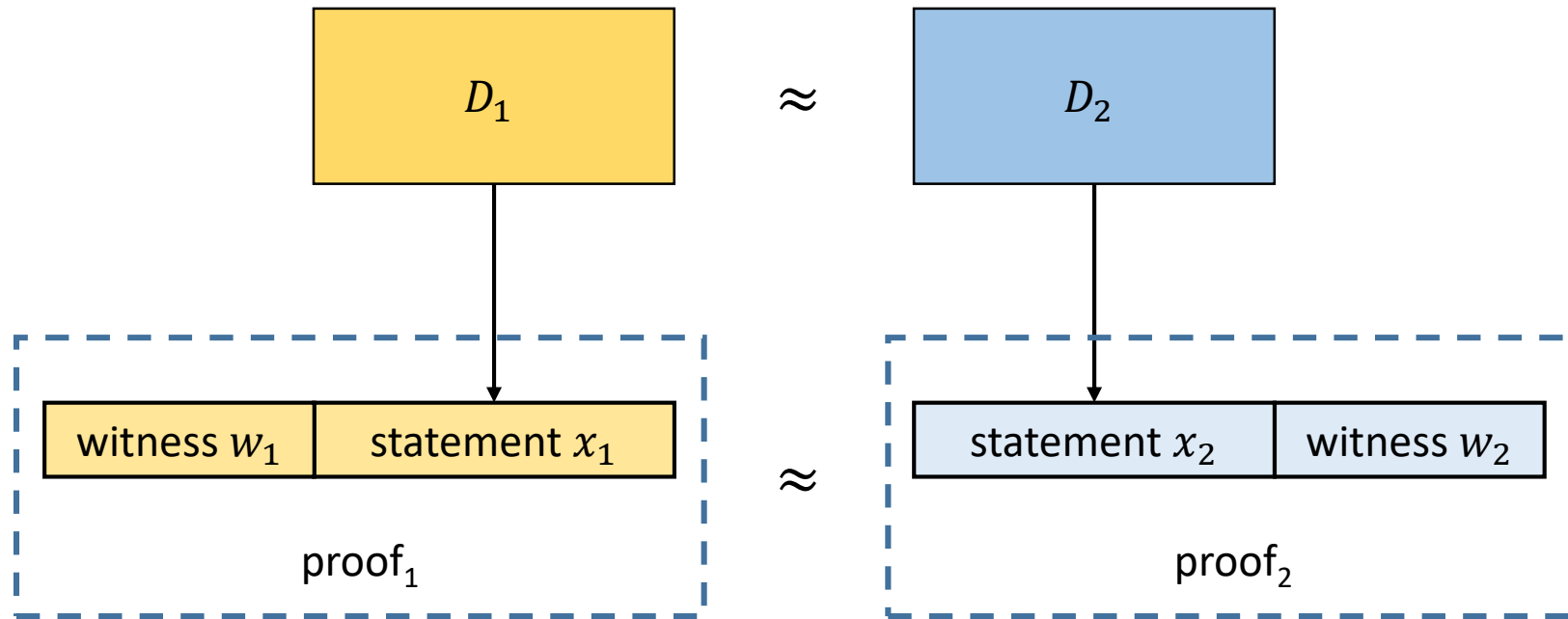
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[JKKR17] constructed **3 round strong WI** from **DDH** in a limited setting.

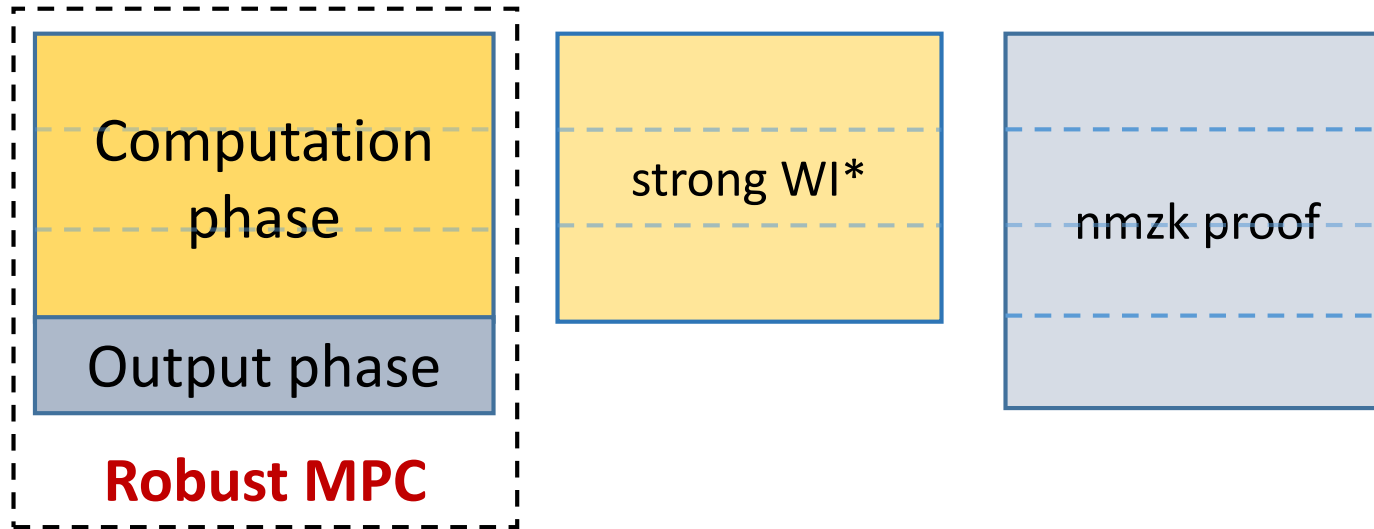
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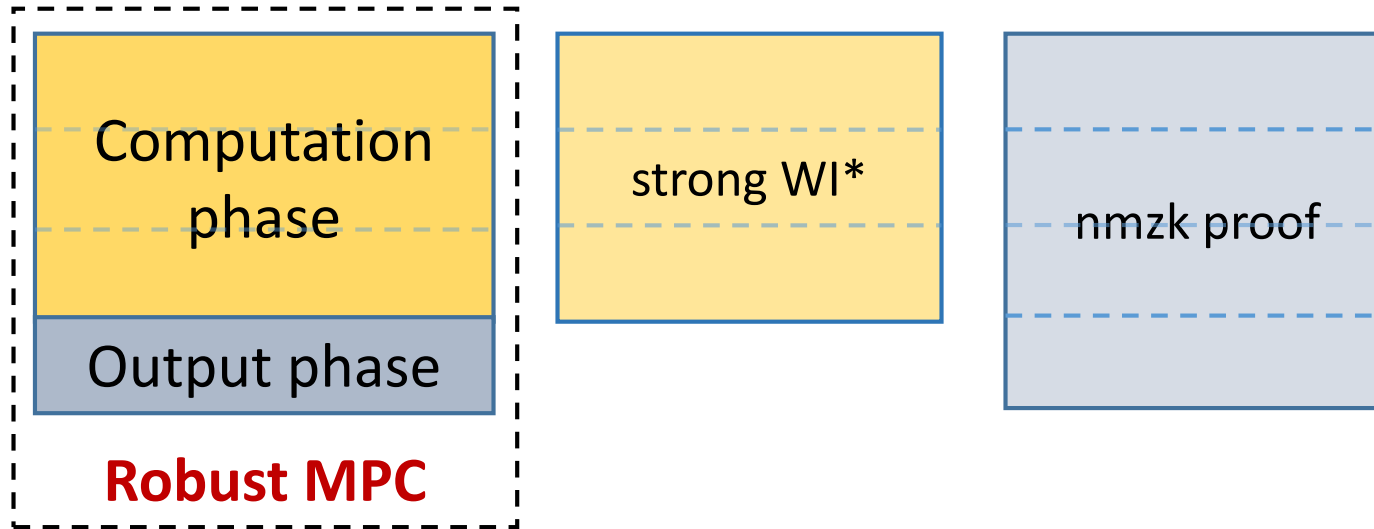
[JKKR17] constructed 3 round **strong WI** from **DDH** in a limited setting. Not applicable to our setting.

# Blueprint of 4 round protocol



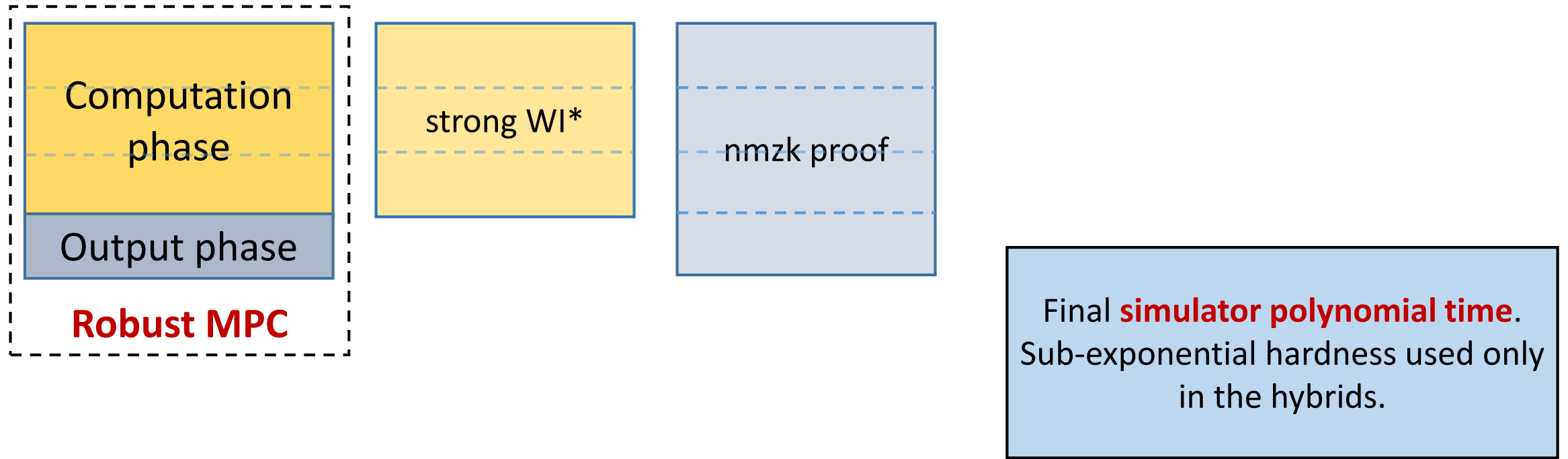
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# Blueprint of 4 round protocol



We construct 3 round strong WI\* assuming **OWP** and **sub-exponentially secure DDH** with **requisite non-malleability** properties [GPR16, KS17].

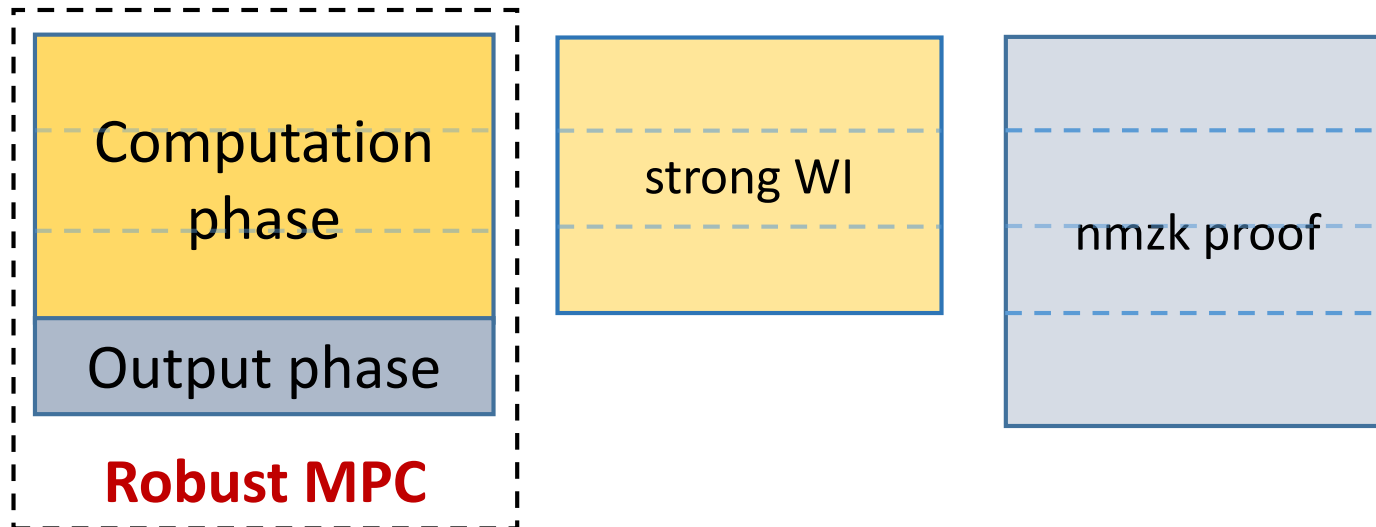
# Blueprint of 4 round protocol



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# Remarks on security proofs

Several challenges, and specific constructions modified accordingly.

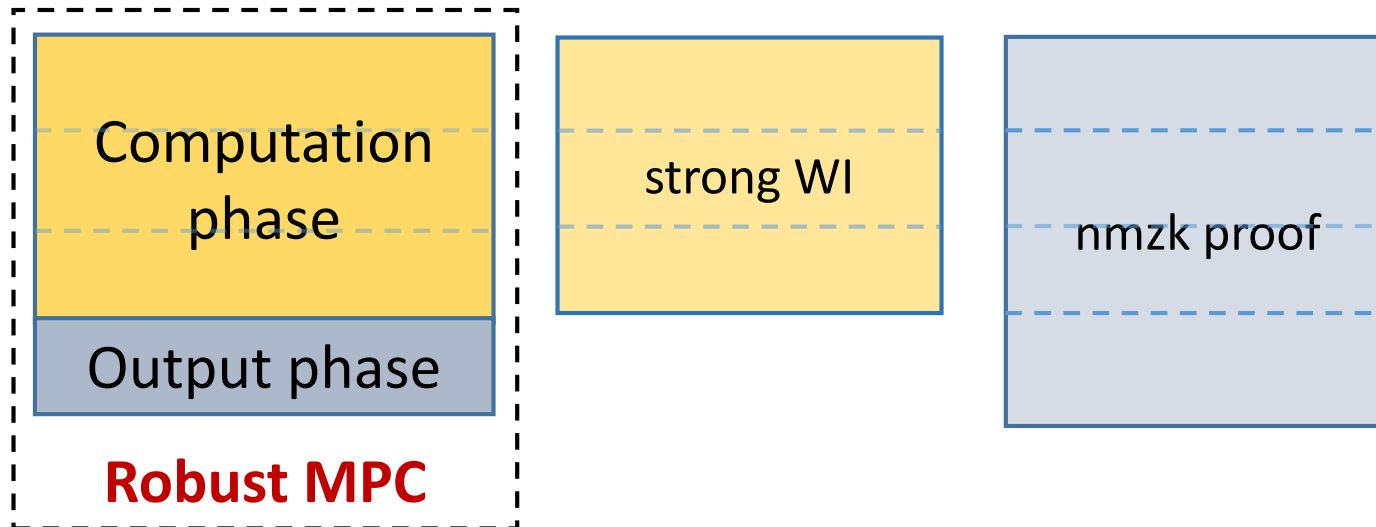




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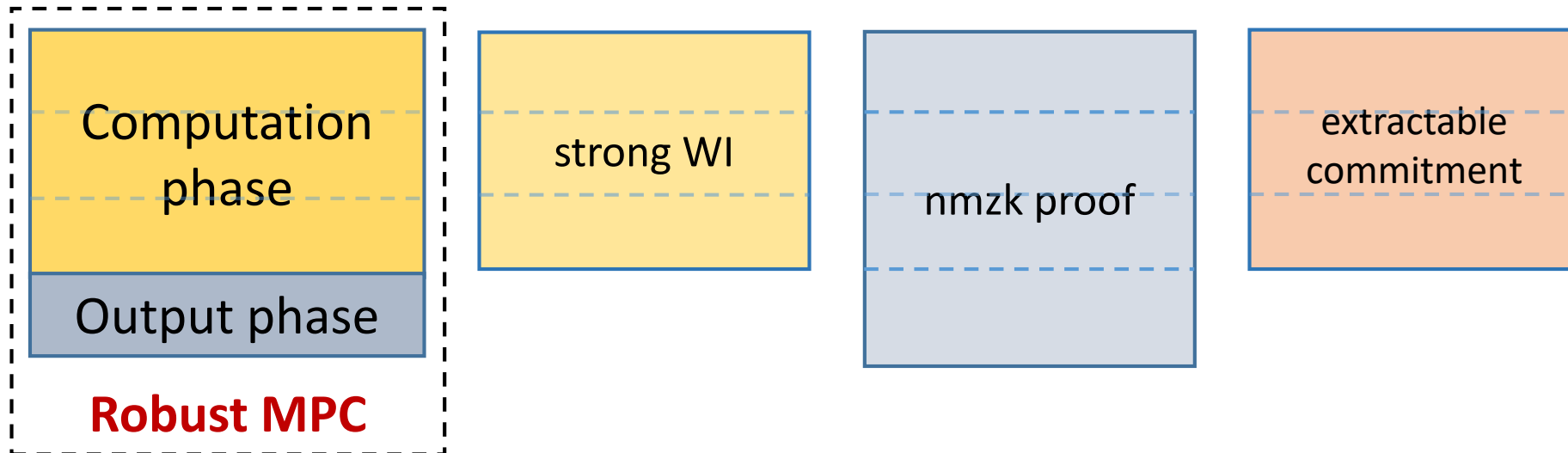
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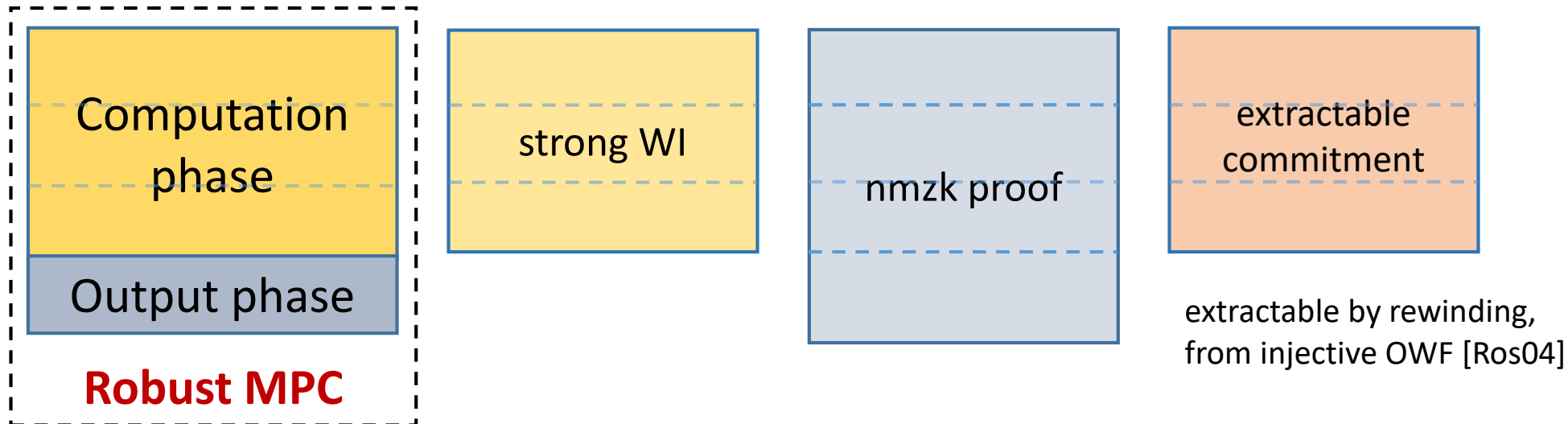
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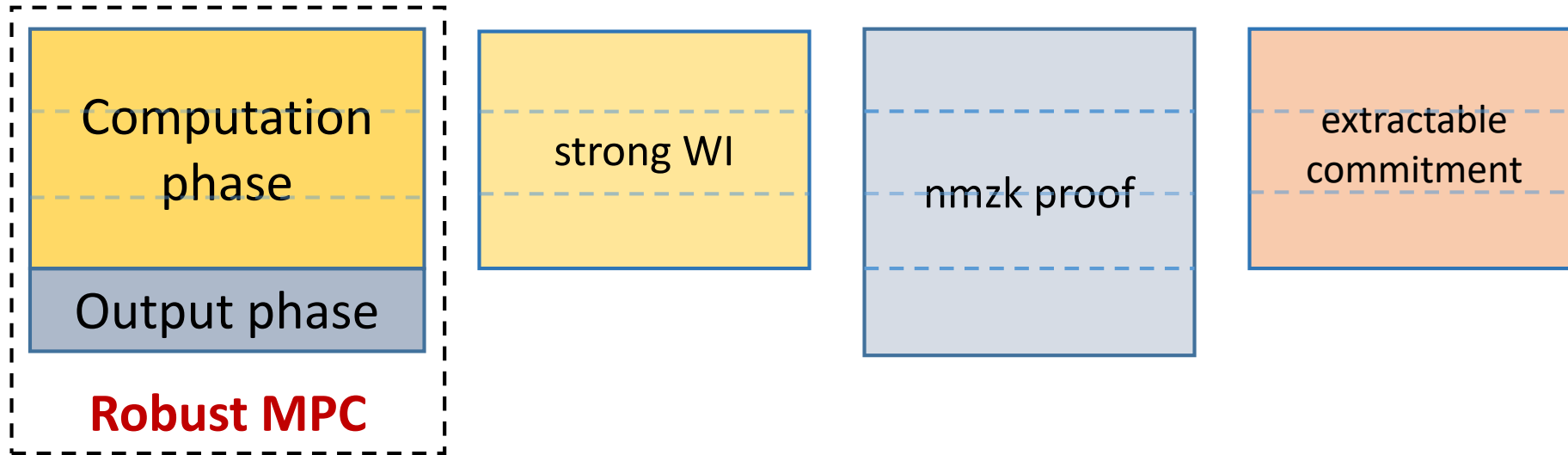
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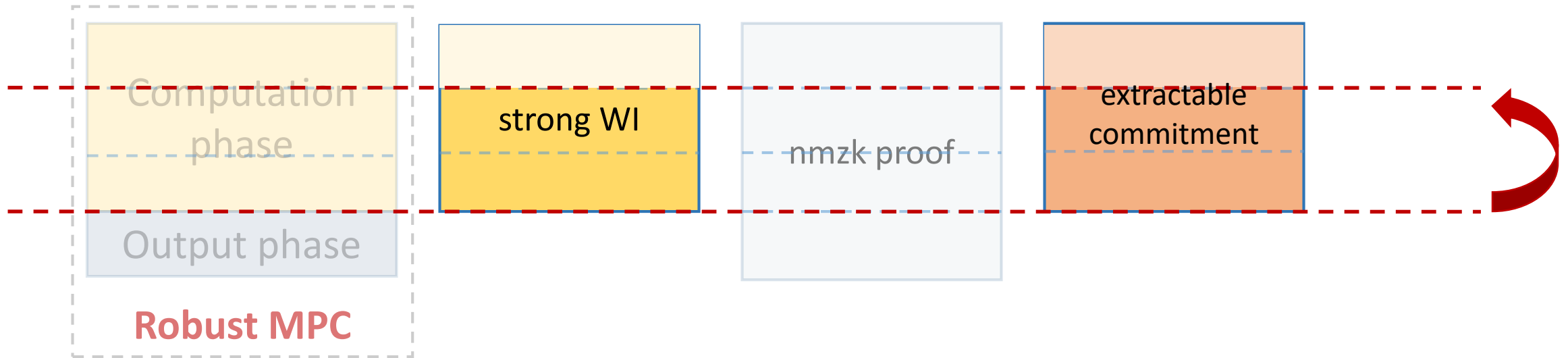
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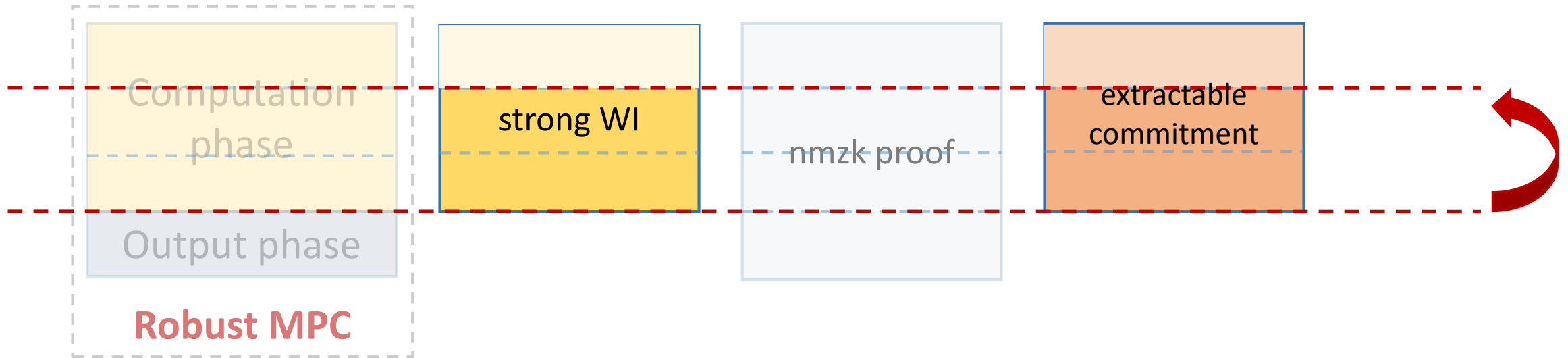
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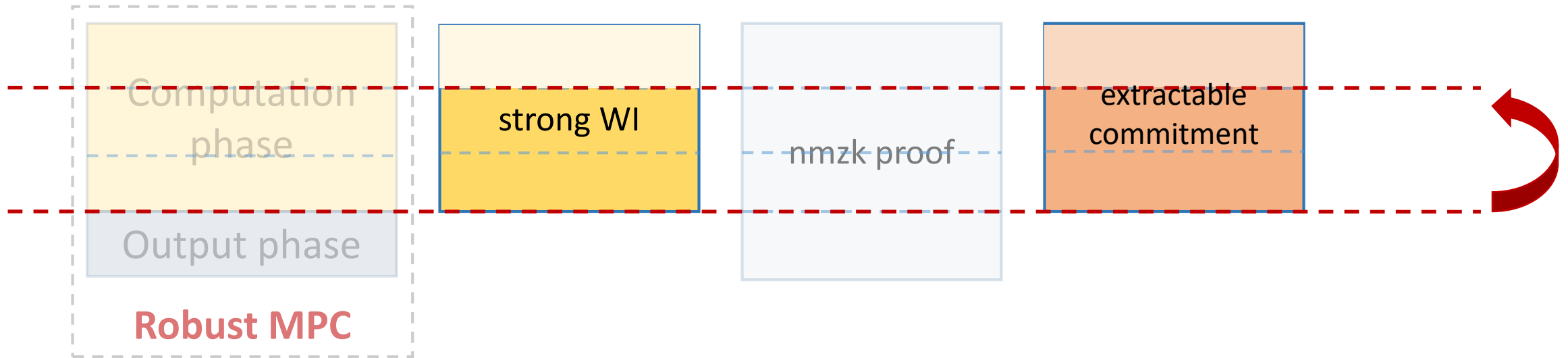


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Overcome this issue by using **rewinding secure primitives**



# Remarks on security proofs



Primitives need to be secure in the presence of rewinding.

Overcome this issue by using **rewinding secure primitives**, or use **complexity leveraging** to bypass it.

# 4 Round Robust MPC

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Round complexity proportional to the depth.

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Main contribution is to bring it down to 4 rounds.

# Thank you. Questions?

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[ia.cr/2017/402](https://ia.cr/2017/402)