Non-Interactive Batch Arguments

and more

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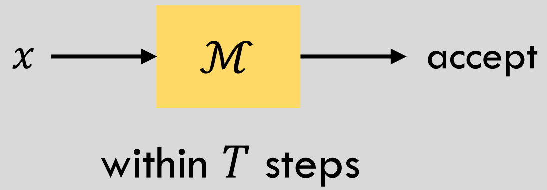
Succinct Non-Interactive Arguments (SNARGs)

$\mathcal{M}$, x

$\mathcal{M}$, x



Succinct Non-Interactive Arguments (SNARGs)

$\mathcal{M}$, x

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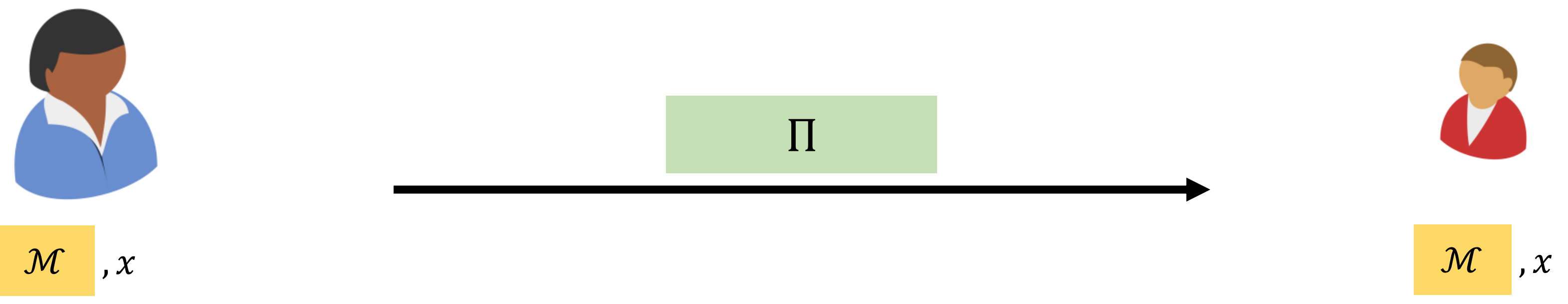
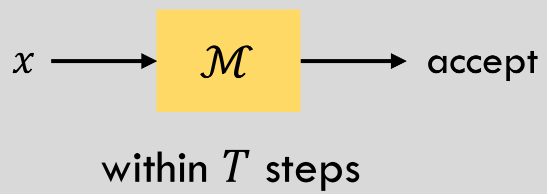
$x \longrightarrow \mathcal{M} \longrightarrow$ accept

within T steps

wants to delegate computation to

Succinct Non-Interactive Arguments (SNARGs)





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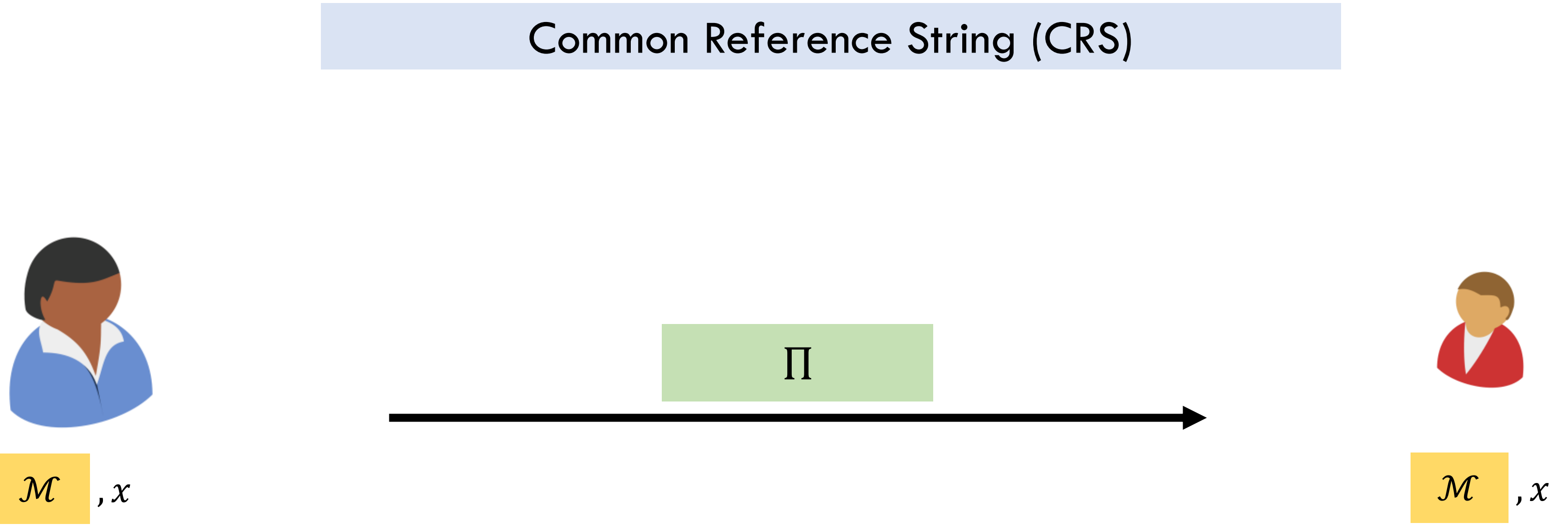

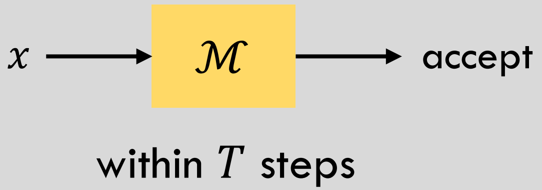

Succinct Non-Interactive Arguments (SNARGs)

Common Reference String (CRS)

Π

$\mathcal{M}$, x

$\mathcal{M}$, x

Π is publicly verifiable

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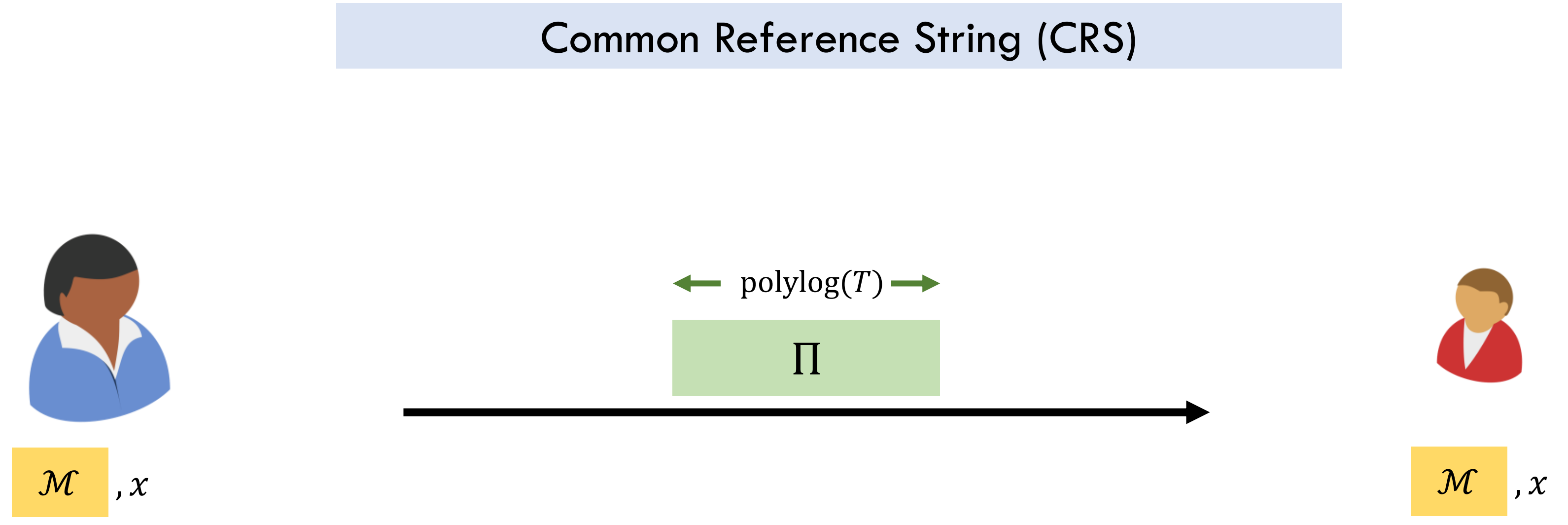
Succinct Non-Interactive Arguments (SNARGs)



Verifier running time: polylog(T)

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Succinct Non-Interactive Arguments (SNARGs)

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$\mathcal{M}$, x

← polylog(T) →

Π

$\mathcal{M}$, x

Verifier running time: polylog(T)

Π is publicly verifiable

x → $\mathcal{M}$ → accept

within T steps

No PPT  can produce accepting Π if

x → $\mathcal{M}$ ✗→ accept

within T steps

Succinct Non-Interactive Arguments (SNARGs)

Common Reference String (CRS)

$\mathcal{M}$, x

$\leftarrow$ polylog(T) $\rightarrow$

Π

$\mathcal{M}$, x

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$x \longrightarrow \mathcal{M} \not\longrightarrow$ accept

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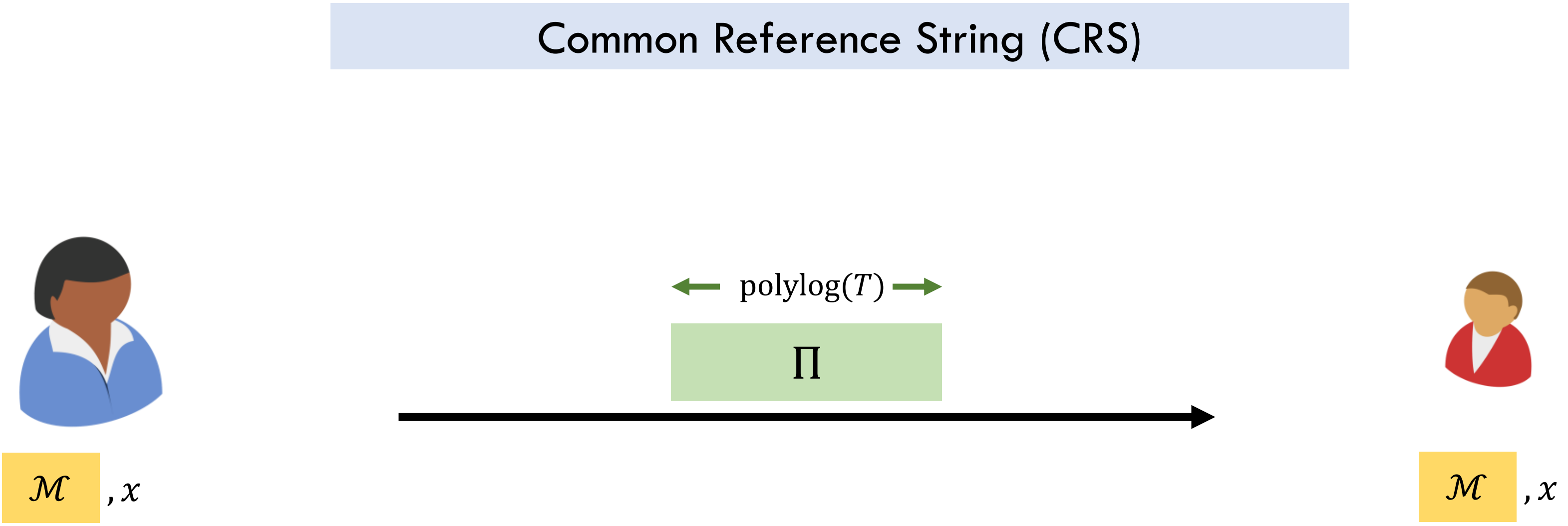
Succinct Non-Interactive Arguments (SNARGs)



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What kind of computation can we hope to delegate based on standard assumptions?

x

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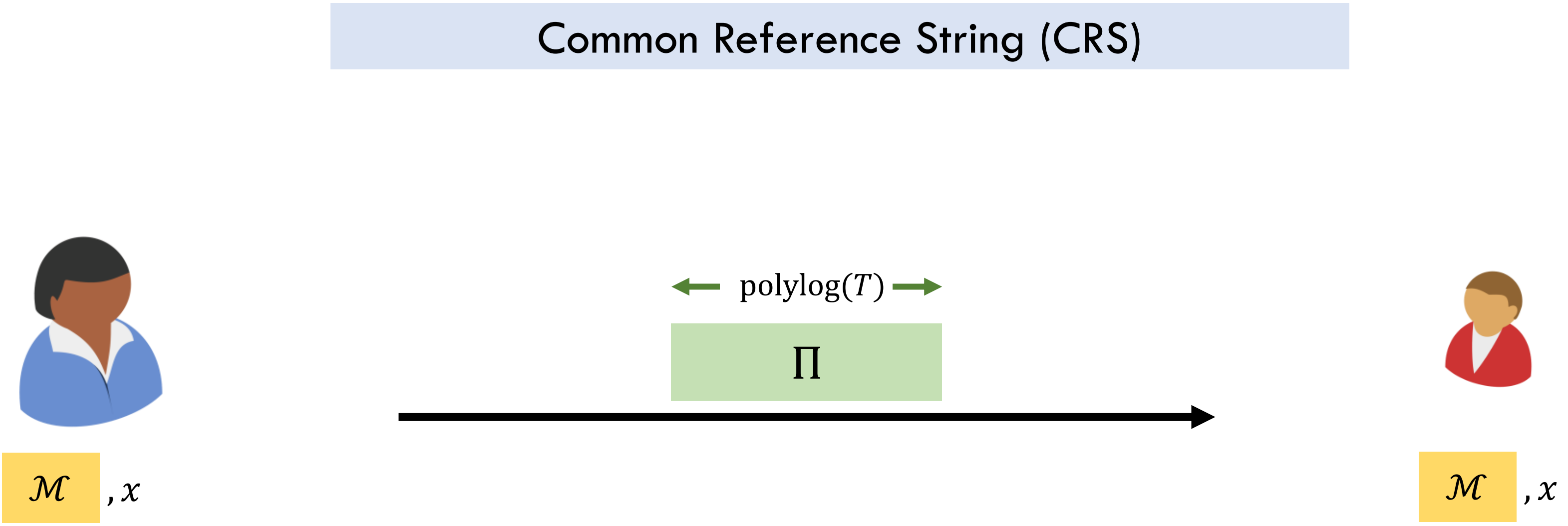

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What kind of computation can we hope to delegate based on standard assumptions?

- Nondeterministic polynomial-time computation (NP)? Unlikely! [Gentry-Wichs'11]

x

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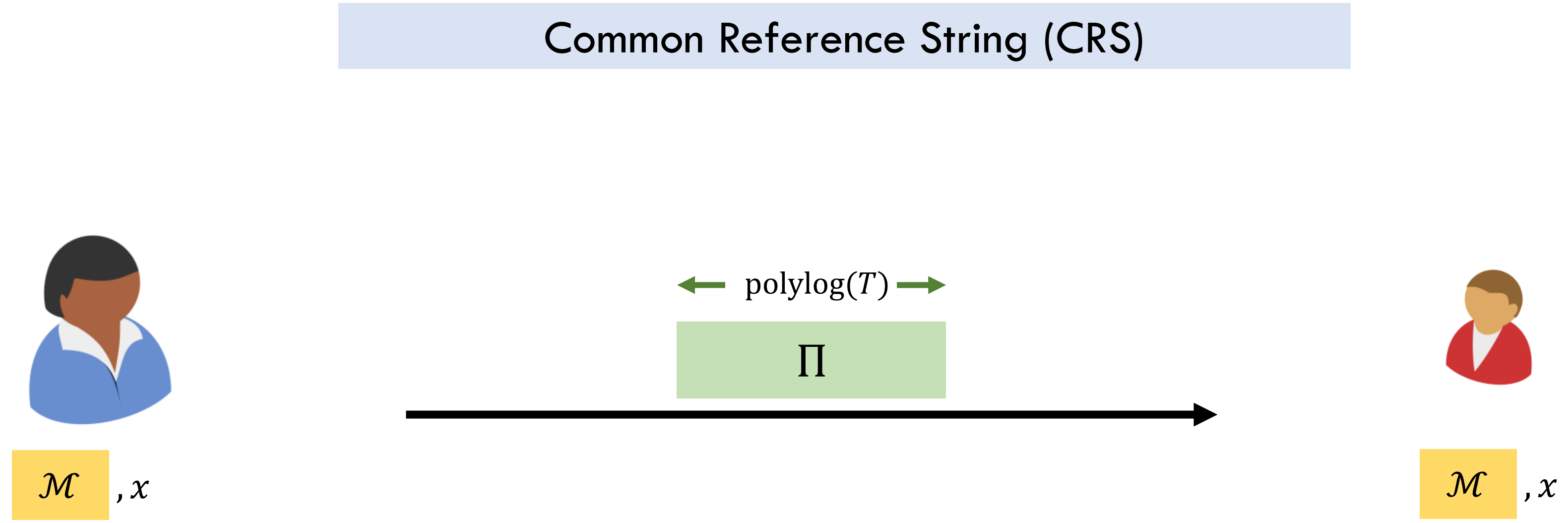

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What kind of computation can we hope to delegate based on standard assumptions?

- Nondeterministic polynomial-time computation (NP)? Unlikely! [Gentry-Wichs'11]
- Deterministic polynomial-time computation (P)?

x

Prior Works

Prior Works

Non-falsifiable assumptions/ Random oracle model

[Micali'94, Groth'10, Lipmaa'12, Damgård-Faust-Hazay'12, Gennaro-Gentry-Parno-Raykova'13, Bitansky-Chiesa-Ishai-Ostrovsky-Paneth'13, Bitansky-Canetti-Chiesa-Tromer'13, Bitanksy-Canetti-Chiesa-Goldwasser-Lin-Rubinstein-Tromer'17]

Some works can delegate NP

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Some works can delegate NP

"Less standard" assumptions

[Canetti-Holmgren-Jain-Vaikuntanathan'15, Koppula-Lewko-Waters'15, Bitansky-Garg-Lin-Pass-Telang'15, Canetti-Holmgren'16, Ananth-Chen-Chung-Lin-Lin'16, Chen-Chow-Chung-Lai-Lin-Zhou'16, Paneth-Rothblum'17, Canetti-Chen-Holmgren-Lombardi-Rothblum-Rothblum-Wichs'19, Kalai-Paneth-Yang'19]

Delegation for P

Prior Works

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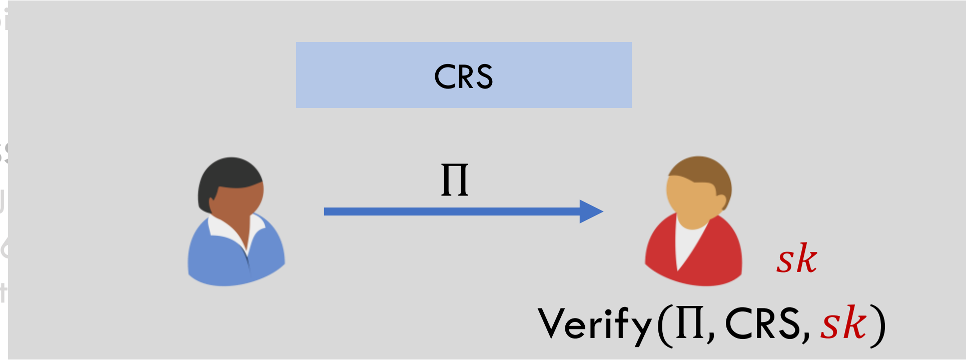

Some works can delegate NP

"Less standard" as

[Canetti-Holmgren-J ... rg-Lin-Pass-Telang'15, Canetti-Holmgren'1 ... '16, Paneth-Rothblum'17, Canett ... Paneth-Yang'19]

Delegation for P

Designated Verifier (standard assumptions)

Prior Works

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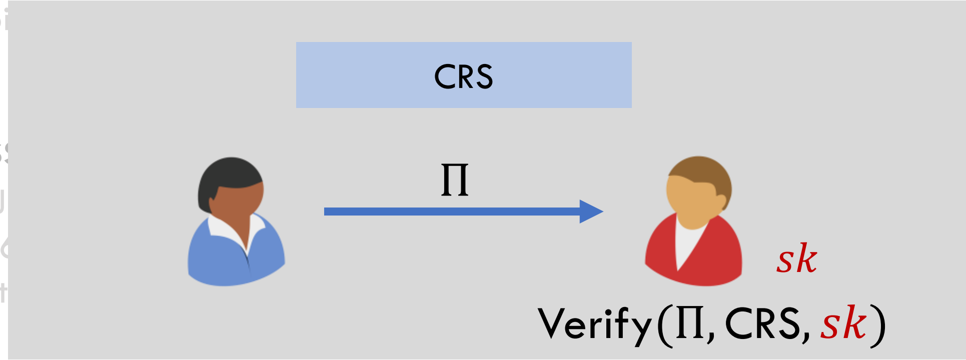

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[Canetti-Holmgren-J ... rg-Lin-Pass-Telang'15, Canetti-Holmgren'1 ... '16, Paneth-Rothblum'17, Canett ... Paneth-Yang'19]

Designated Verifier (standard assumptions)

[Kalai-Raz-Rothblum'13, Kalai-Raz-Rothblum'14, Kalai-Paneth'16, Brakerski-Holgren-Kalai'17, Badrinarayanan-Kalai-Khurana-Sahai-Wichs'18, Holmgren-Rothblum'18, Brakerski-Kalai'20]

Some works can delegate NP

Delegation for P

Delegation for P

Do there exists SNARGs for P based on standard assumptions?

Previously best known: [Jawale-Kalai-Khurana-Zhang'21] for depth bounded computation based on sub-exponential hardness of LWE.

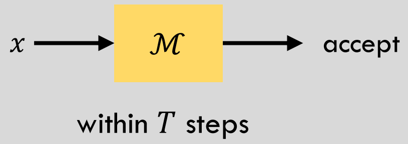
Builds on [Canetti-Chen-Holmgren-Lombardi-Rothblum-Rothblum-Wichs'19]

Our Result

Theorem

Assuming LWE there exists a SNARG for P where

$|\mathsf{CRS}|, |\Pi|, |$  $| = \mathrm{polylog}(T)$



LWE – Learning with Errors

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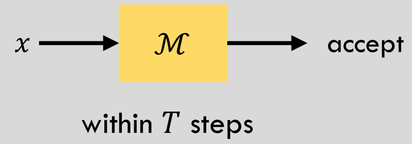
LWE – Learning with Errors



Tool: Fiat-Shamir (FS) Methodology

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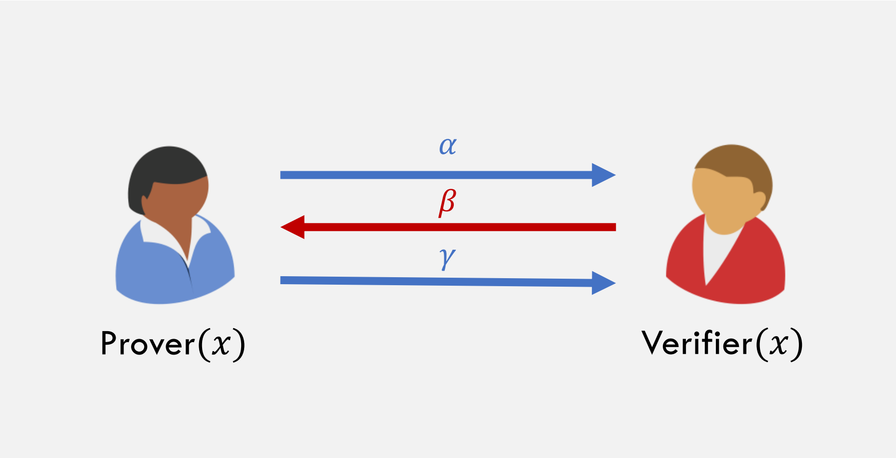
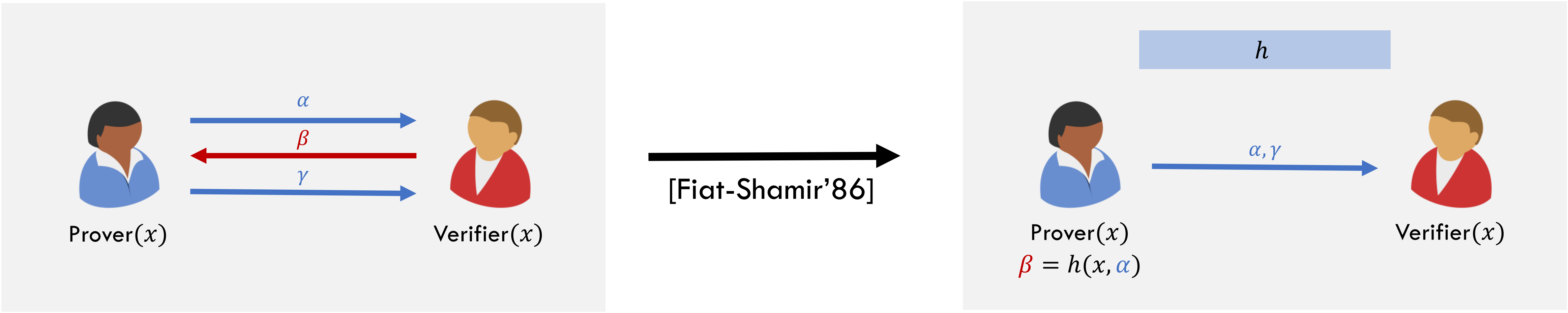


β is a random string

Fiat-Shamir (FS) Methodology



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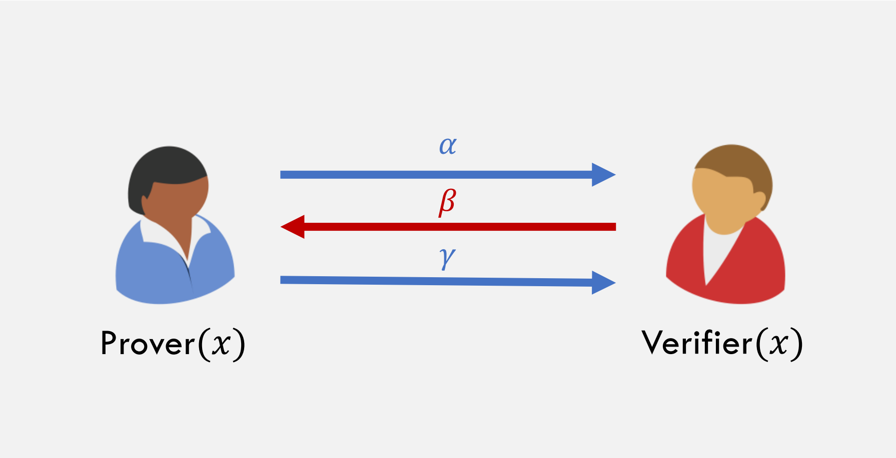

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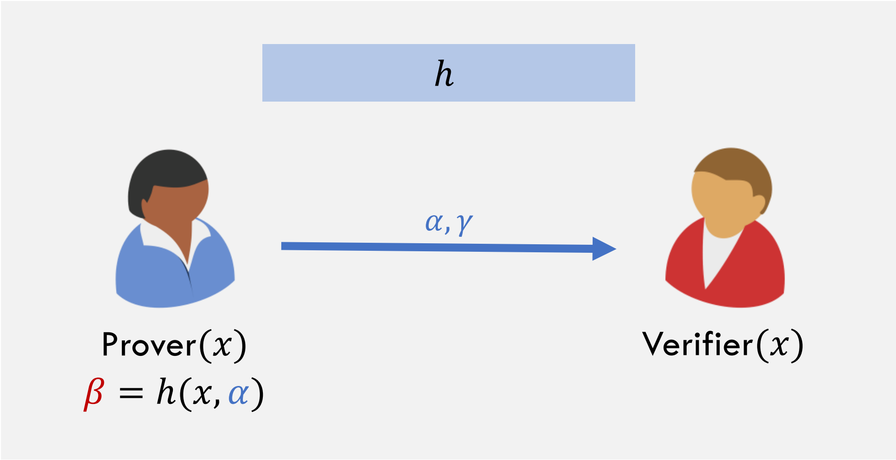
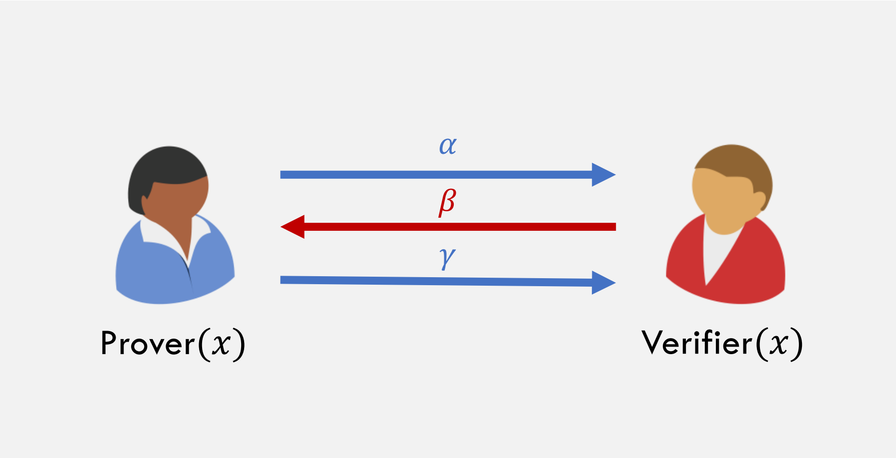
[Fiat-Shamir'86]



$\forall\, x \notin \mathcal{L}$

$$\mathrm{BAD}_{x,\alpha} = \{\beta \mid \exists \gamma \text{ s.t. Verifier accepts } (\alpha, \beta, \gamma)\}$$

Fiat-Shamir (FS) Methodology



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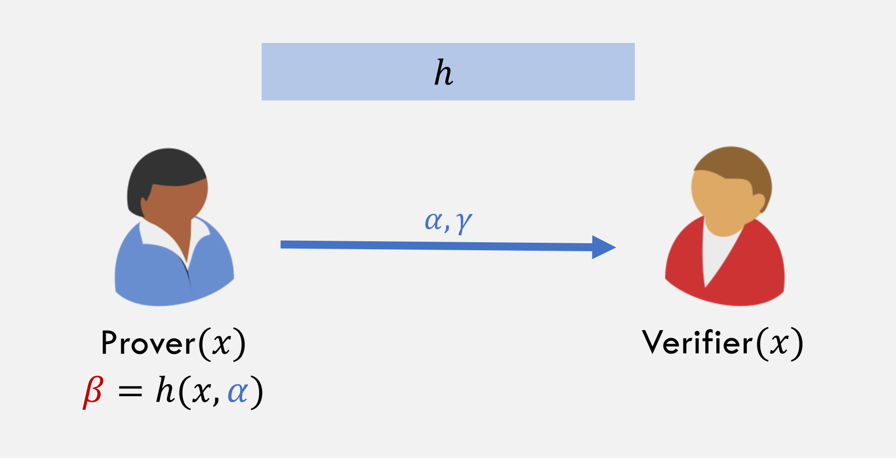

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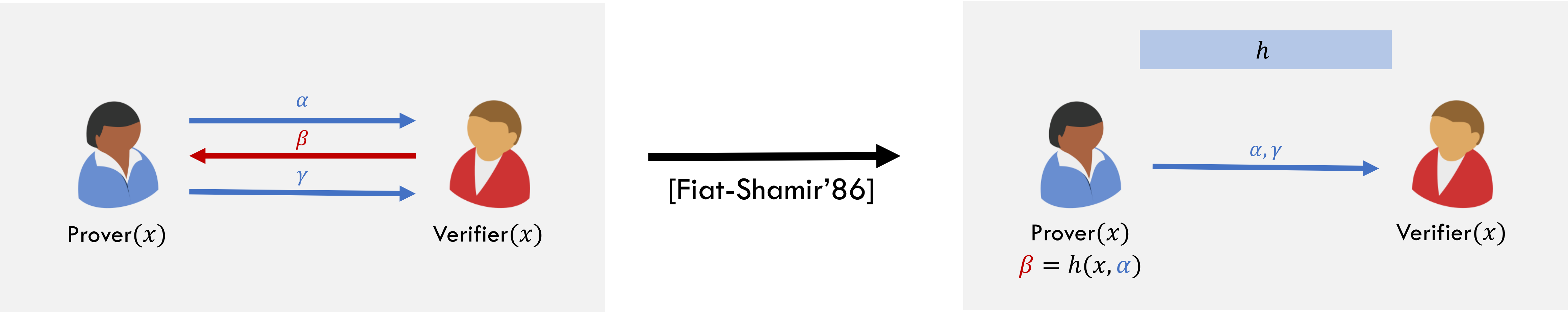
If $x \notin \mathcal{L}$, no PPT  can find α such that

$$h(x, \alpha) \in \mathrm{BAD}_{x,\alpha}$$

Correlation Intractability [Canetti-Goldreich-Halevi'98]



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If $x \notin \mathcal{L}$, no PPT  can find α such that

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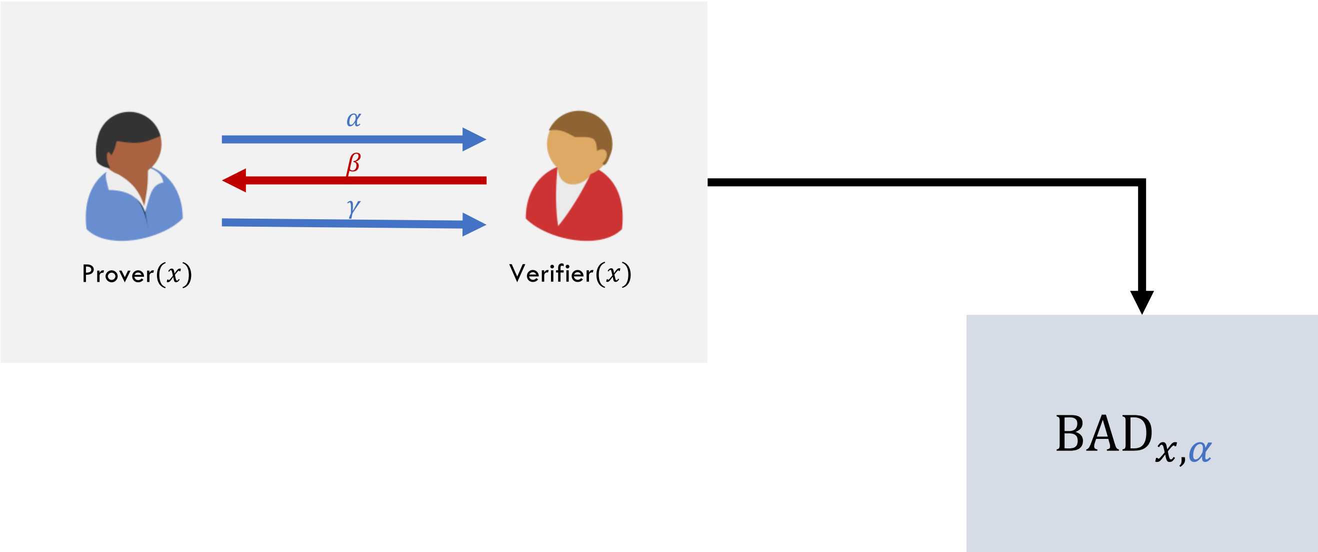
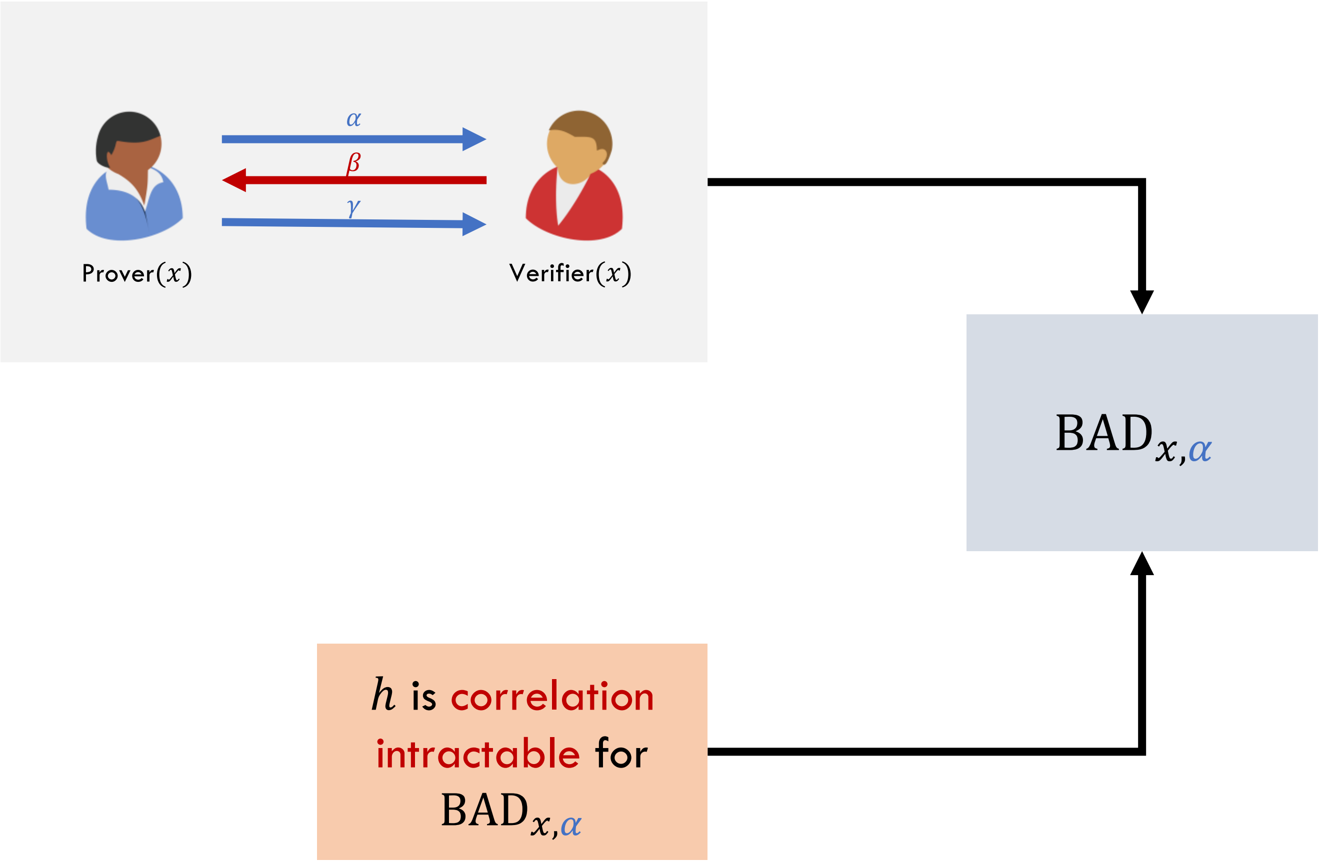
h is **correlation intractable (CI)** for $\mathrm{BAD}_{x,\alpha}$

Instantiating the FS Transform



Instantiating the FS Transform



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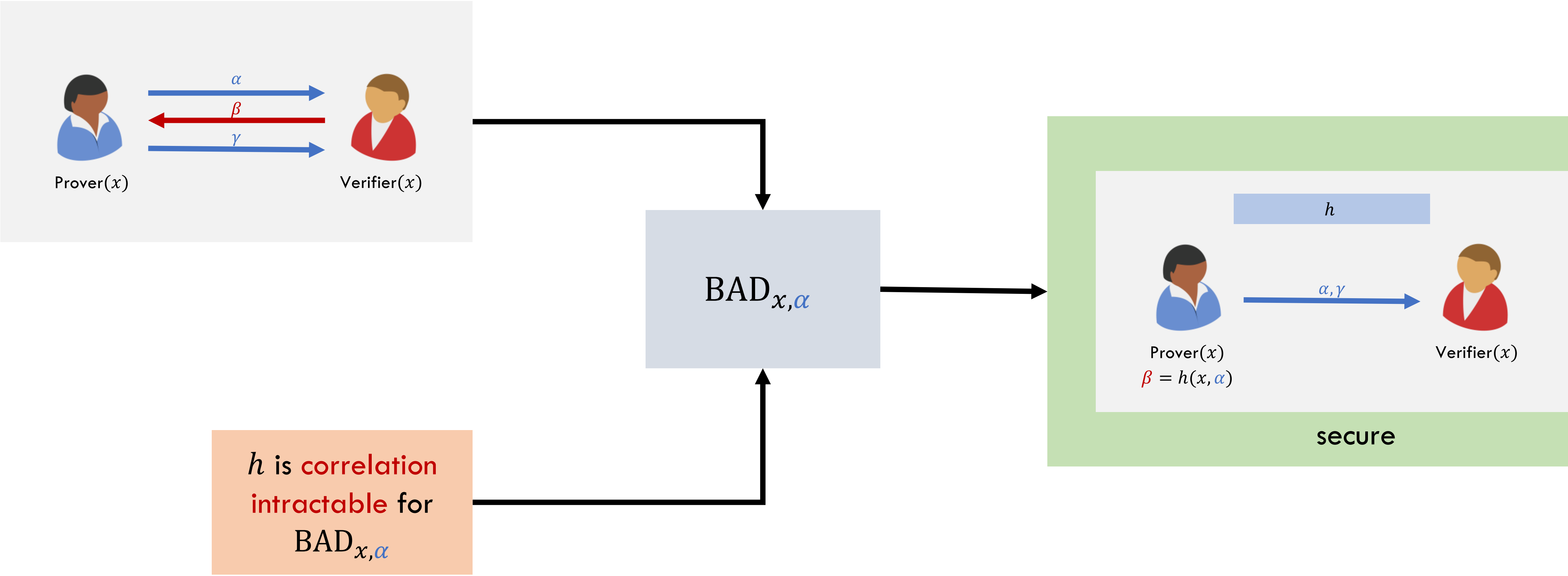

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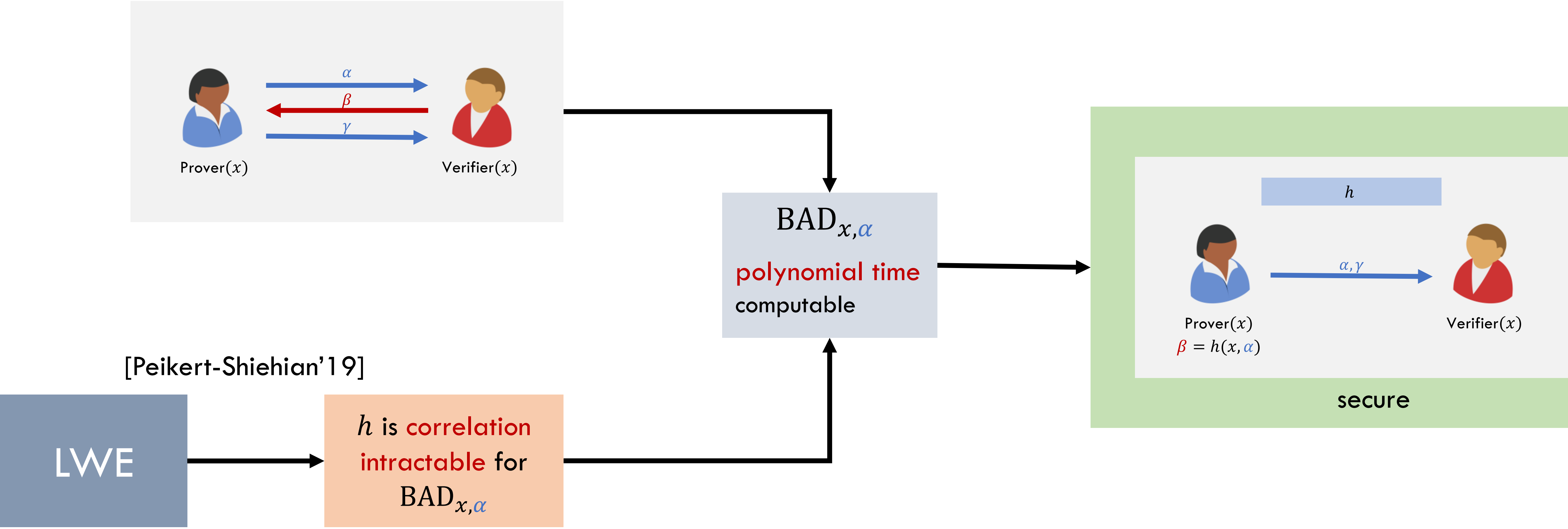

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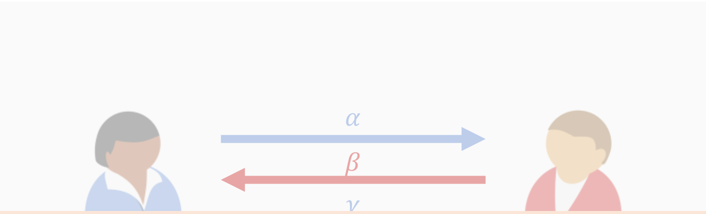


FS methodology is secure for certain protocols under a variety of assumptions (via correlation intractable hash functions)

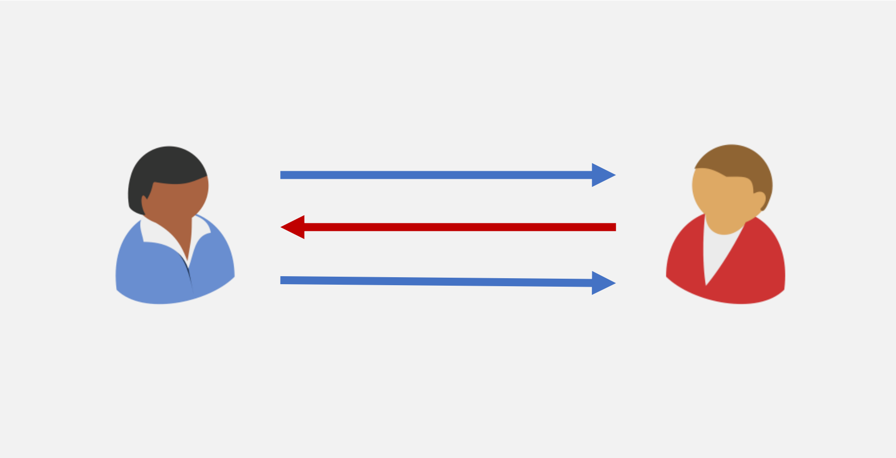
[Kalai-Rothblum-Rothblum'17, Canetti-Chen-Reyzin-Rothblum'18, Holmgren-Lombardi'18, Canetti-Chen-Holmgren-Lombardi-Rothblum-Rothblum-Wichs'19, Peikert-Sheihian'19, Brakerski-Koppula-Mour'20, Couteau-Katsumata-Ursu'20, Jain-Jin'21, Jawale-Kalai-Khurana-Zhang'21, Holmgren-Lombardi-Rothblum'21]

secure

LWE

h is correlation intractable for $\mathrm{BAD}_{x,\alpha}$

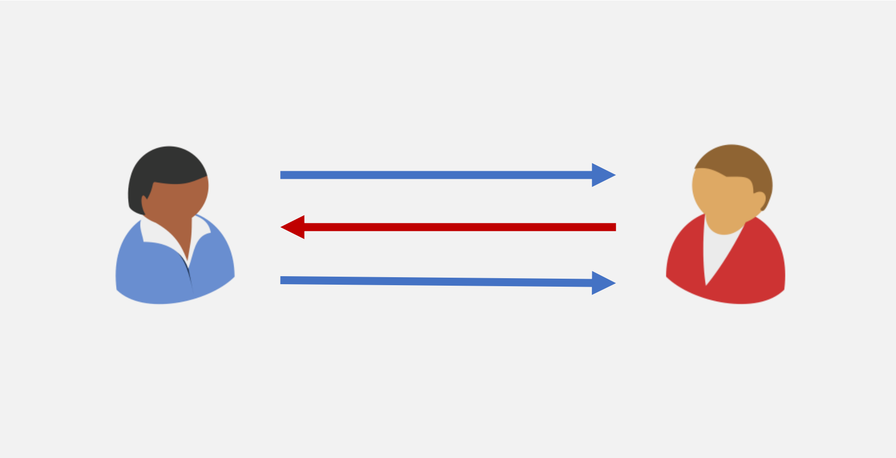
Fiat-Shamir (FS) Methodology



[Kilian'92]

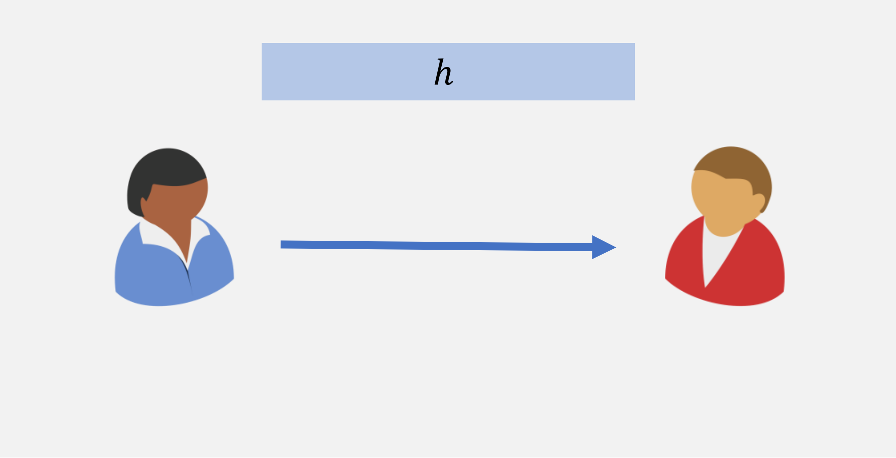
Succinct interactive arguments for all polynomial time computation.

Fiat-Shamir (FS) Methodology



[Kilian'92]

Instantiate with CIH

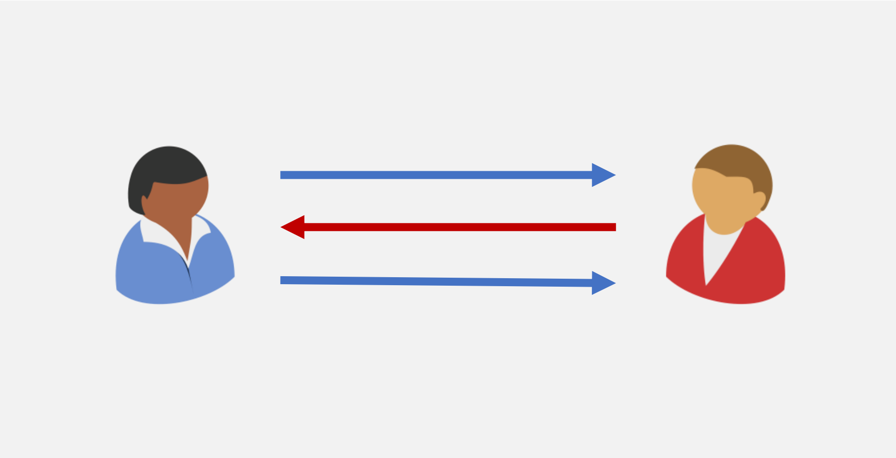


Succinct interactive arguments for all polynomial time computation.

[Bartusek-Bronfman-Holmgren-Ma-Rothblum'19]

Instantiating hash function for Fiat-Shamir transformation of Kilian's protocol is hard.

Fiat-Shamir (FS) Methodology



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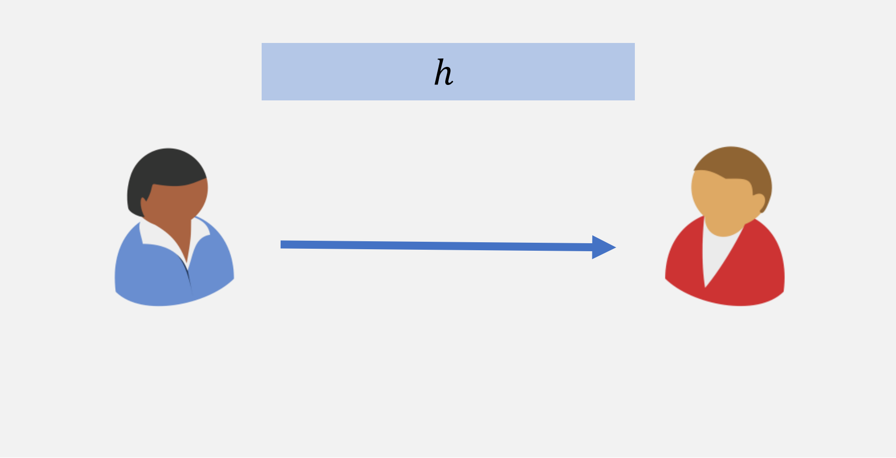

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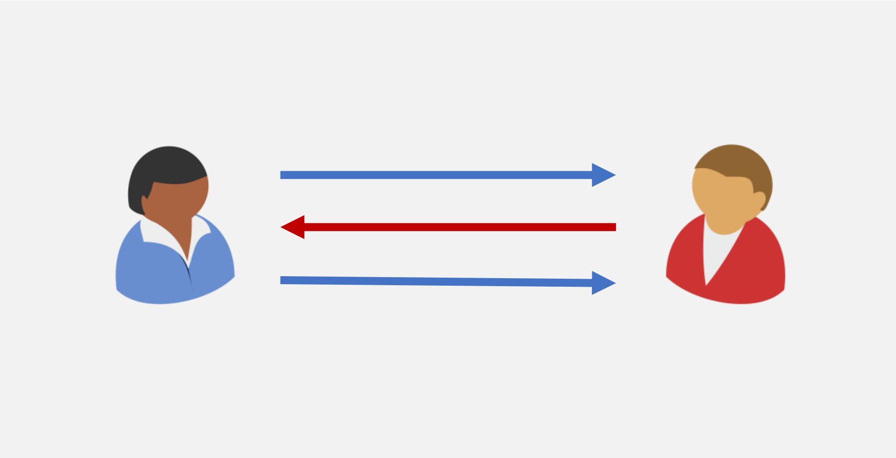

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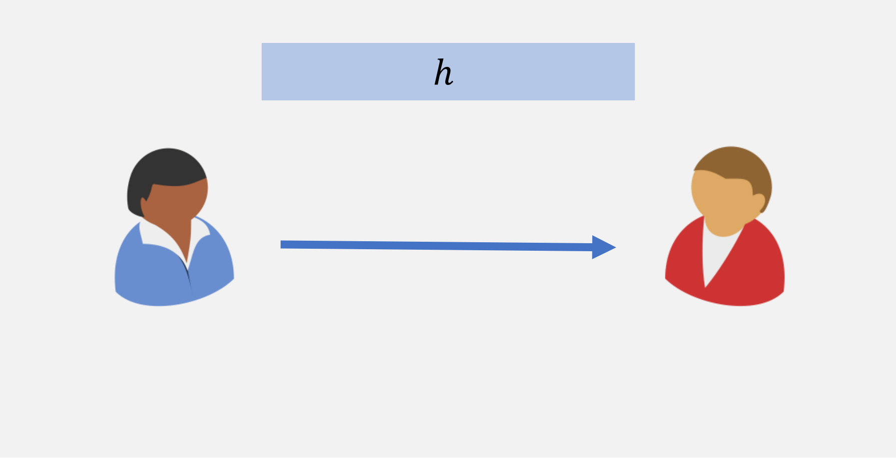
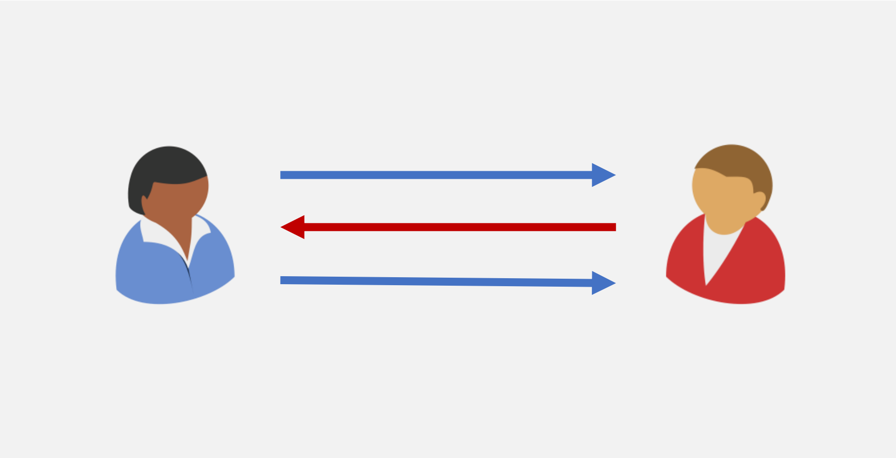


Succinct interactive arguments for all polynomial time computation.

Known instantiations of CI Hash for Fiat-Shamir transform are for proofs.

[Canetti-Sarkar-Wang'20] instantiate Fiat-Shamir transform for specific Sigma protocol that is an argument.

Fiat-Shamir (FS) Methodology



[Goldwasser-Kalai-Rothblum'08]

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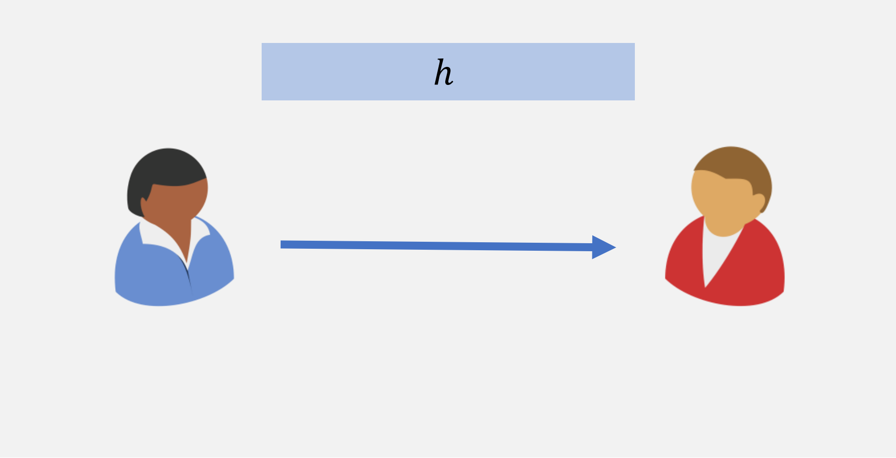

Succinct interactive proof for depth bounded computation.

[Canetti-Chen-Holmgren-Lombardi-Rothblum-Rothblum-Wichs'19, Jawale-Kalai-Khurana-Zhang'21]

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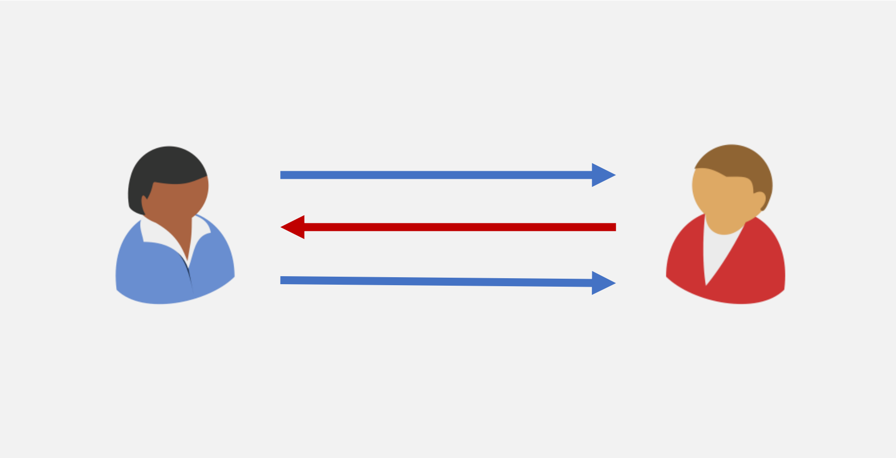

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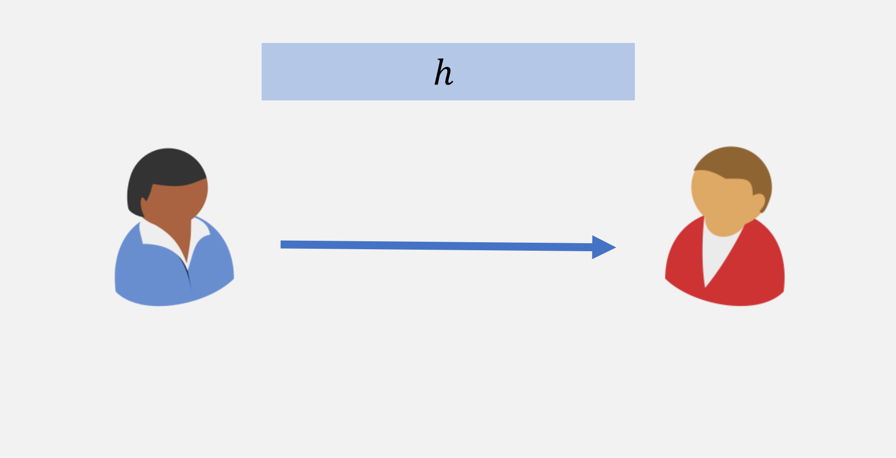
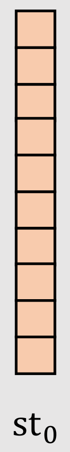


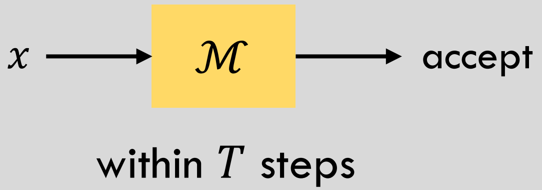
Succinct interactive proof for depth bounded computation.

[Canetti-Chen-Holmgren-Lombardi-Rothblum-Rothblum-Wichs'19, Jawale-Kalai-Khurana-Zhang'21]

Interactive proofs for all polynomial time computation unlikely to exist.

Delegation via Batching [Reingold-Rothblum-Rothblum'16]





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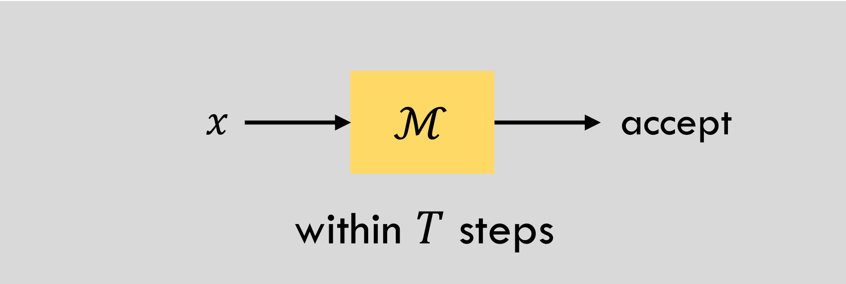

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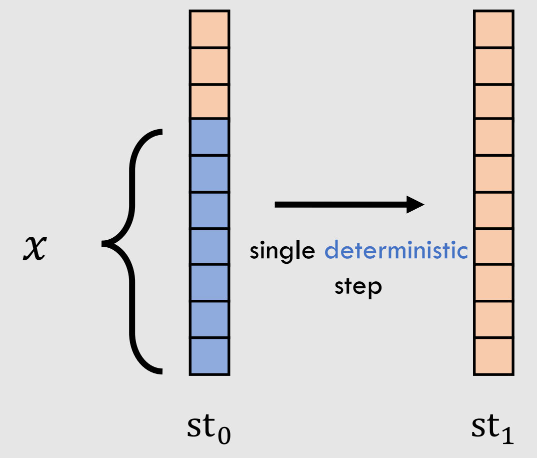

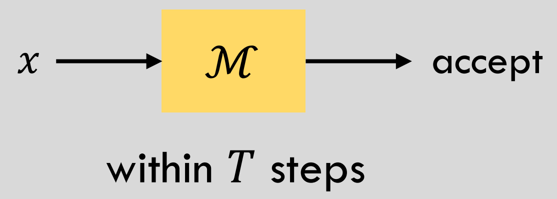

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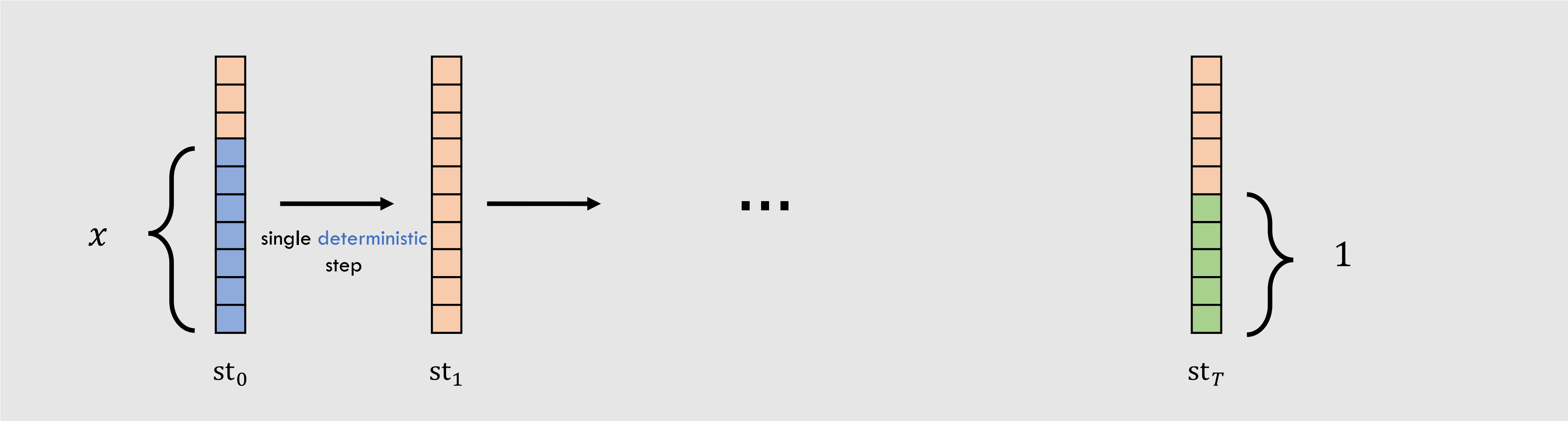

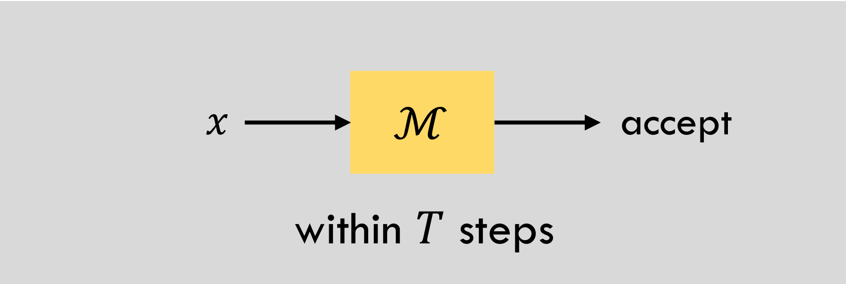

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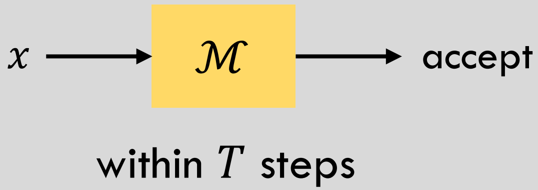
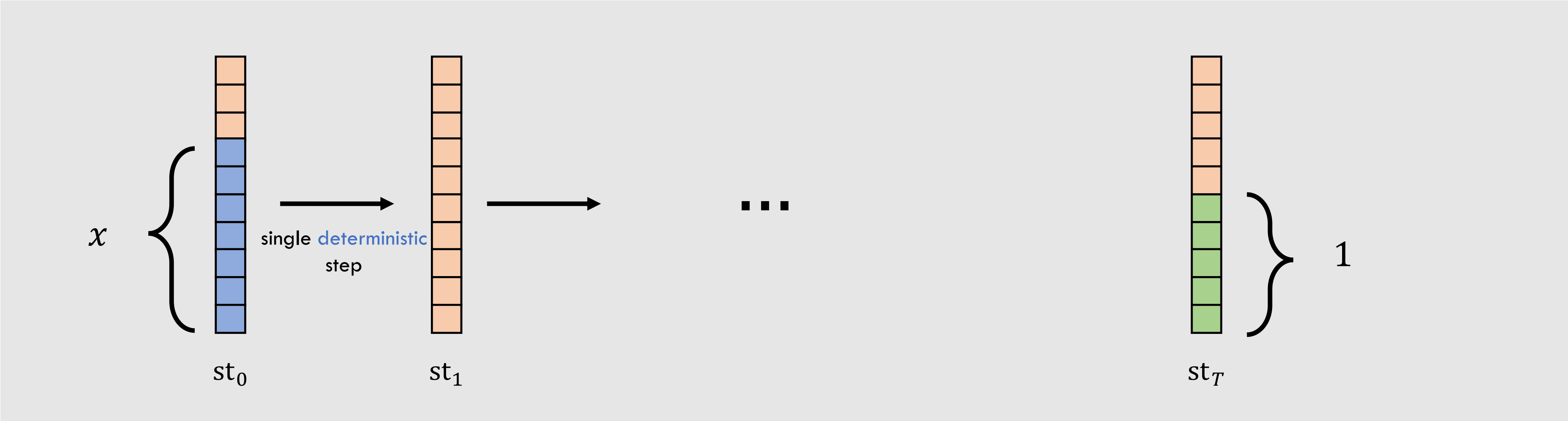
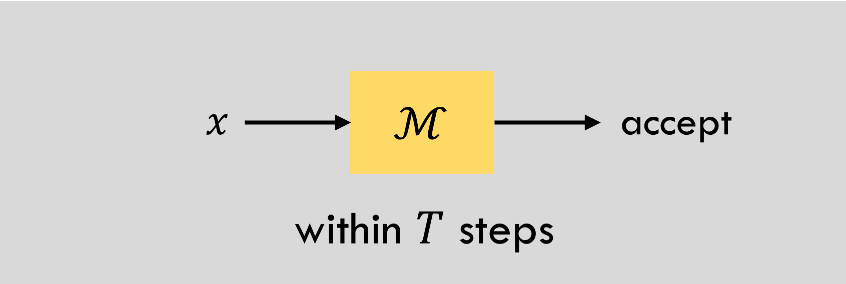


This talk: Bounded space computation

Delegation via Batching [Reingold-Rothblum-Rothblum'16]





Prove for every $i \in [0, \ldots, T-1]$

$$\mathrm{st}_i \to \mathrm{st}_{i+1}$$

is the correct transition.

SNARGs for Batch NP

CRS

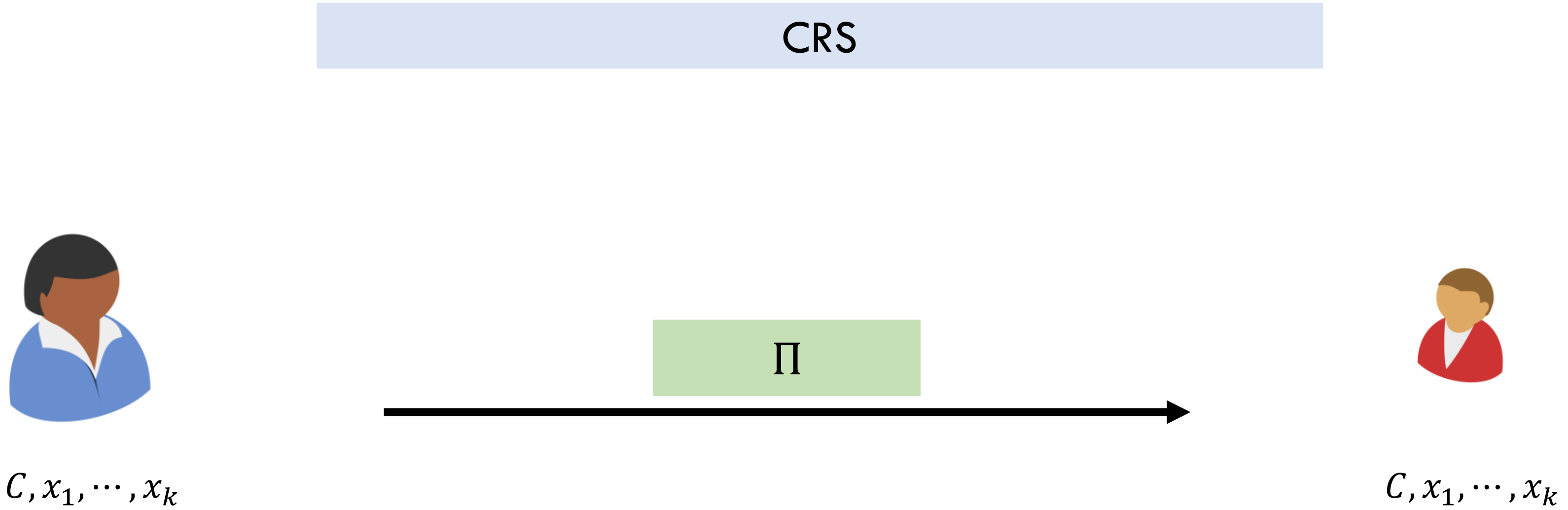
$C, x_1, \cdots, x_k$

$C, x_1, \cdots, x_k$

$\text{SAT} = \{(C, x) \mid \exists w \; s.t. \; C(x, w) = 1\}$

$\forall i \in [k], (C, x_i) \in \text{SAT}$

SNARGs for Batch NP



Π is publicly verifiable

$$\text{SAT} = \{(C, x) \mid \exists w \; s.t. \; C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

SNARGs for Batch NP

CRS

$C, x_1, \cdots, x_k$

Π

$C, x_1, \cdots, x_k$

Π is publicly verifiable

$\text{SAT} = \{(C, x) \mid \exists w \; s.t. \; C(x, w) = 1\}$

$\forall i \in [k], (C, x_i) \in \text{SAT}$

No PPT  can produce accepting Π if

$\exists i^* \in [k], (C, x_{i^*}) \notin \text{SAT}$

SNARGs for Batch NP

CRS

Π

$C, x_1, \cdots, x_k$

$C, x_1, \cdots, x_k$

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SNARGs for Batch NP

CRS

$C, x_1, \cdots, x_k$

$w_1, \ldots, w_k$

$C, x_1, \cdots, x_k$

Π is publicly verifiable

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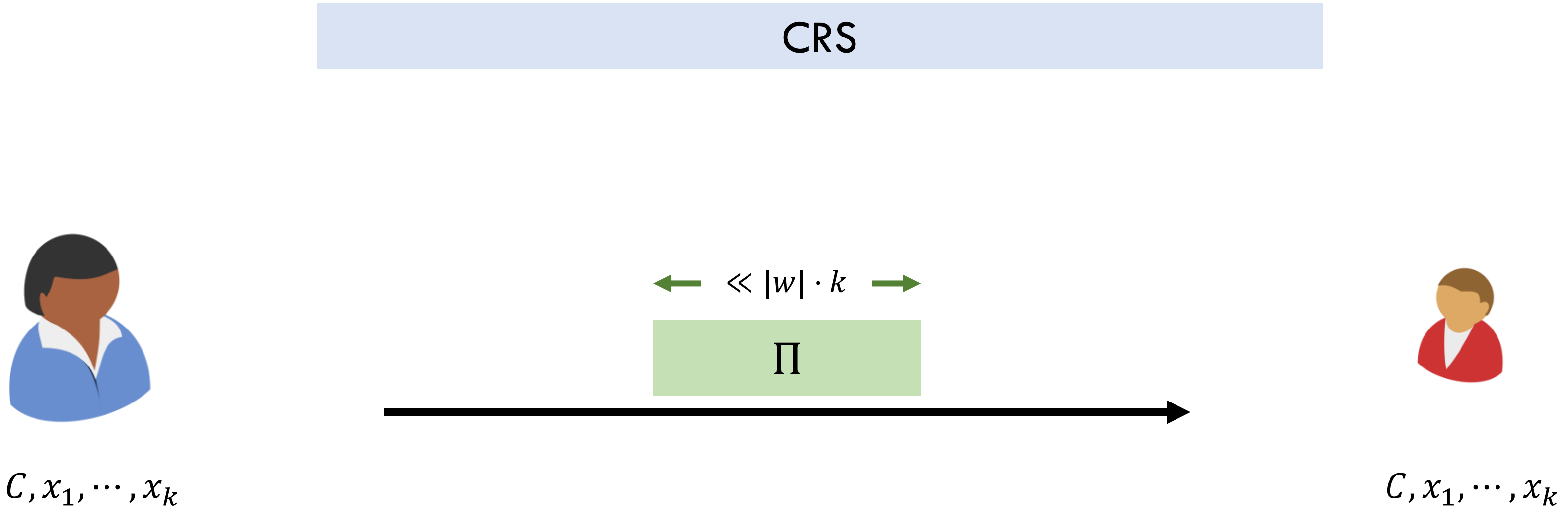

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SNARGs for Batch NP

CRS

$C, x_1, \cdots, x_k$

$\ll |w| \cdot k$

Π

$C, x_1, \cdots, x_k$

Verifier running time:
$k \cdot |x| + |\Pi|$

Π is publicly verifiable

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Prior Works

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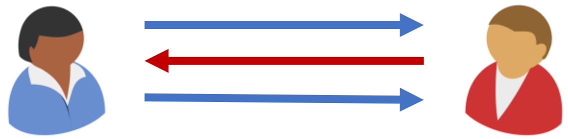
Interactive batch proofs for UP

[Reingold-Rothblum-Rothblum'16, Reingold-Rothblum-Rothblum'18, Rothblum-Rothblum'20]



UP – each statement has a unique witness.

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Succinct Non-interactive Arguments (SNARGs) for NP

[Micali’94, Damgård-Faust-Hazay’12, Bitansky-Canetti-Chiesa-Tromer’13, Bitanksy-Canetti-Chiesa-Goldwasser-Lin-Rubinstein-Tromer’16]

SNARGs

$|\Pi| \ll |w|$

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SNARGs

$|\Pi| \ll |w|$

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Designated Verifier SNARGs for Batch NP

[Brakerski-Holmgren-Kalai'17, Brakerski-Kalai'20]

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Designated Verifier from standard assumptions

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SNARGs for Batch NP from new non-standard assumption

[Kalai-Paneth-Yang'19]

Falsifiable assumption on groups with bilinear maps.

Do there exists SNARGs for Batch NP based on standard assumptions?

[Gentry-Wichs'11] lower bound doesn't apply.

Our Result

Theorem

There exists SNARGs for Batch NP

Assuming QR + sub-exp DDH

$$|\Pi| = \tilde{O}(|C| + \sqrt{k|C|})$$

[C-Jain-Jin'21a]

QR – Quadratic residuosity, LWE – Learning with Error, DDH – Decisional Diffie-Hellman

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Our Result

Theorem

There exists SNARGs for Batch NP

Assuming LWE

$$|\Pi| = \mathrm{poly}(\log k, |C|)$$

[C-Jain-Jin'21b]

Assuming QR + sub-exp DDH

$$|\Pi| = \tilde{O}(|C| + \sqrt{k|C|})$$

[C-Jain-Jin'21a]

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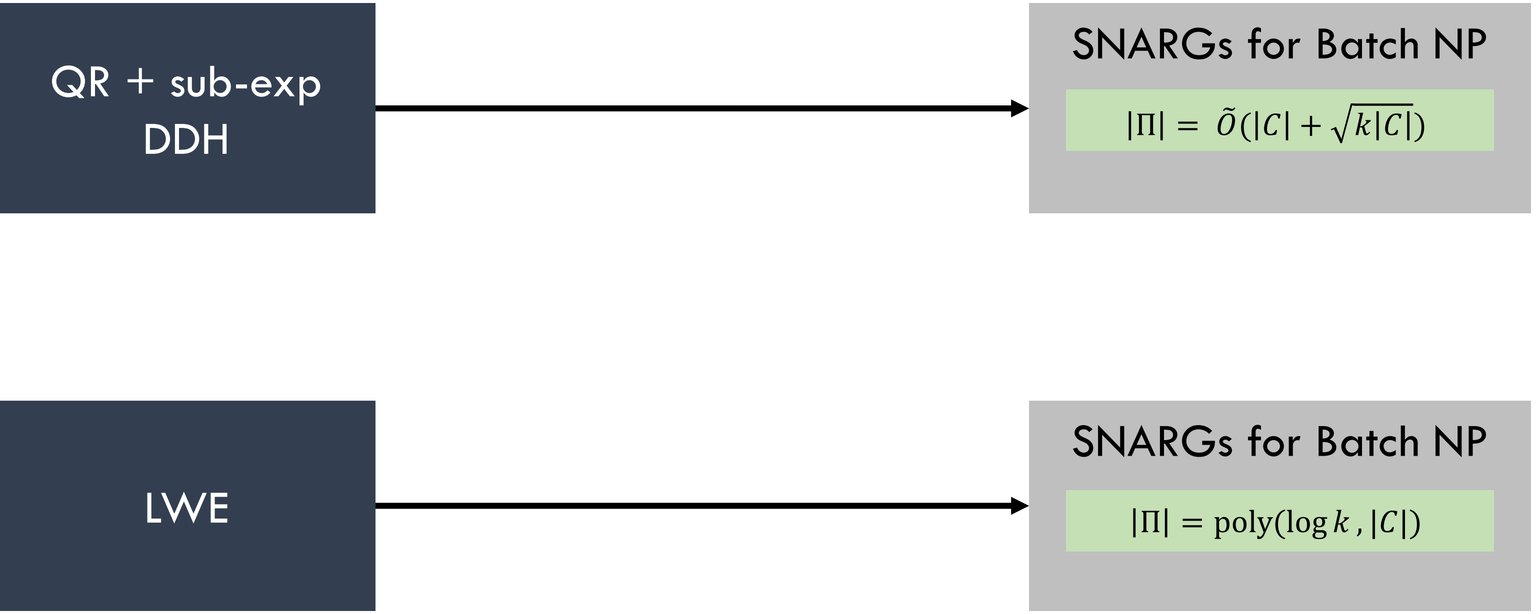

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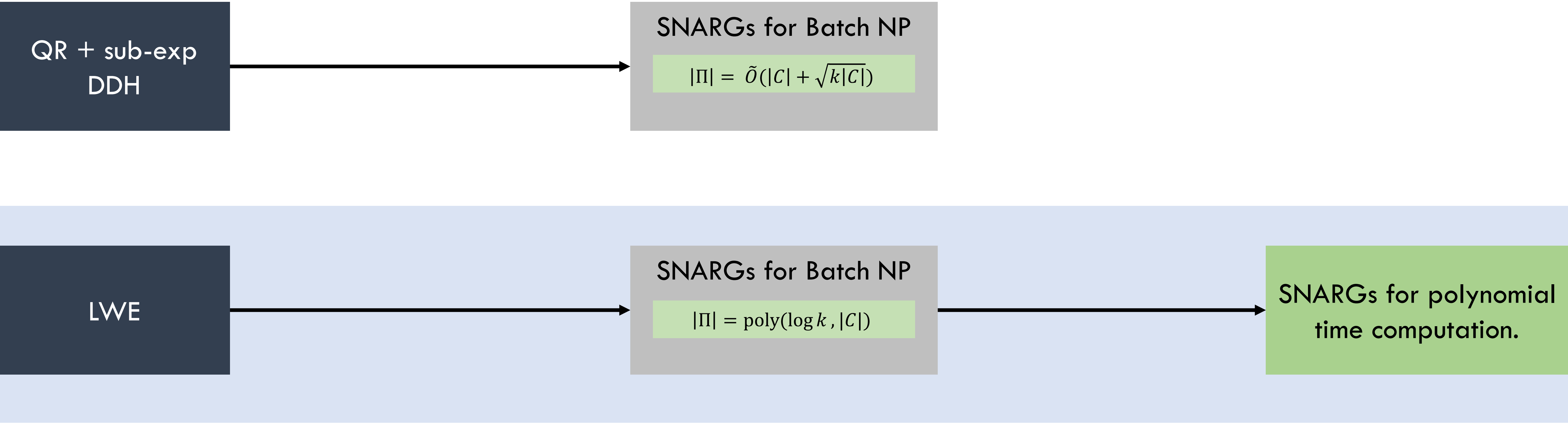

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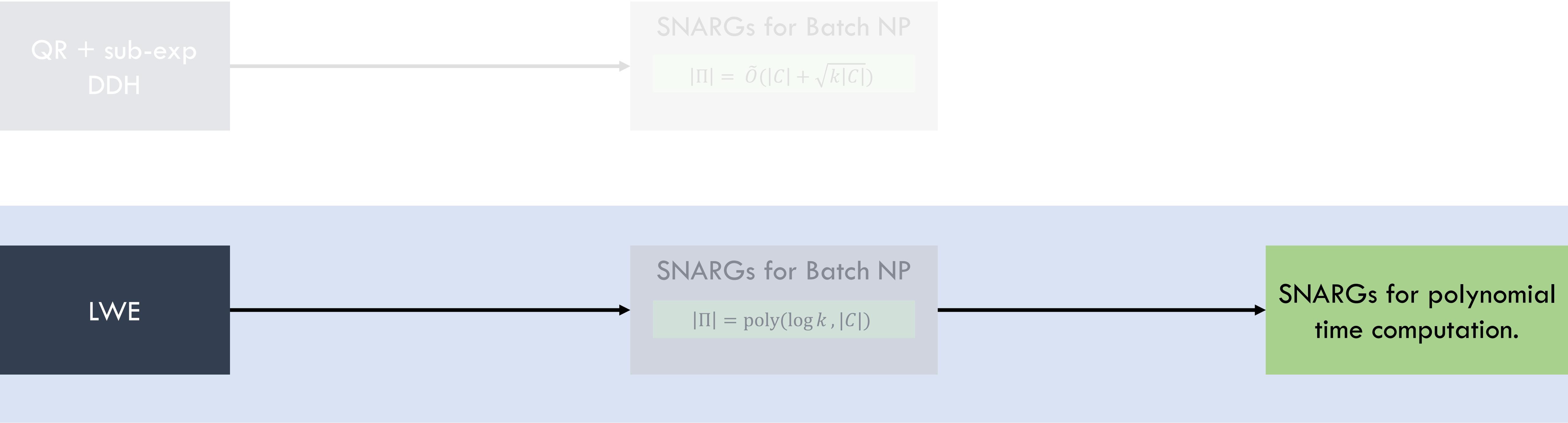

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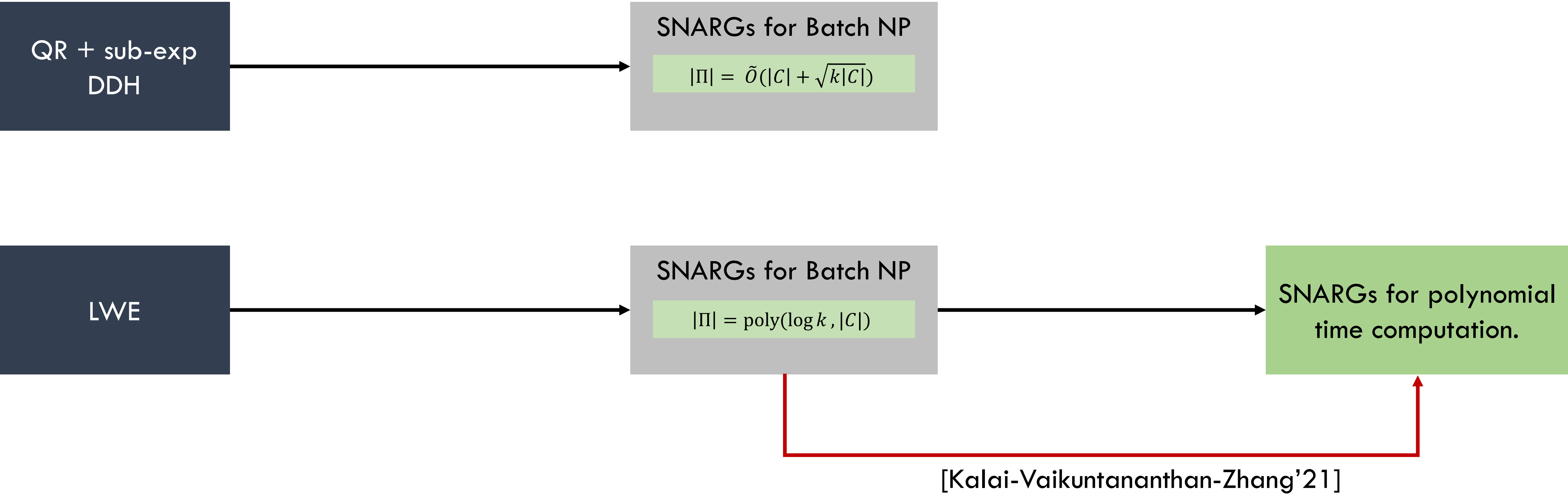

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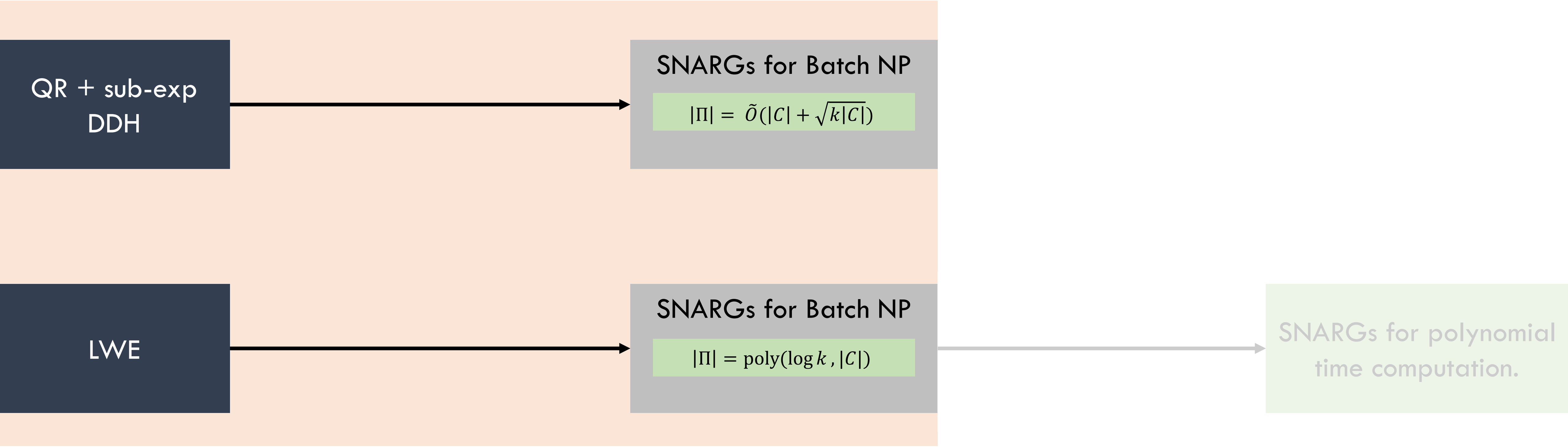
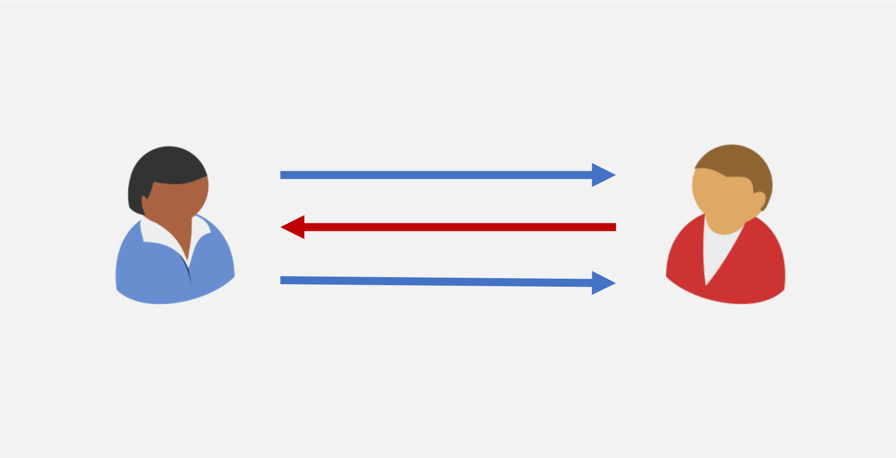
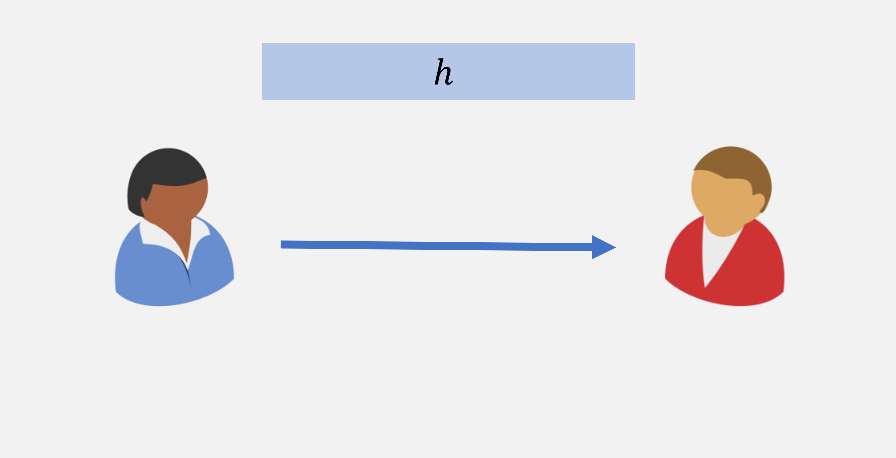


SNARGs for Batch NP

Fiat-Shamir (FS) Methodology



Instantiate with CIH



No known interactive proof for batch NP

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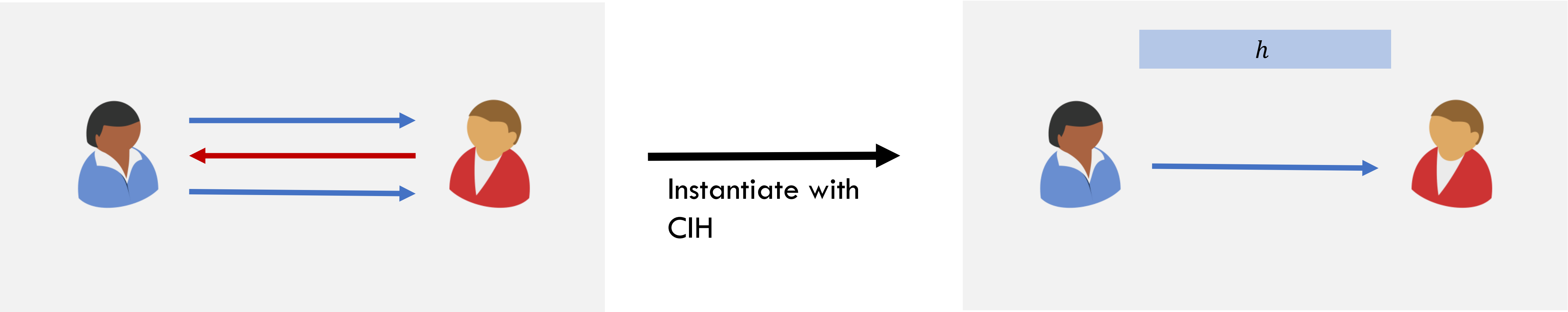

A Different Starting Point for Fiat-Shamir Methodology

No known interactive proof for batch NP

Dual-Mode Interactive Batch Arguments

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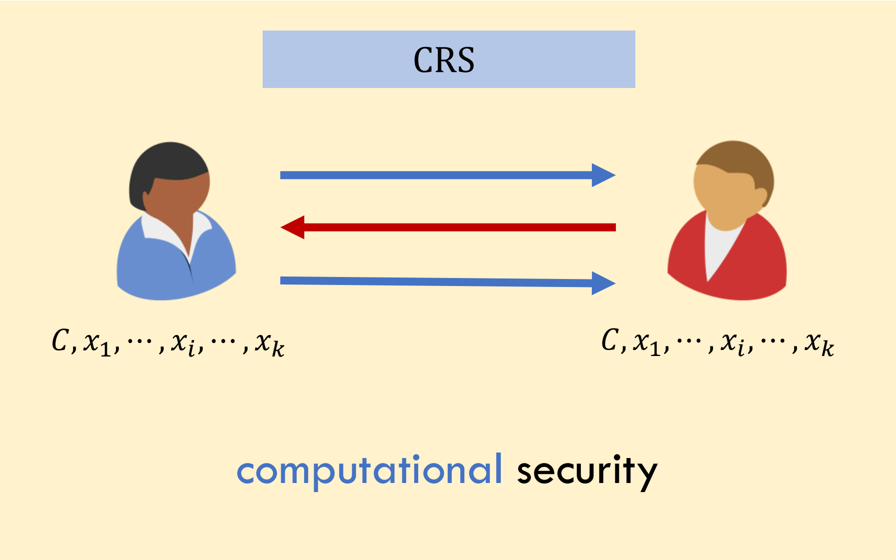
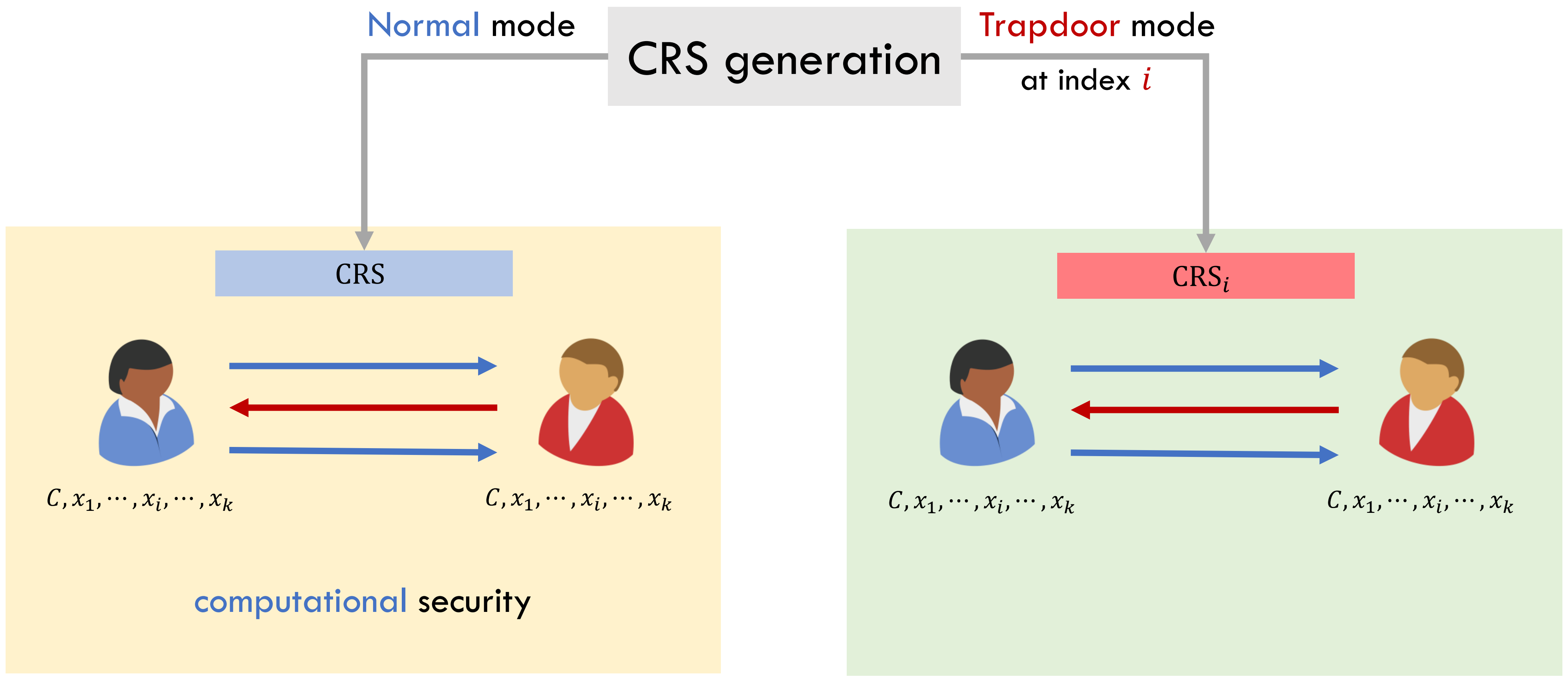


Dual-Mode Interactive Batch Arguments



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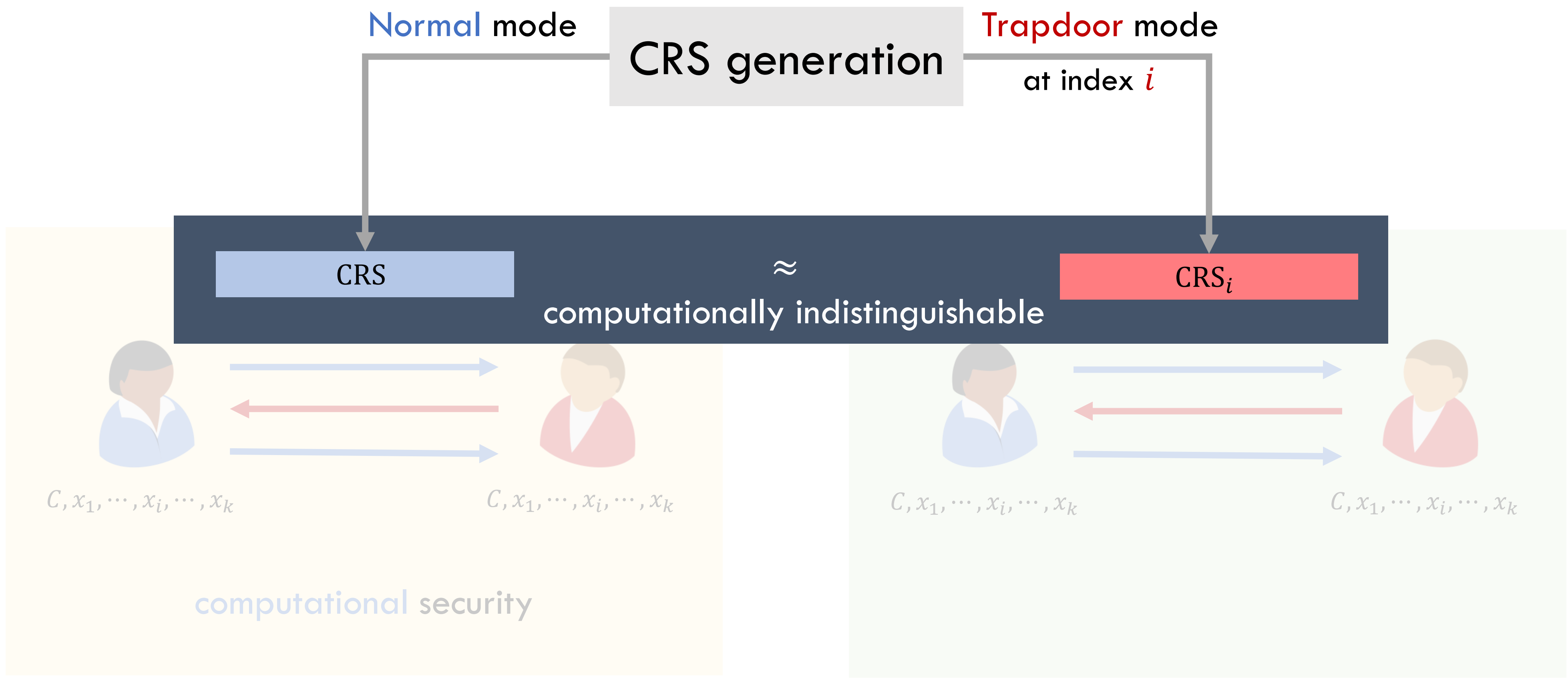

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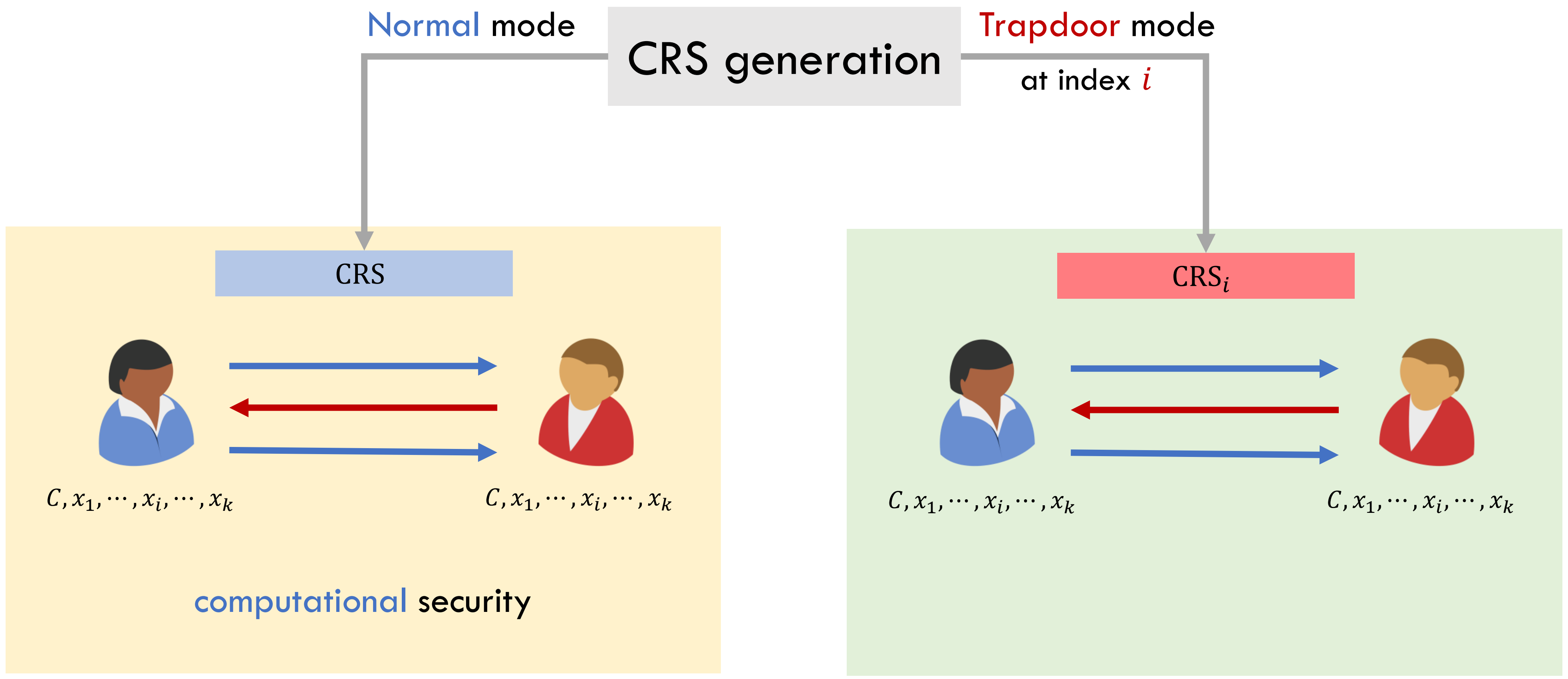

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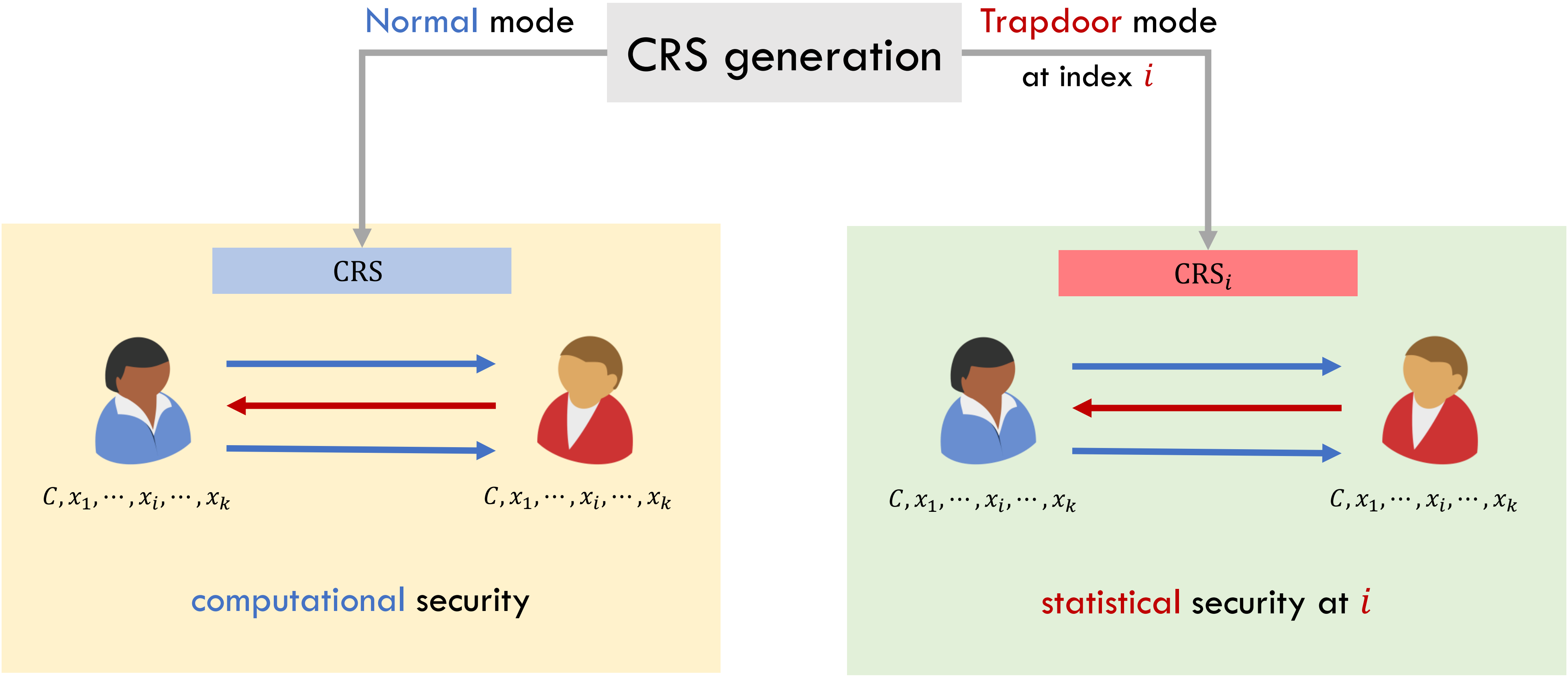

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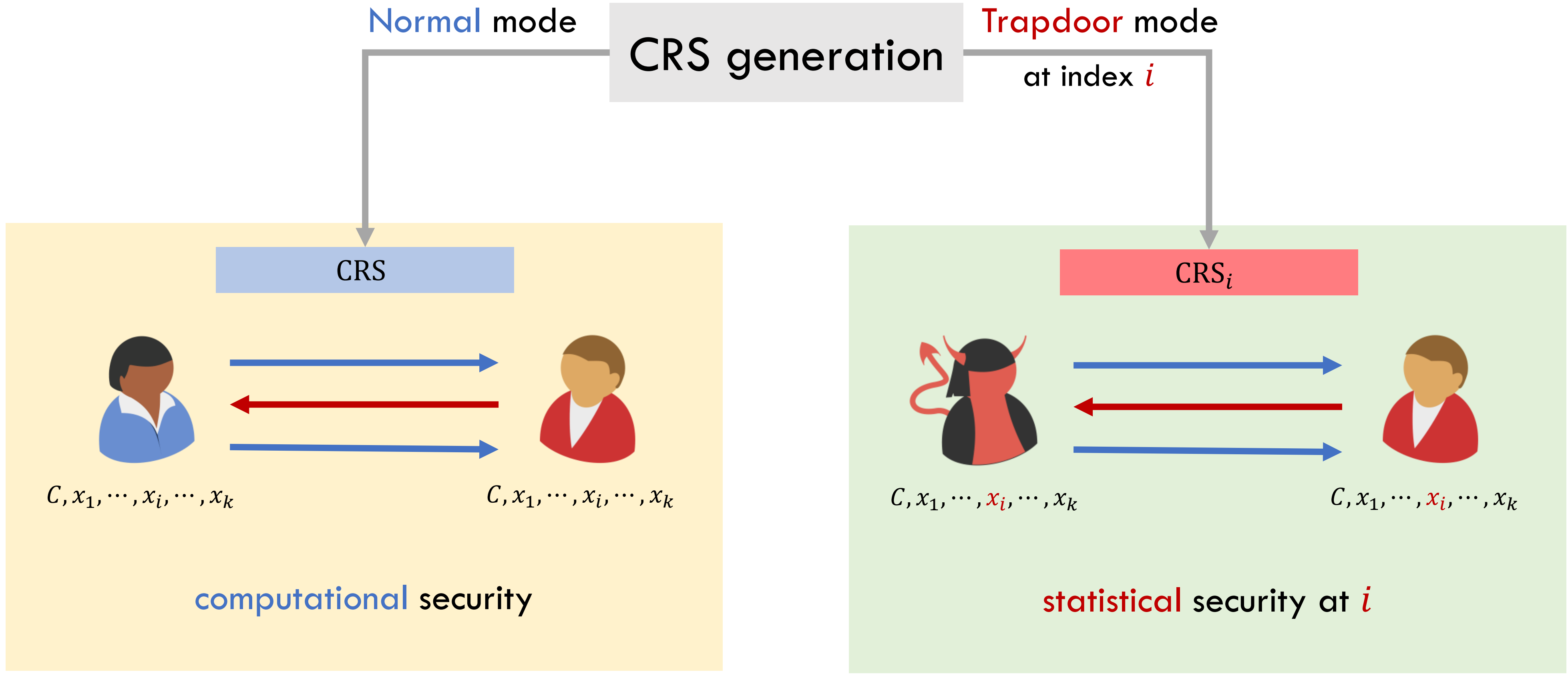
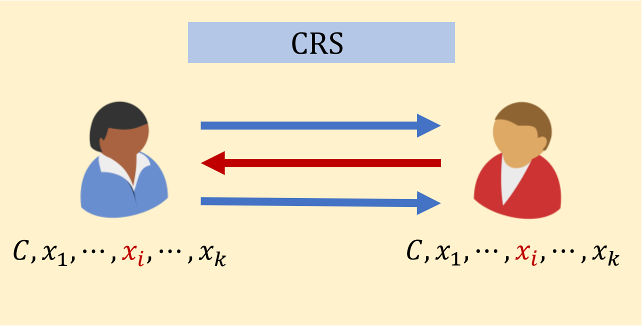


Even unbounded cannot make accept if $(C, x_i) \notin \mathrm{SAT}$

Security Intuition



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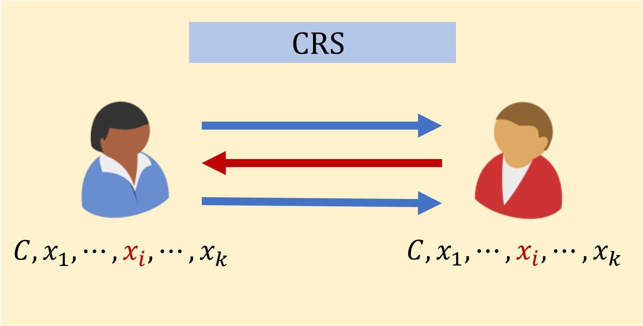
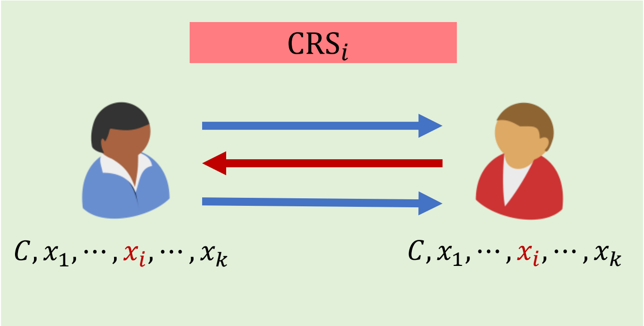


Switch to trapdoor mode at i



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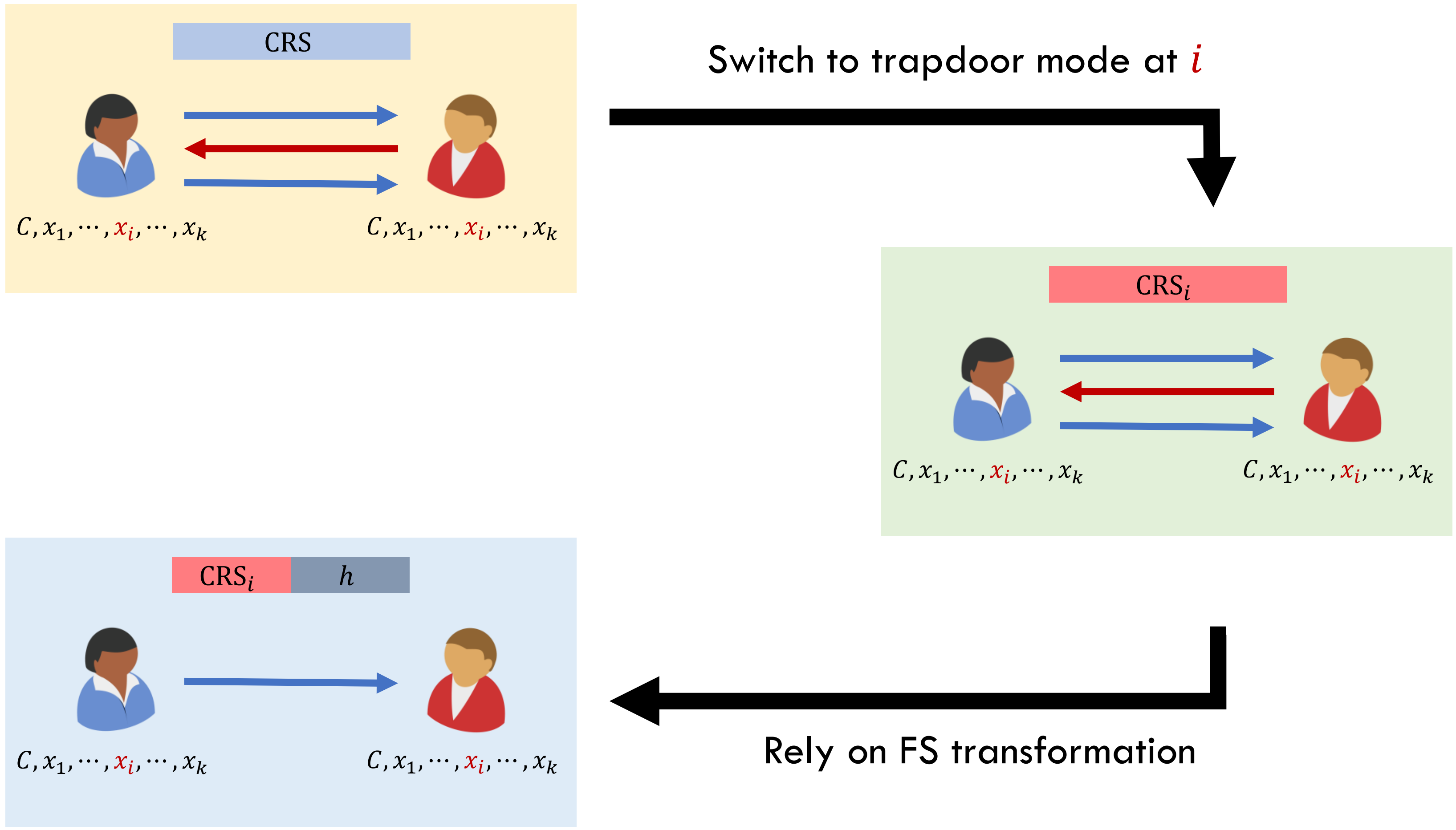
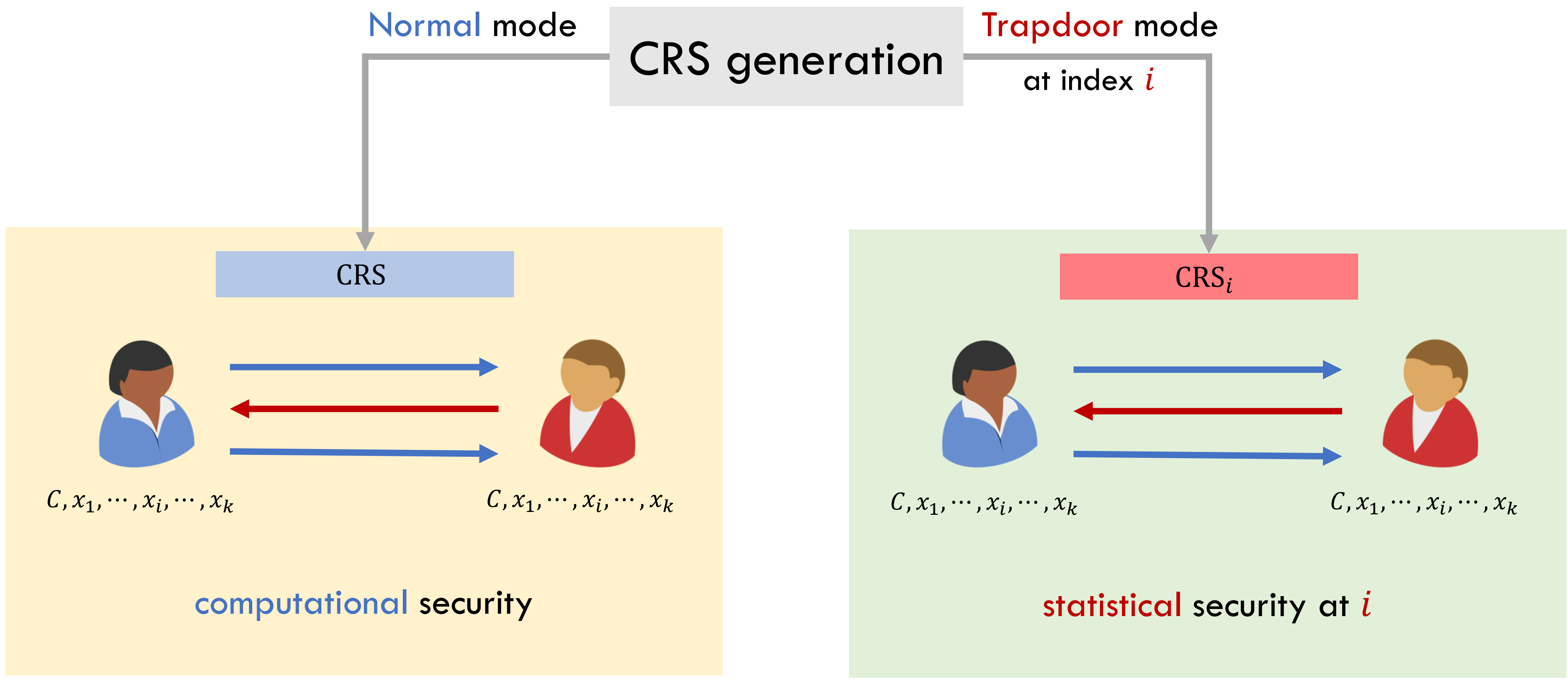
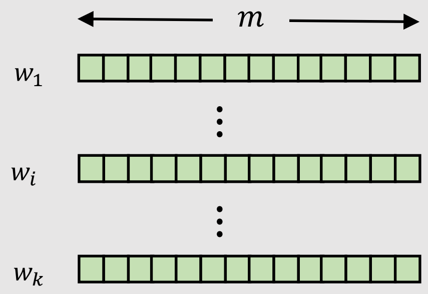


Dual-Mode Interactive Batch Arguments



Dual Mode Batch Argument

Protocol Template



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Dual Mode Batch Argument

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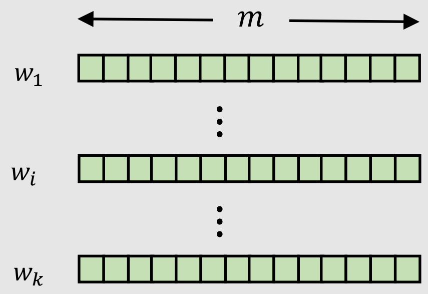

commitment key K

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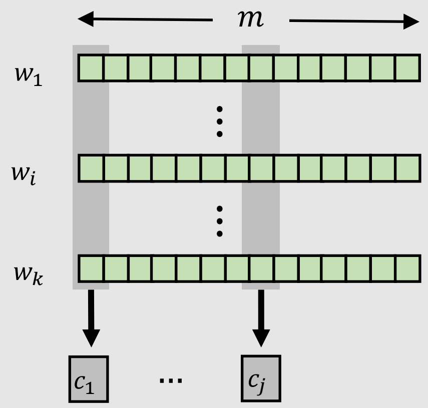

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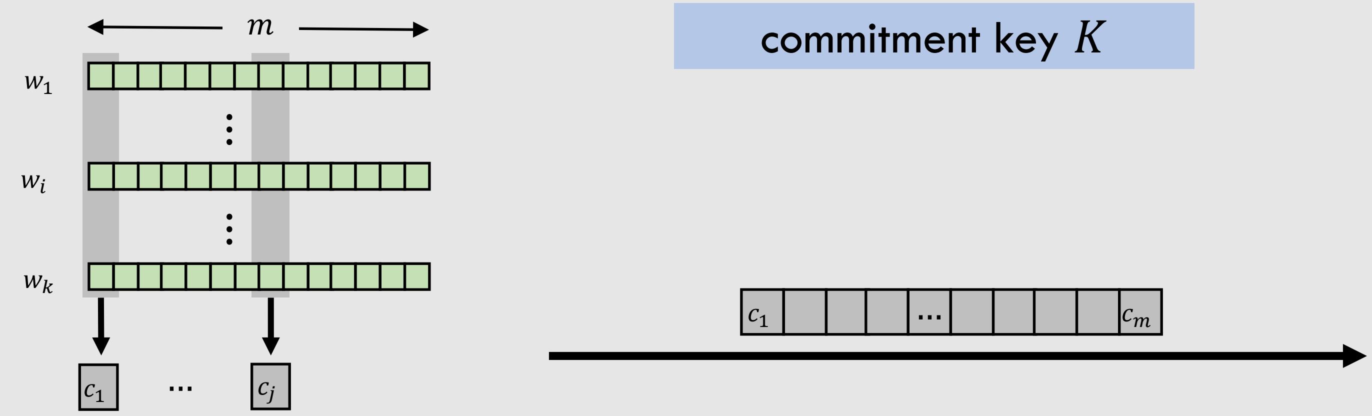

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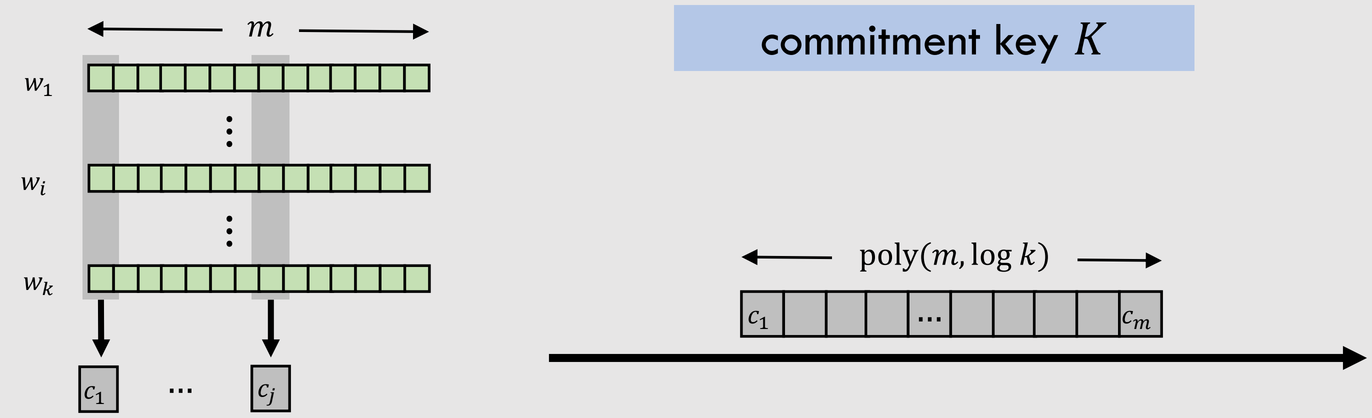

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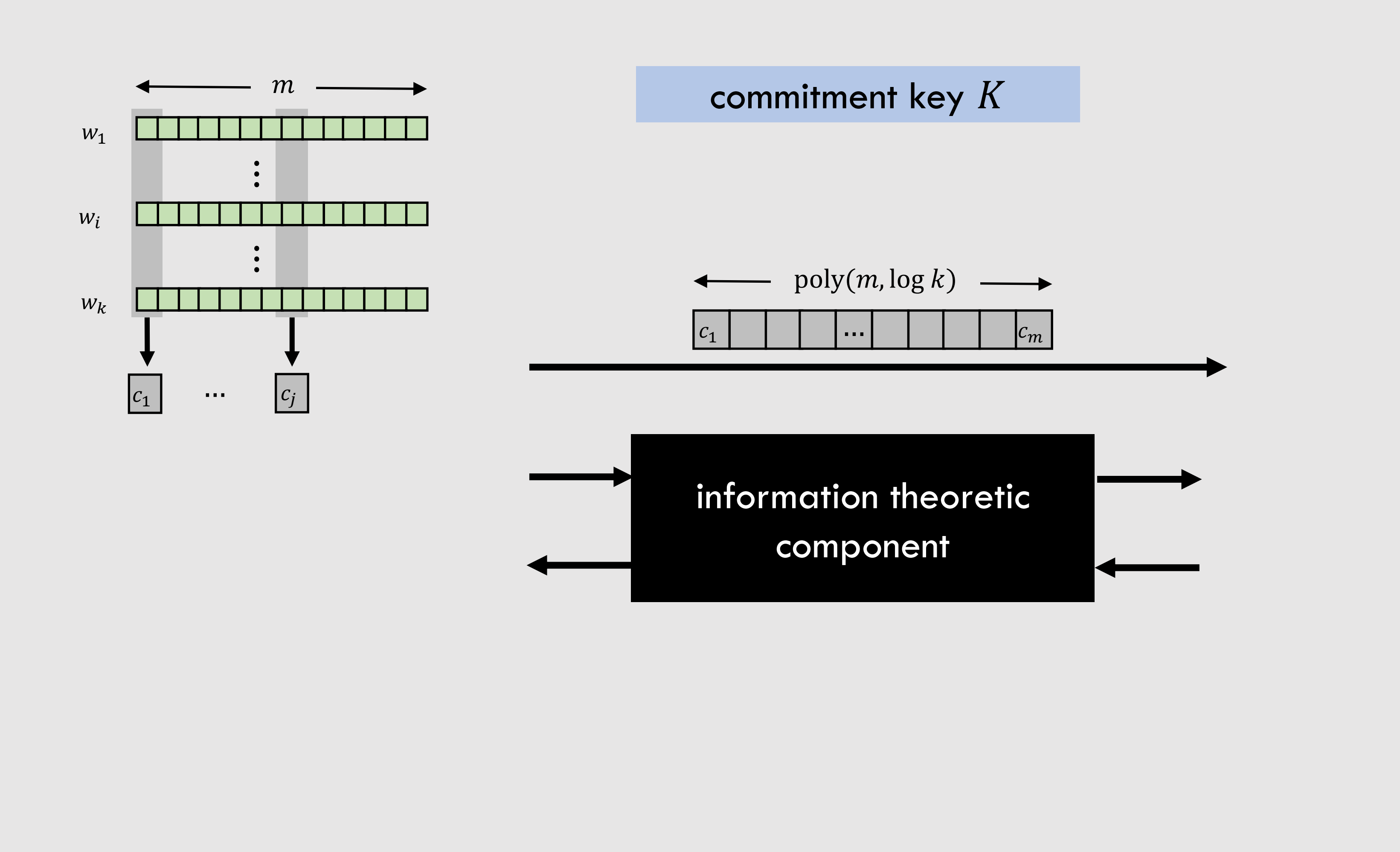

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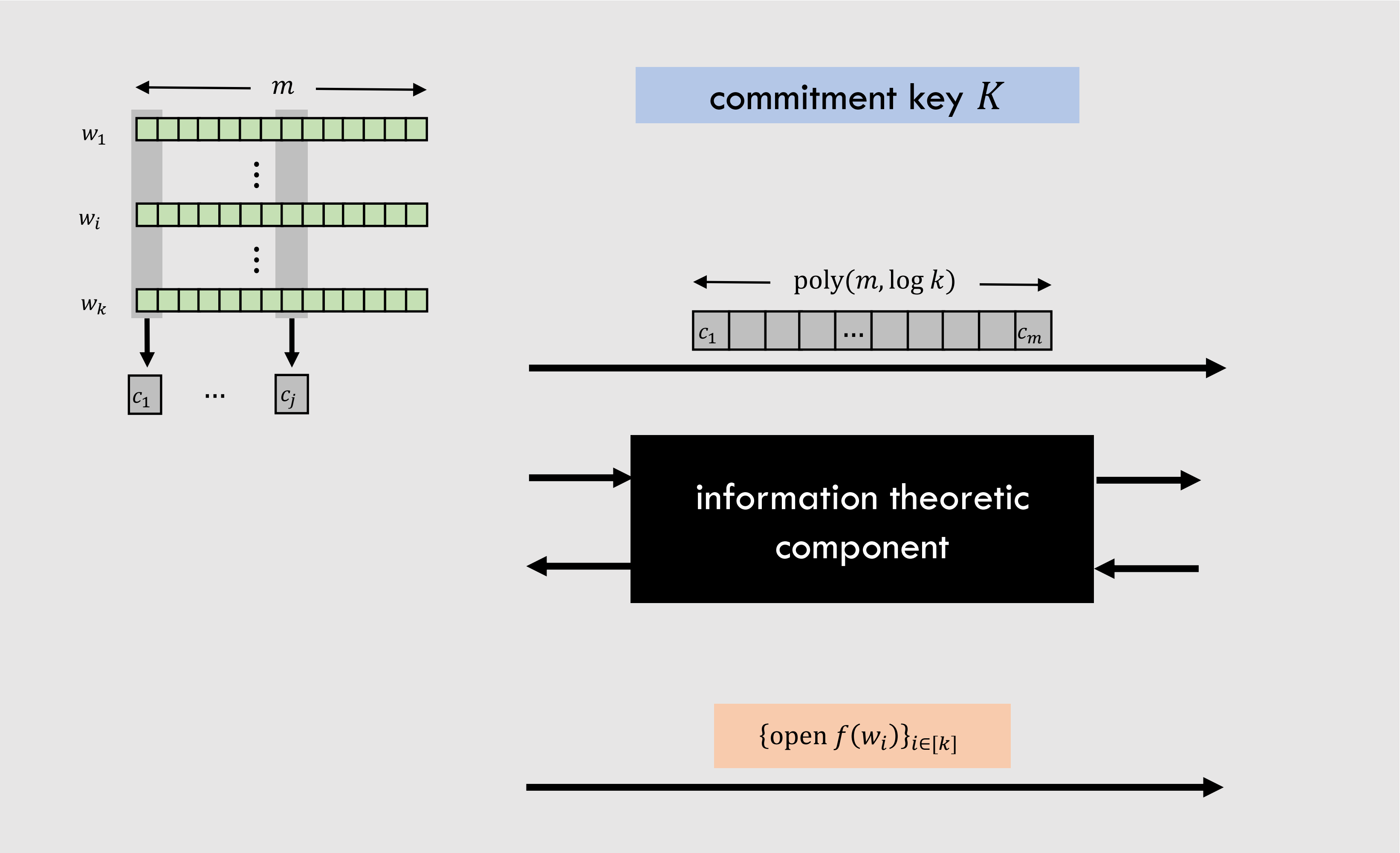

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f determined by the information theoretic component.

Dual Mode Batch Argument

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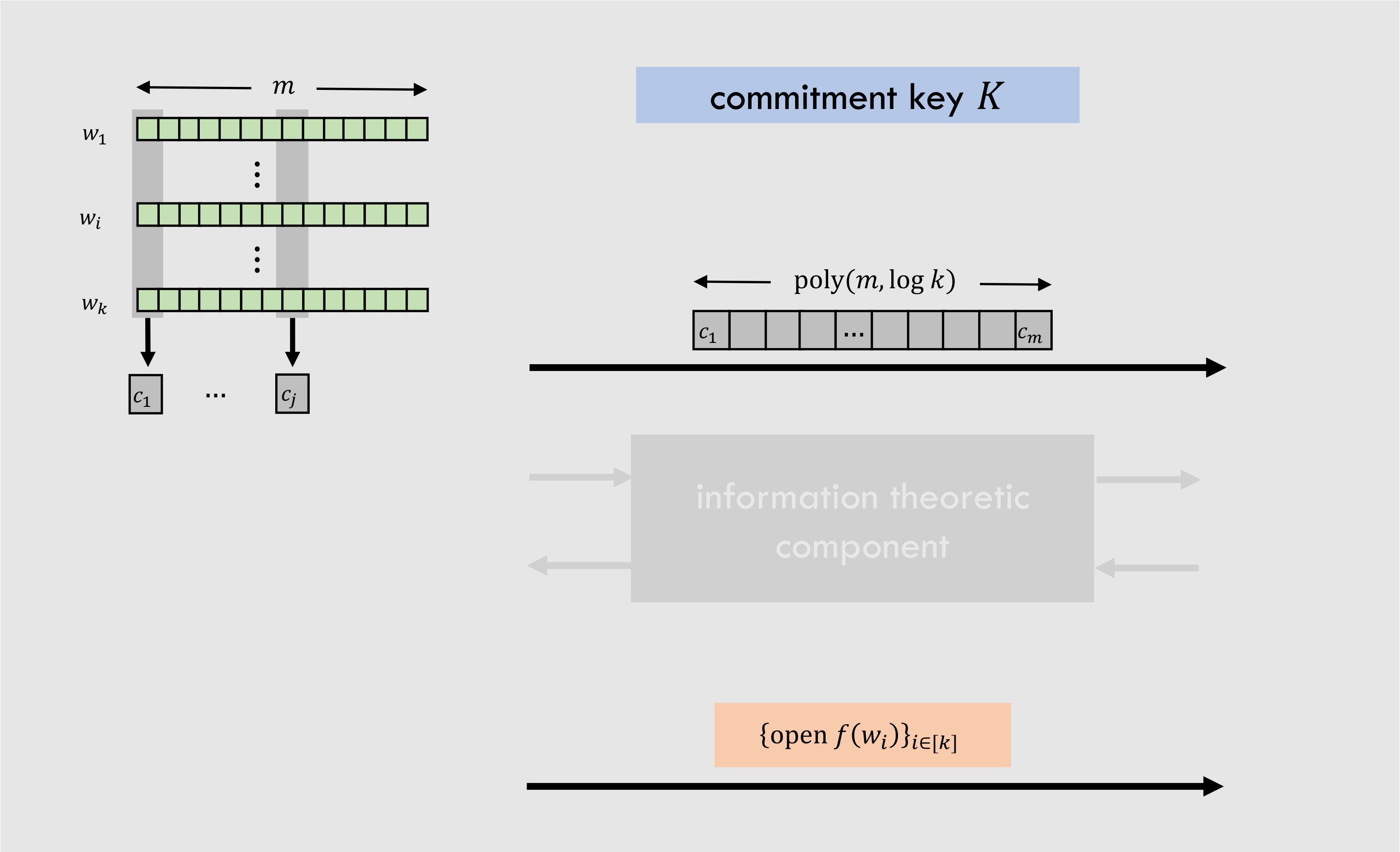

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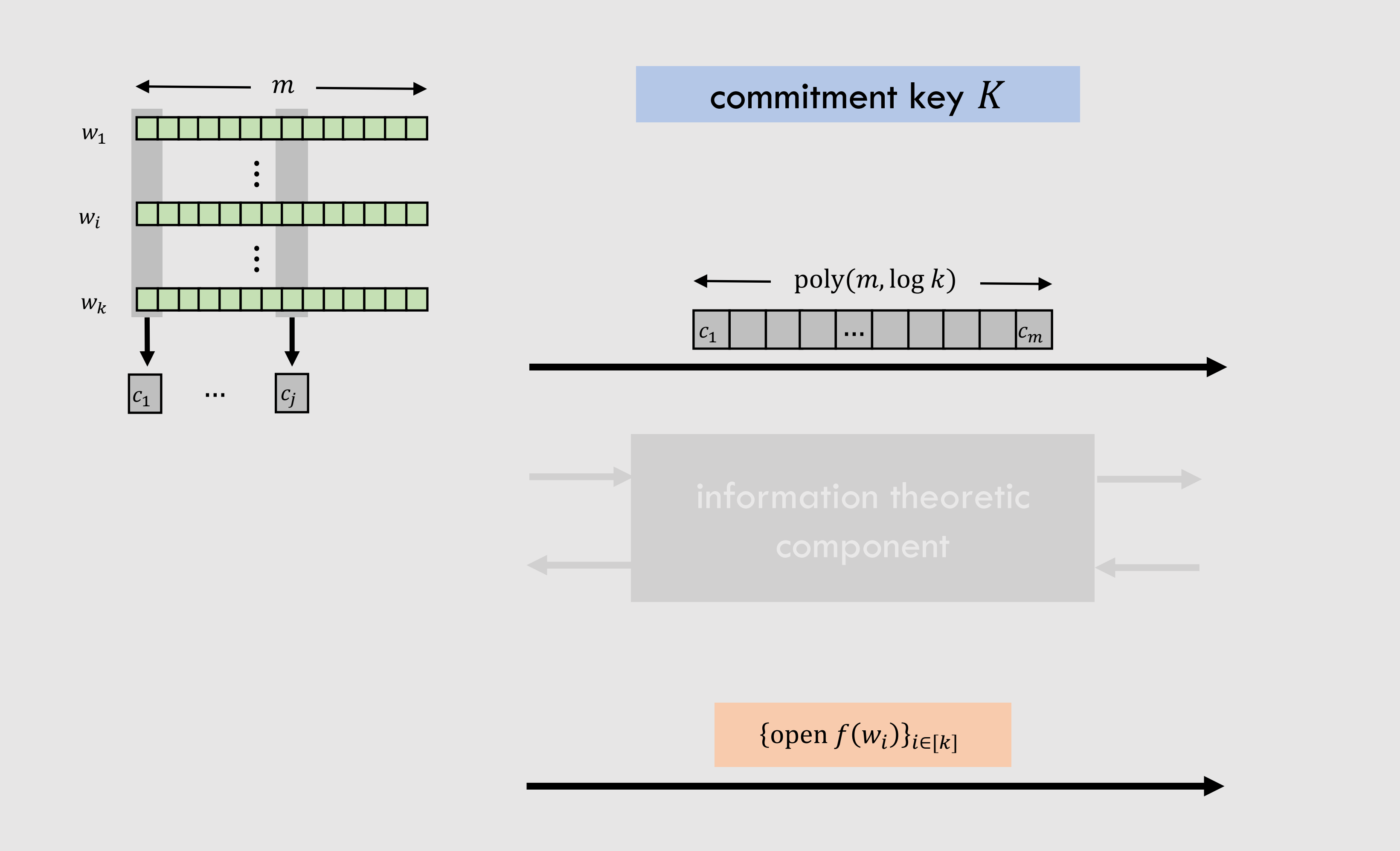

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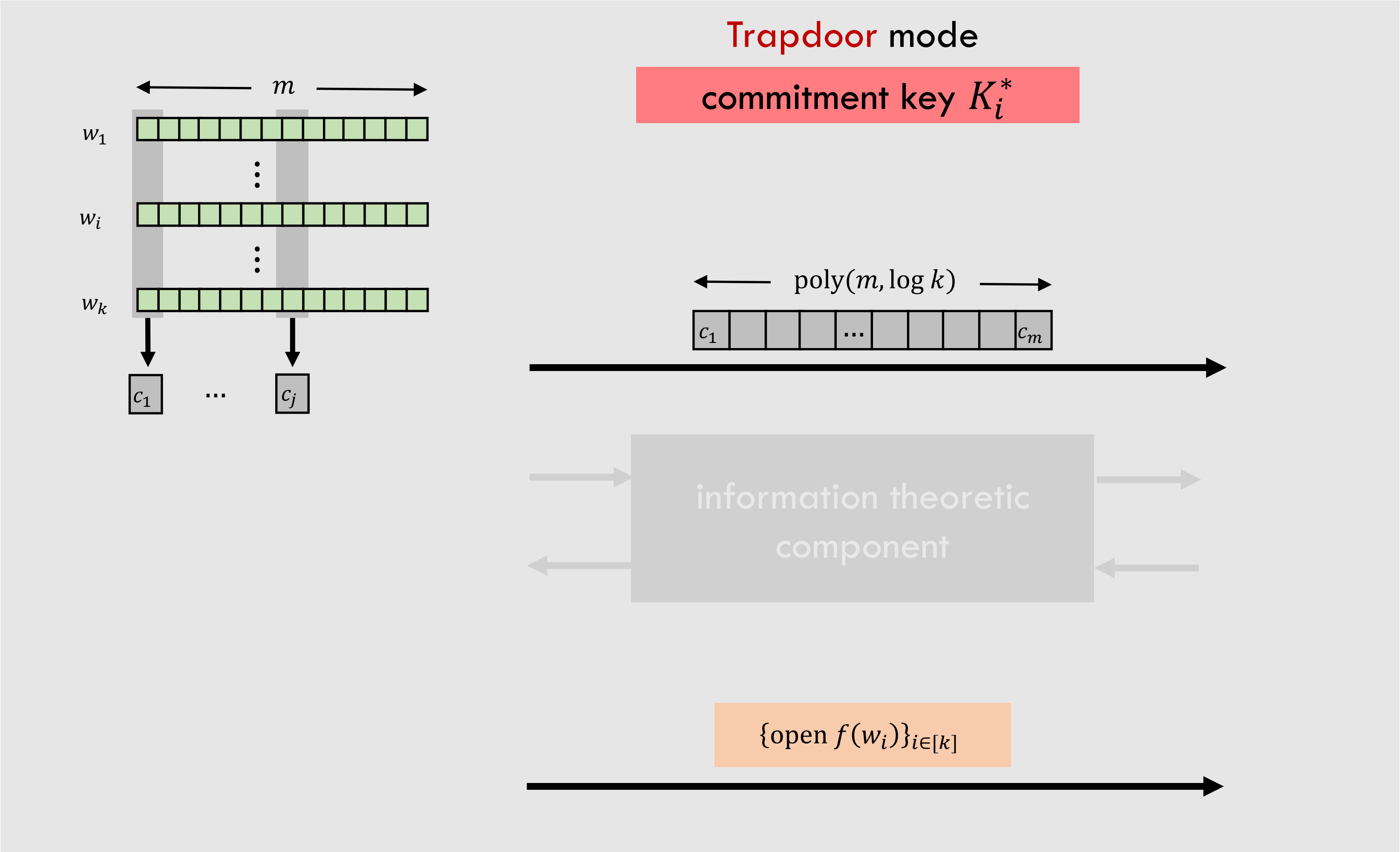
Somewhere Statistically Binding (SSB) Commitment Scheme [Hubáček -Wichs'15]

Dual Mode Batch Argument

Protocol Template



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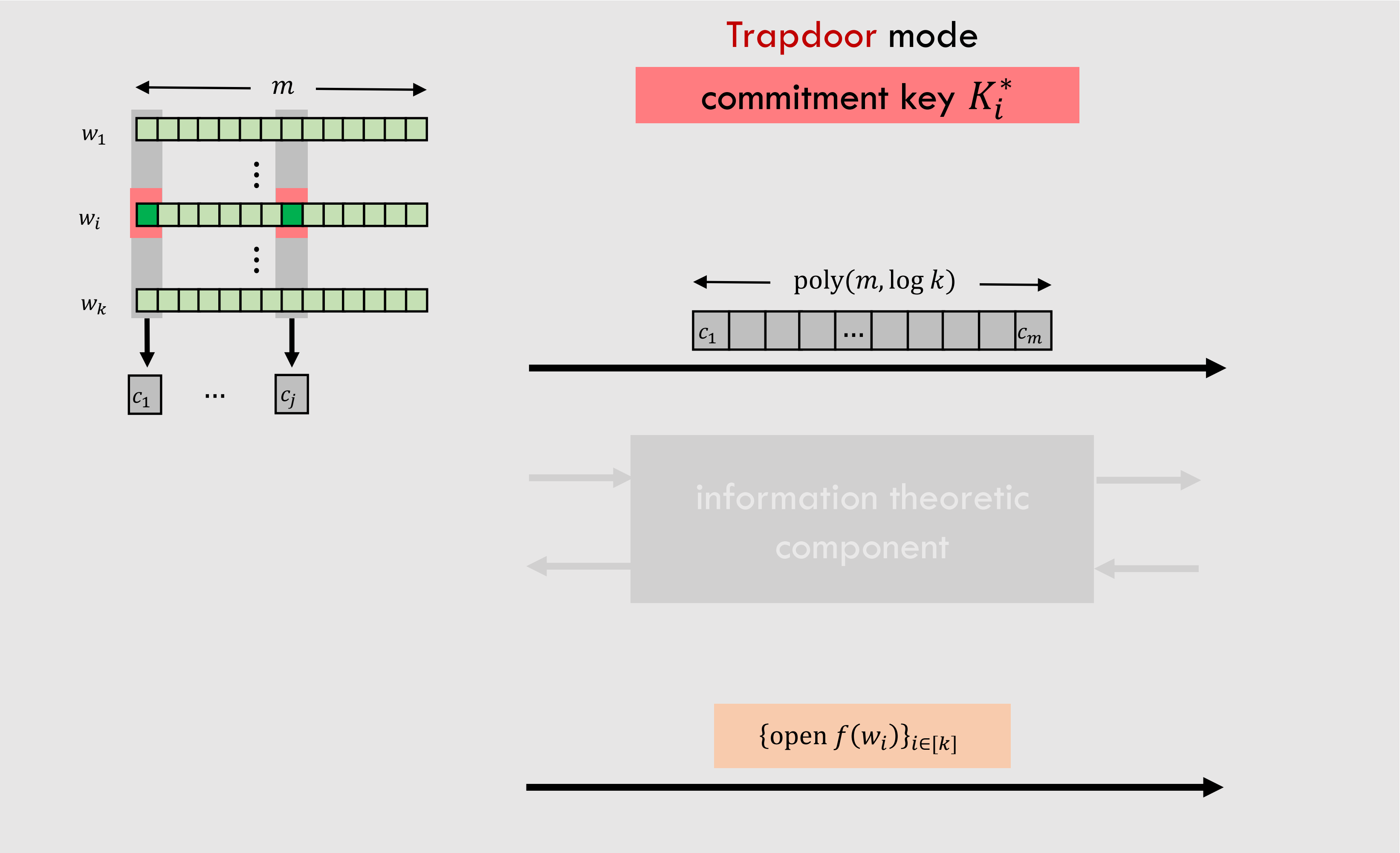

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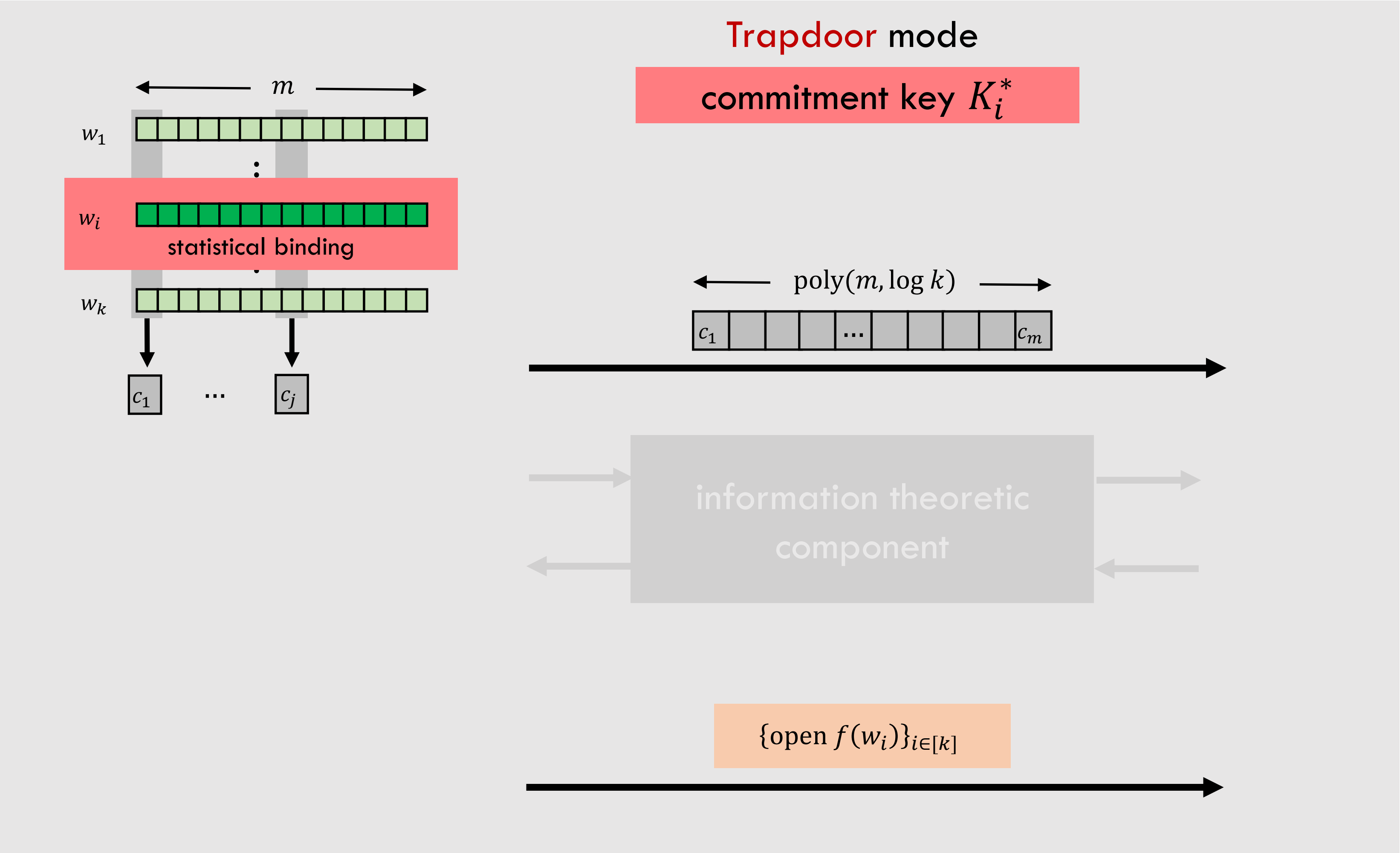

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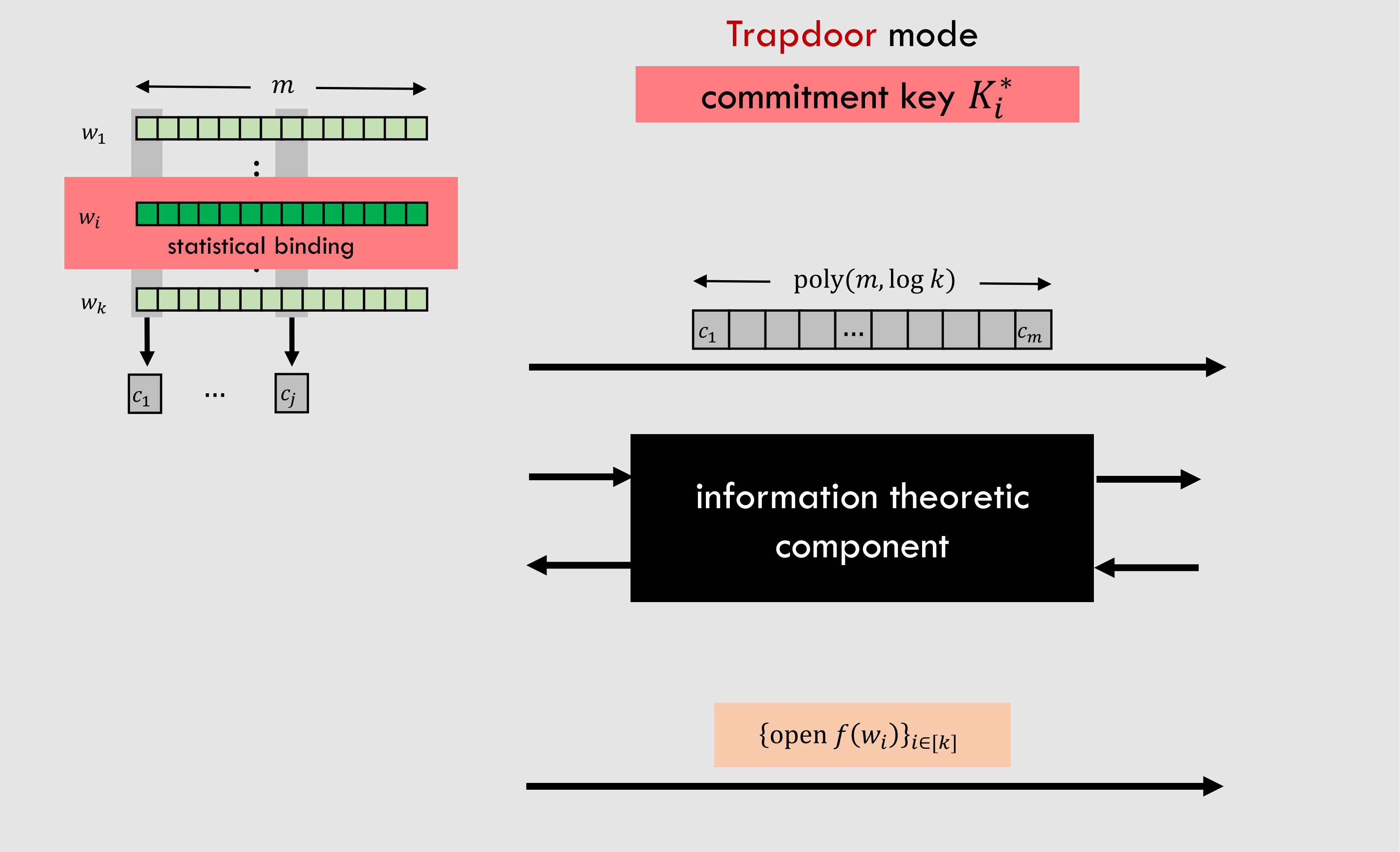

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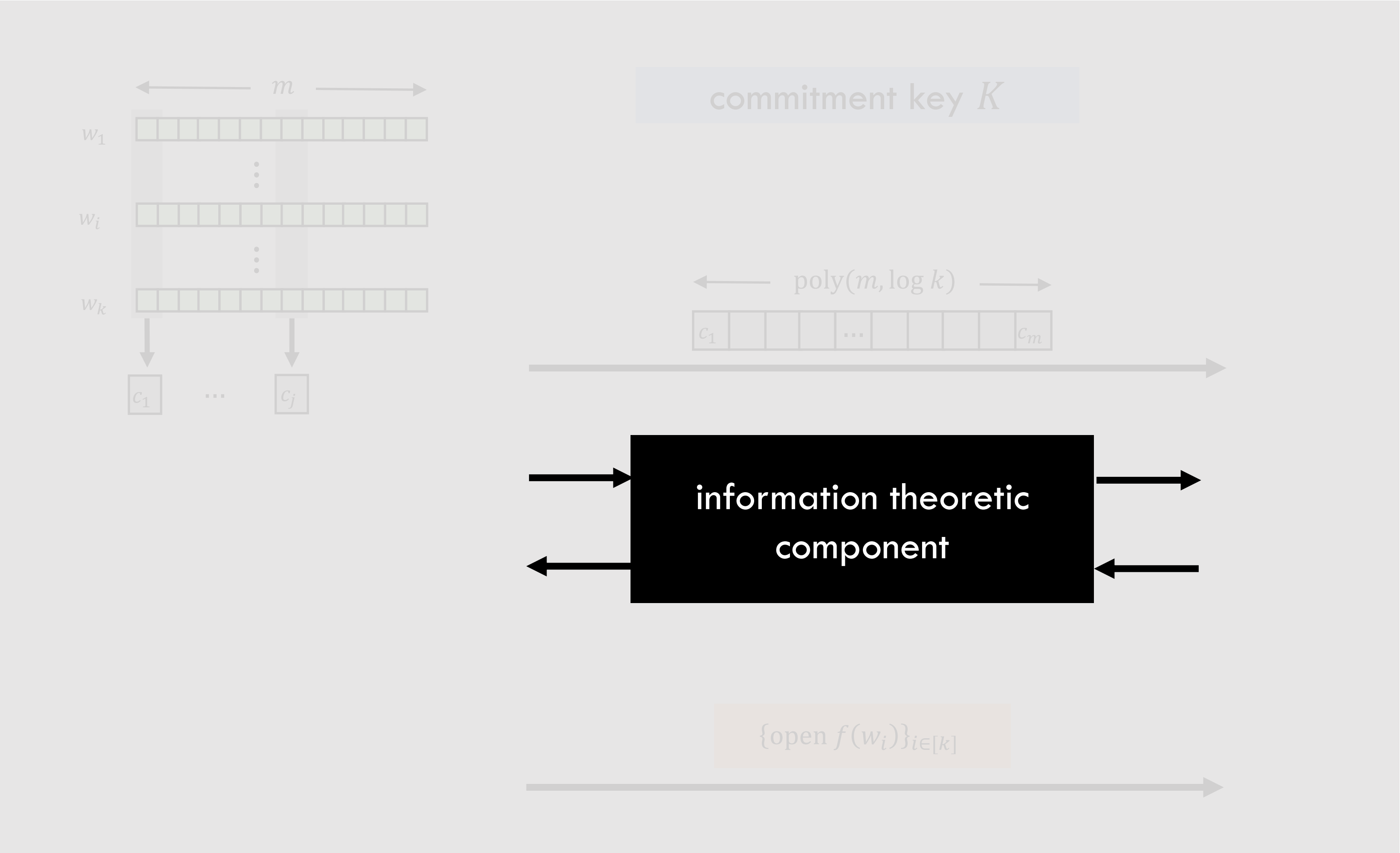

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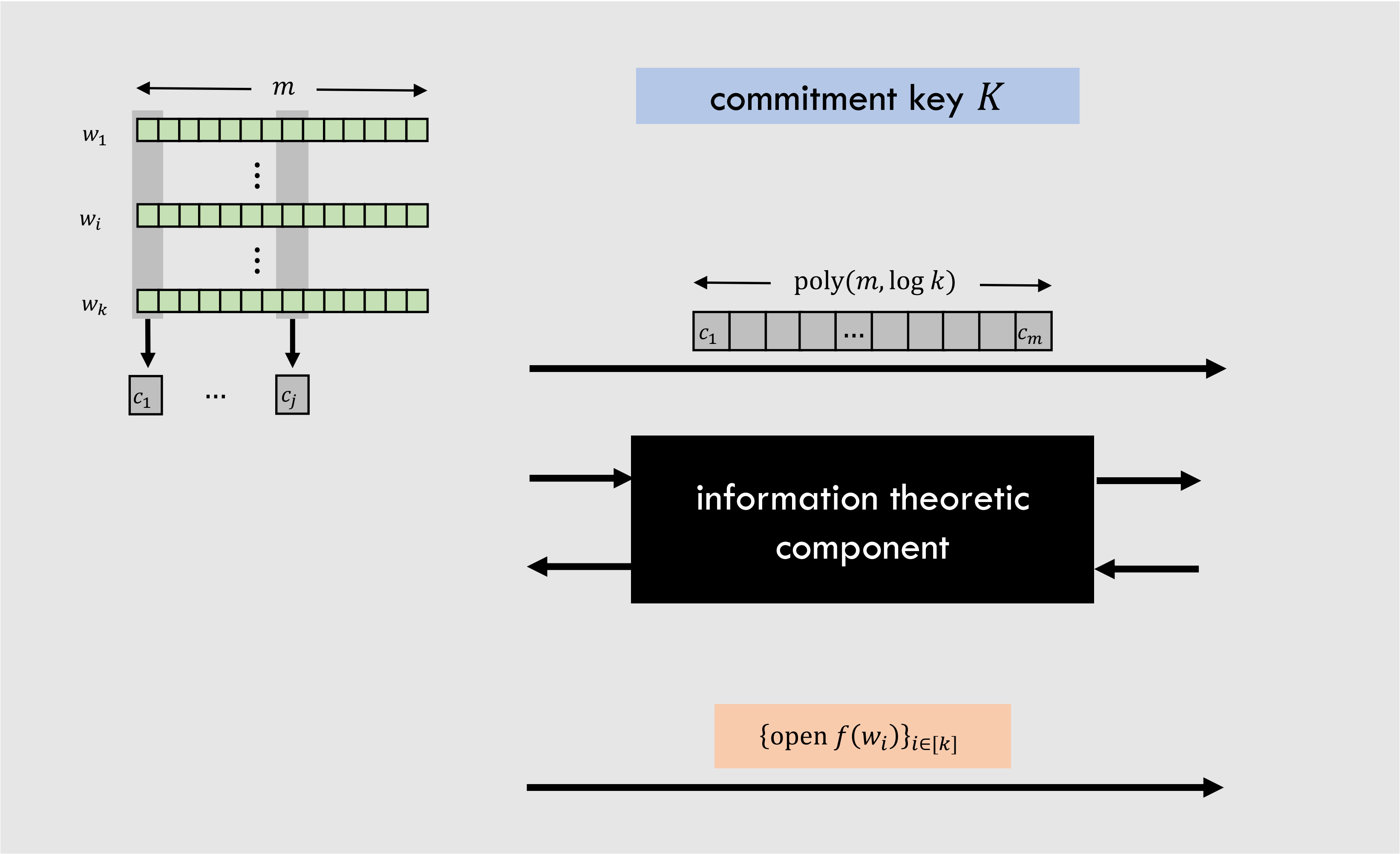
Somewhere Statistically Binding (SSB) Commitment Scheme

Needs to be Fiat-Shamir friendly.

Based on LWE/sub-exp DDH

Dual Mode Batch Argument

Protocol Template



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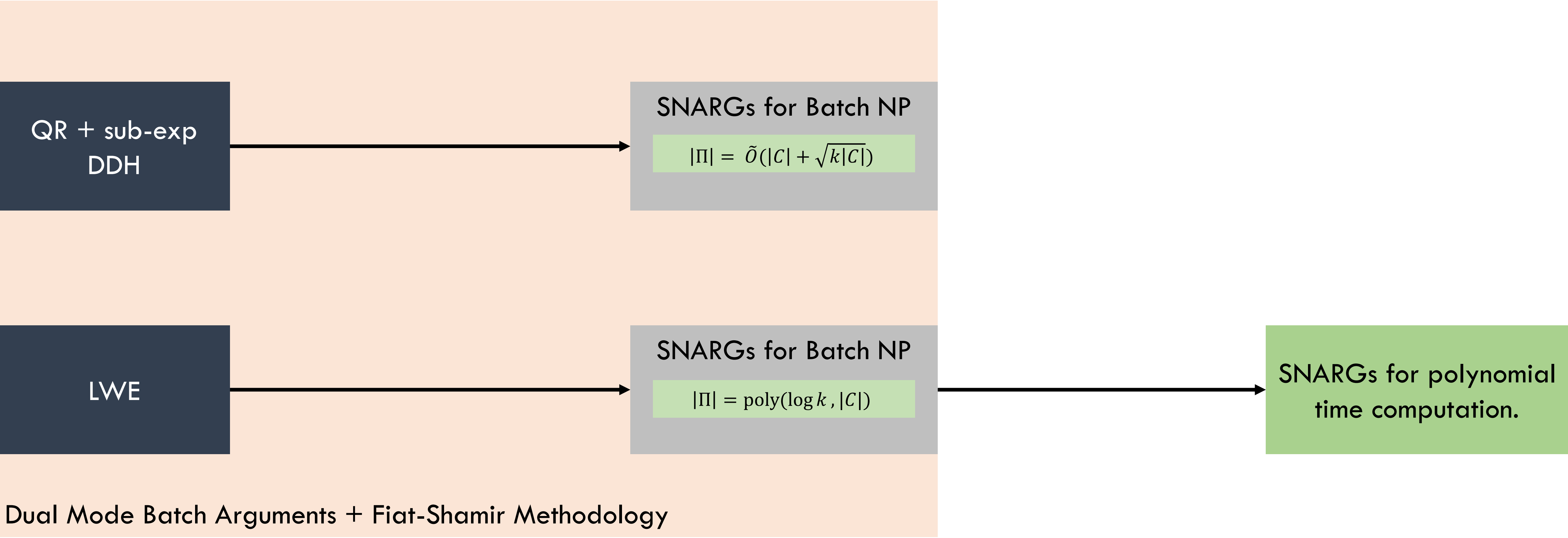
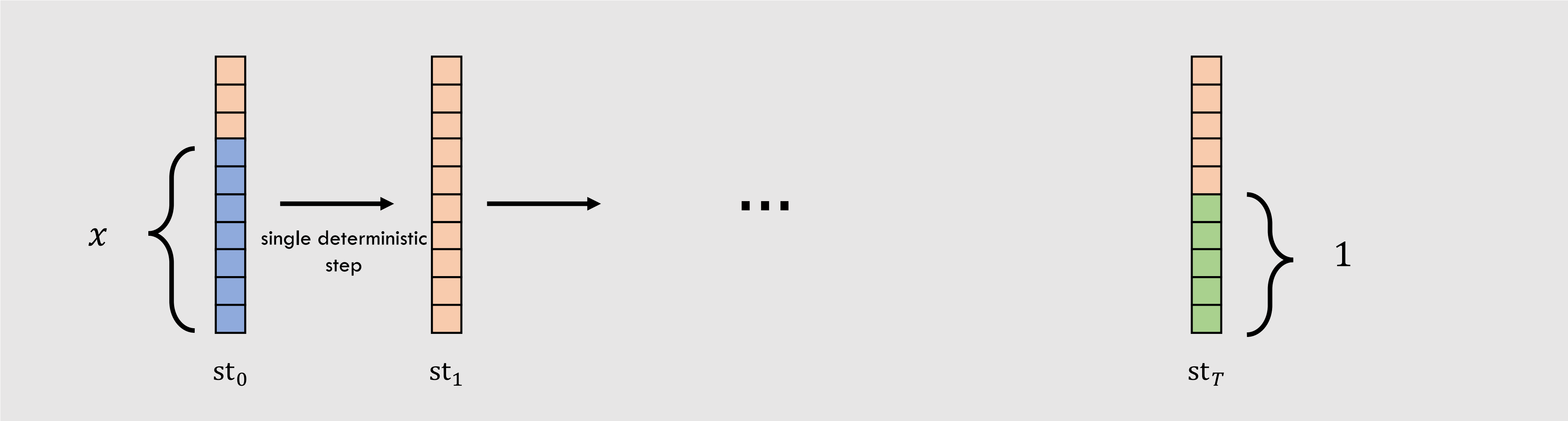
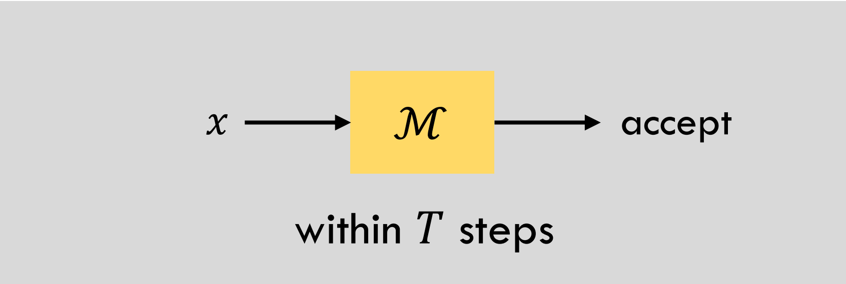


Delegation via Batching [Reingold-Rothblum-Rothblum'16]



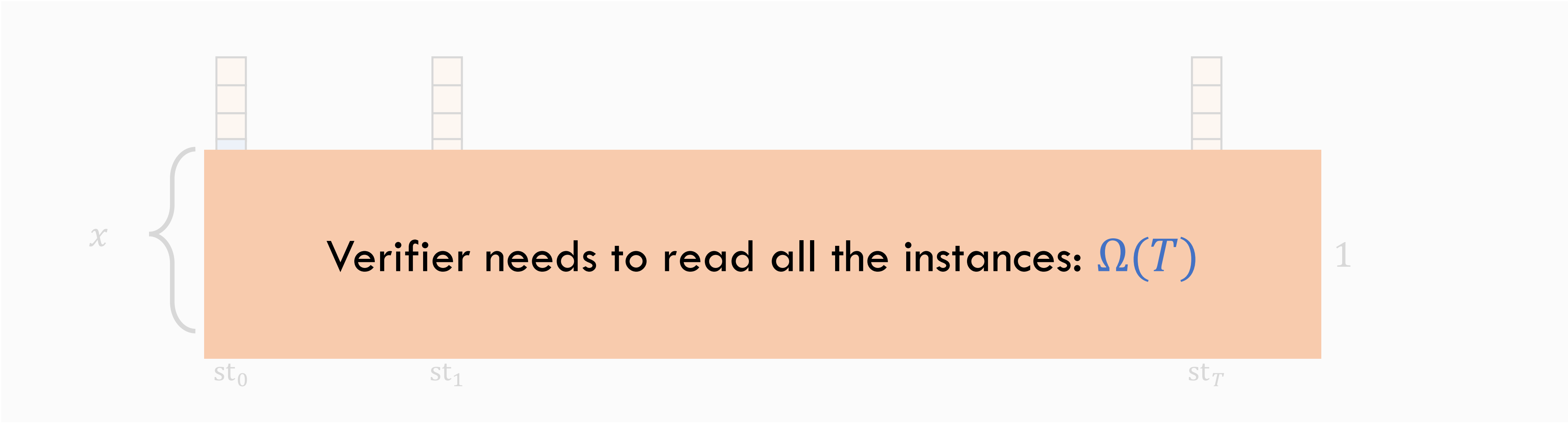


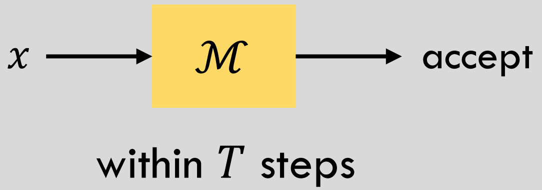
Prove for every $i \in [0, \ldots, T-1]$

$$\mathrm{st}_i \to \mathrm{st}_{i+1}$$

is the correct transition.

Delegation via Batching [Reingold-Rothblum-Rothblum'16]





Prove for every $i \in [0, \dots, T-1]$

$$\mathrm{st}_i \to \mathrm{st}_{i+1}$$

is the correct transition.

SNARGs for Batch Index

$$L_C = \{i \mid \exists w \; s.t. \; C(i, w) = 1\}$$

$$\forall i \in [k], i \in L_C$$

C, k

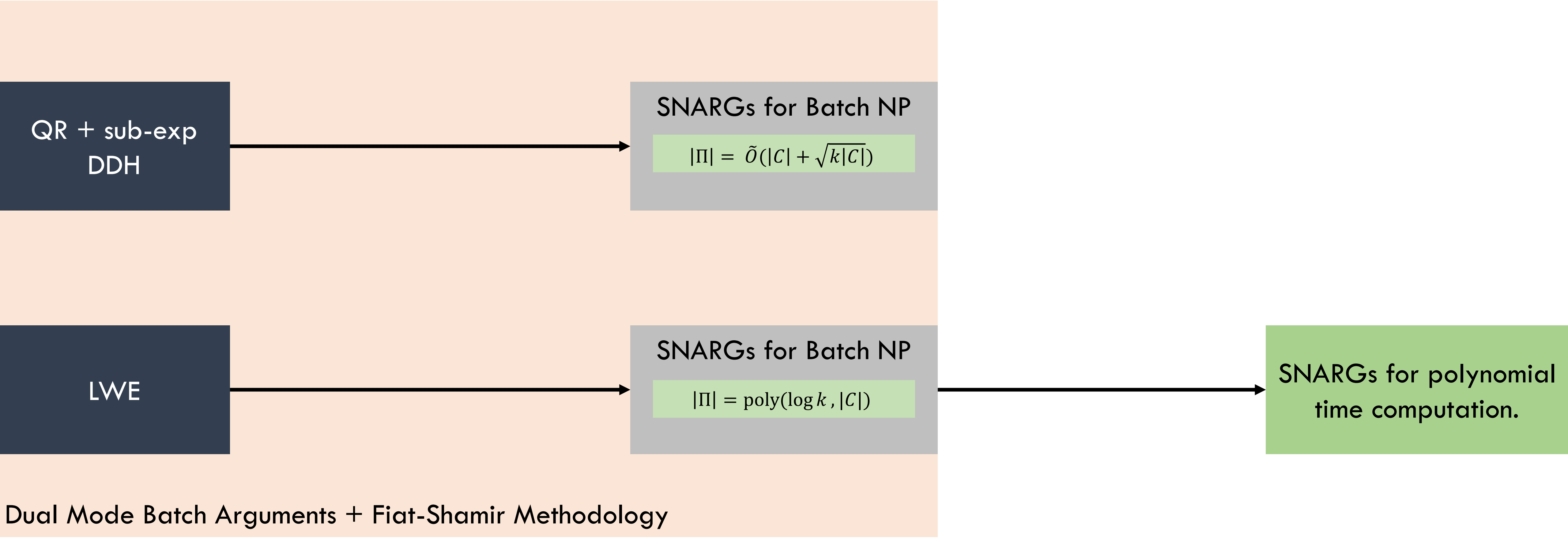
Π

C, k

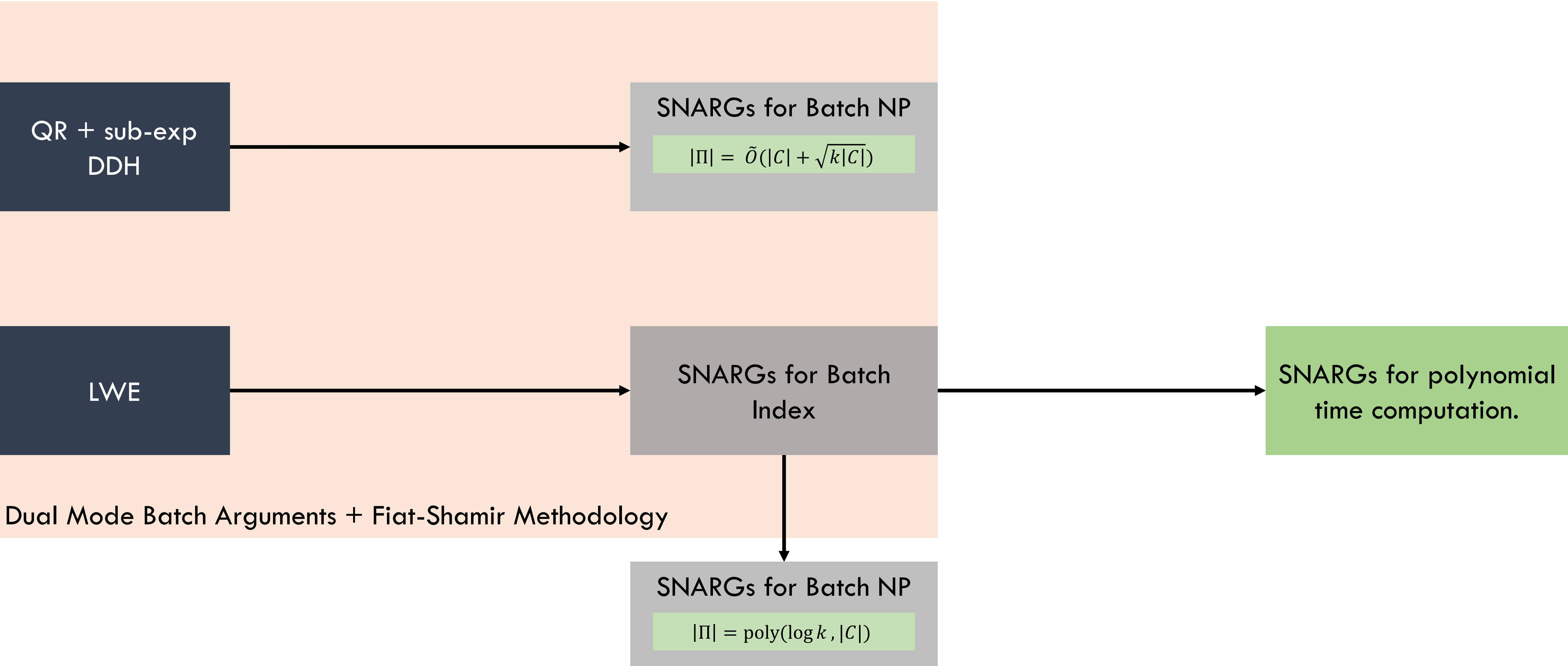
Verifier running time:

$$\mathrm{poly}(\log k, |C|)$$

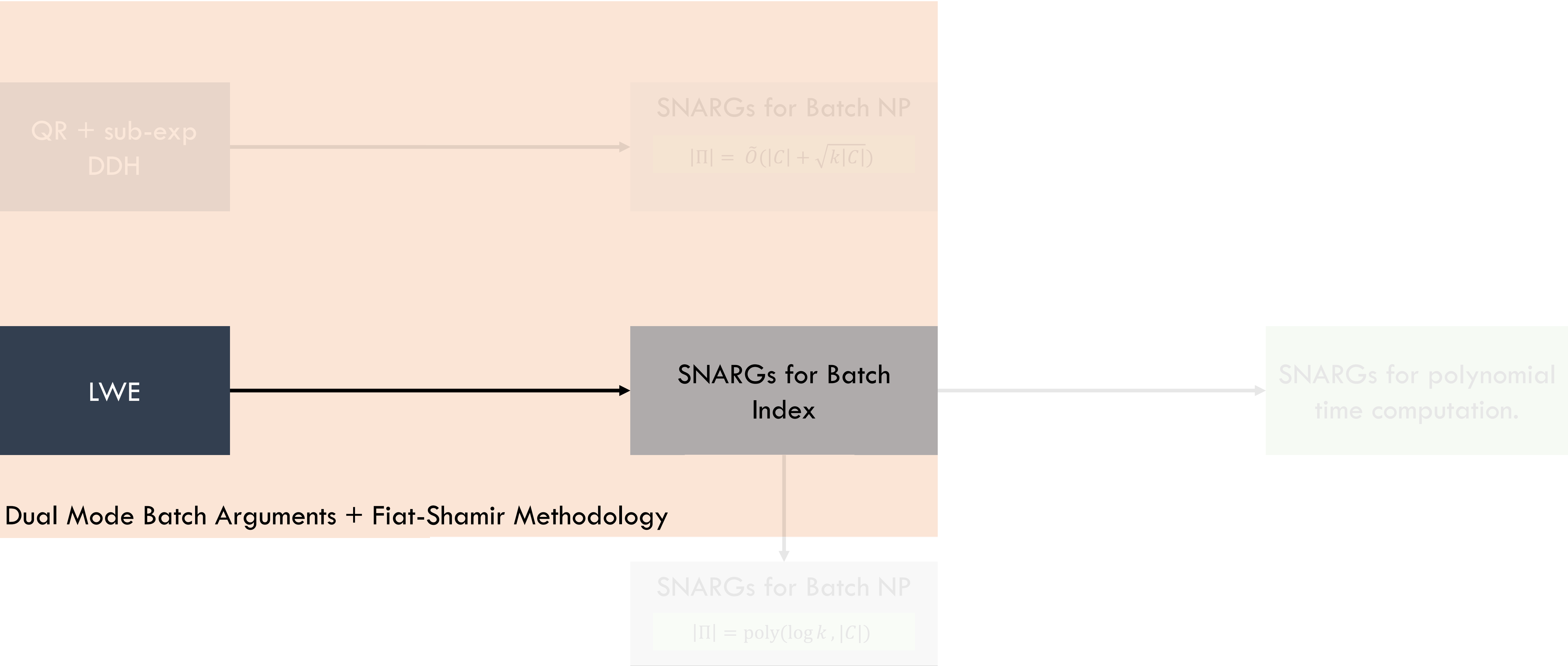
Results Overview



Results Overview



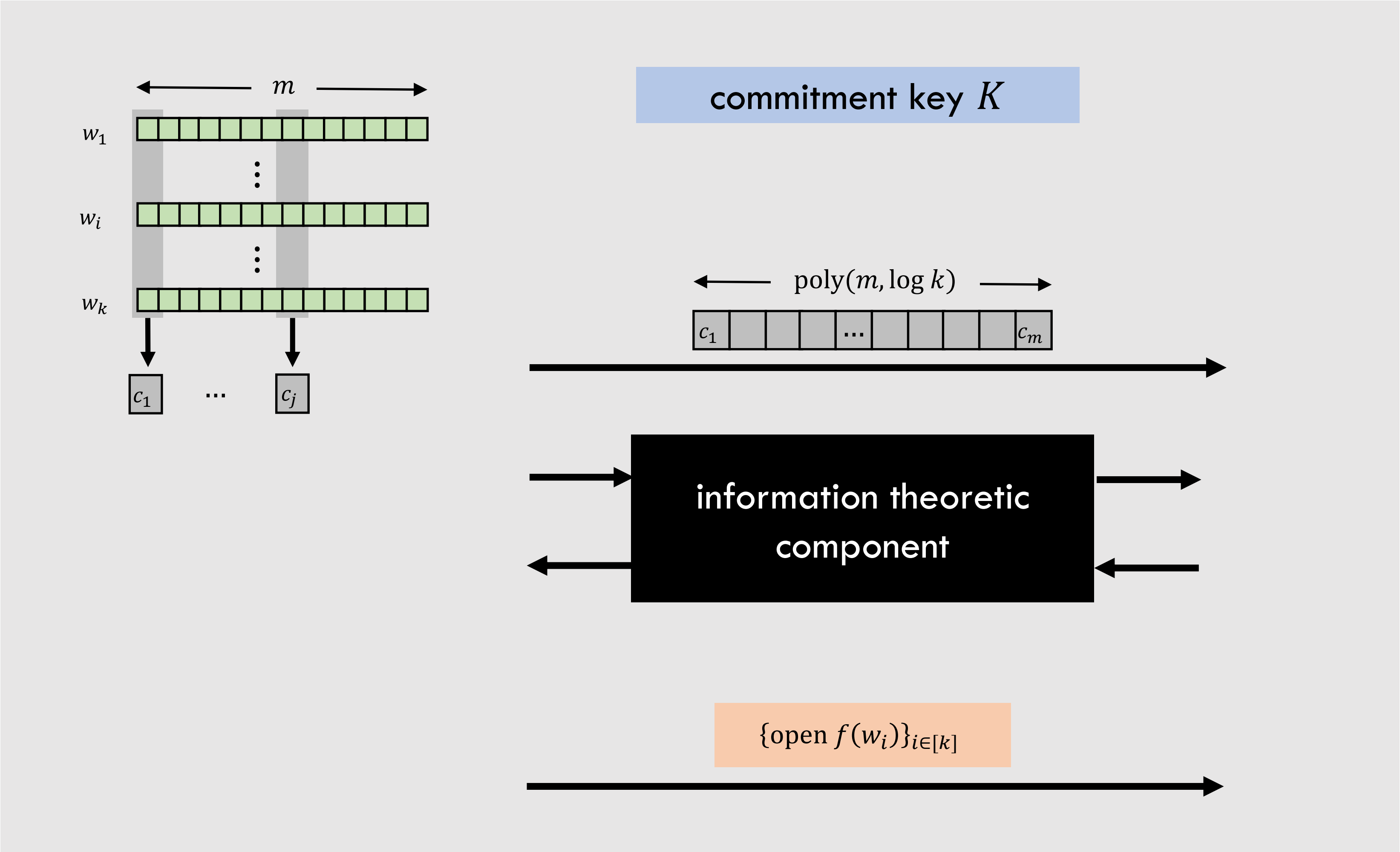
Results Overview



Dual Mode Argument for Batch Index

$L_C = \{i \mid \exists w \text{ s.t. } C(i,w) = 1\}$

$\forall i \in [k], i \in L_C$



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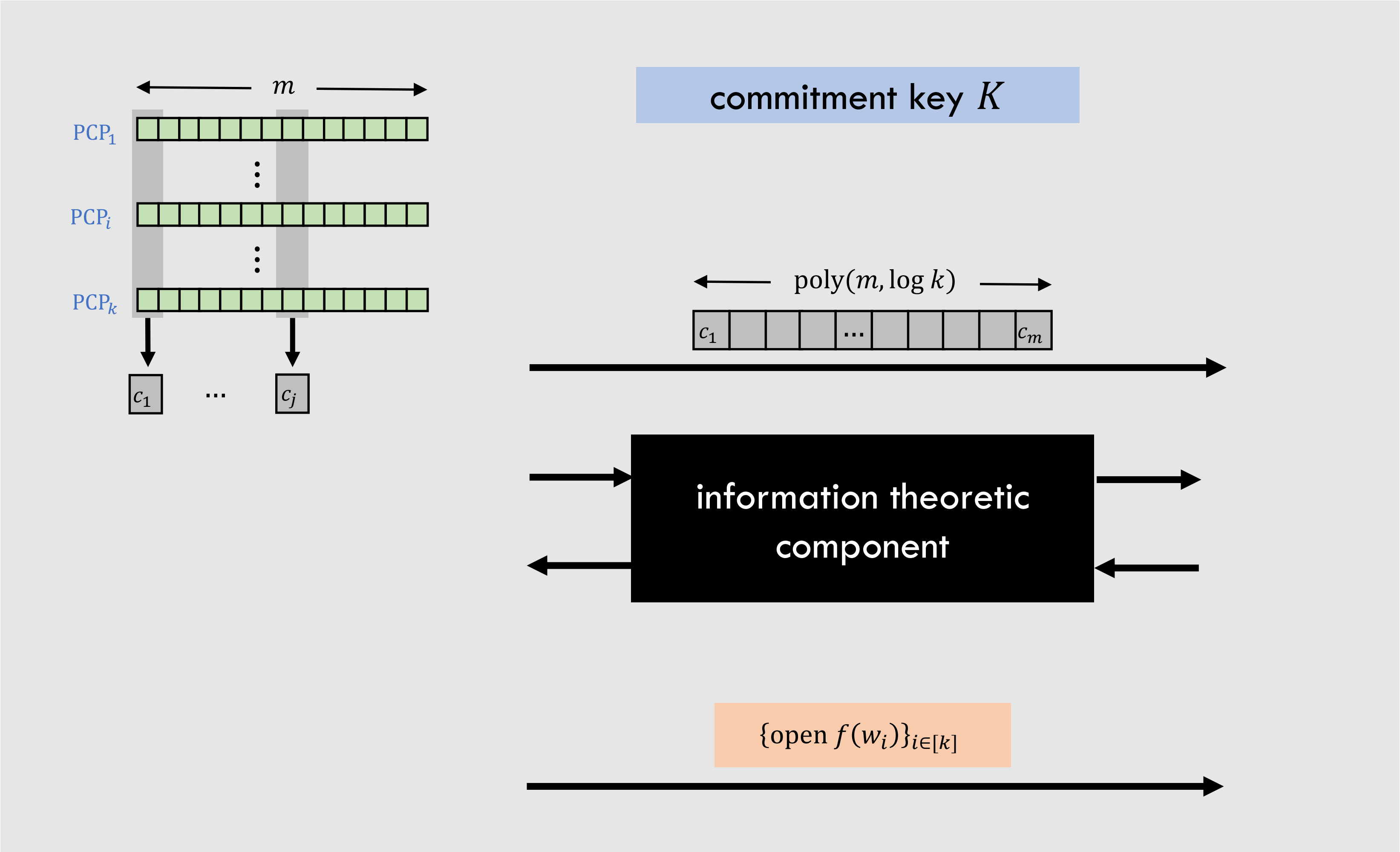

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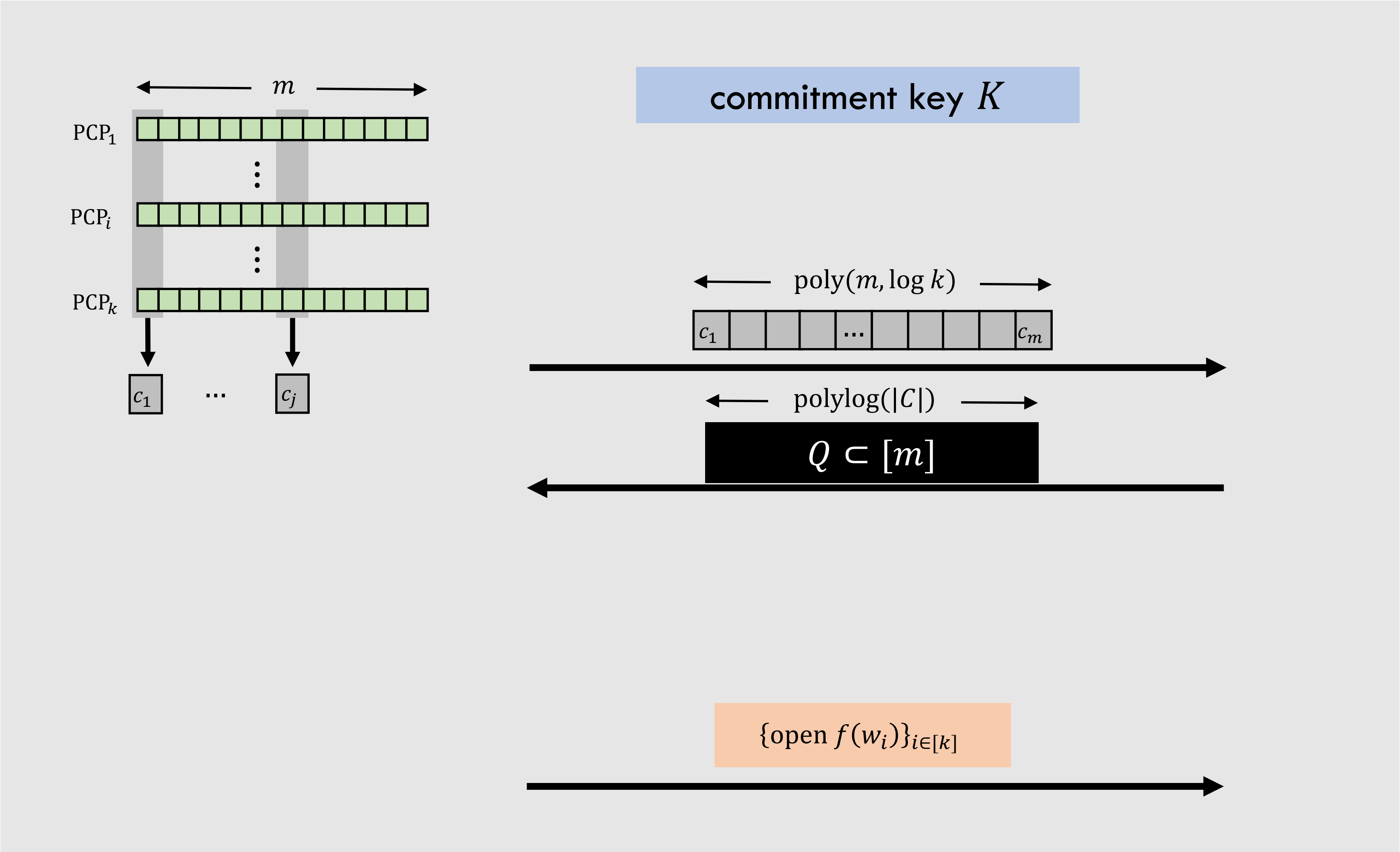

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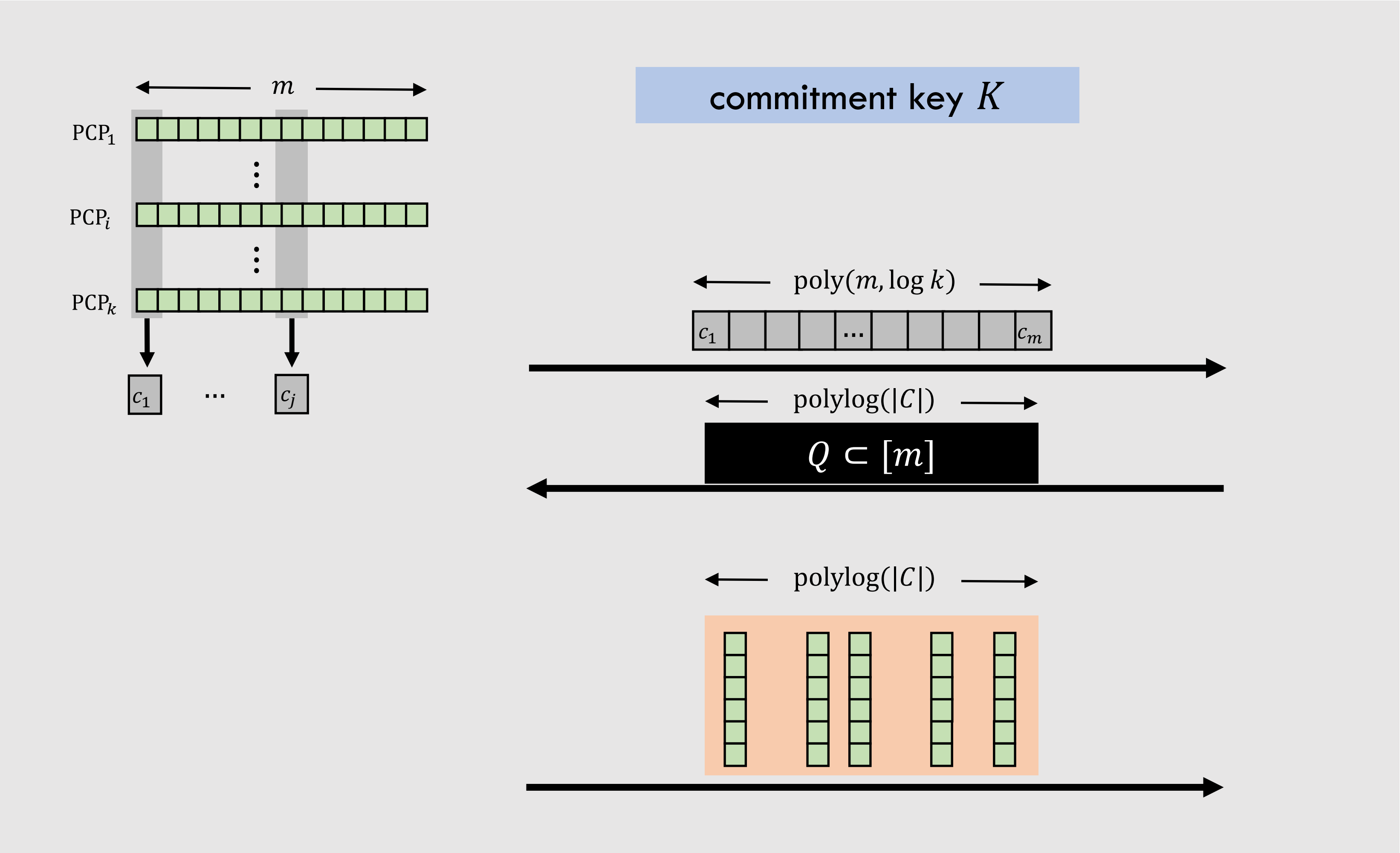

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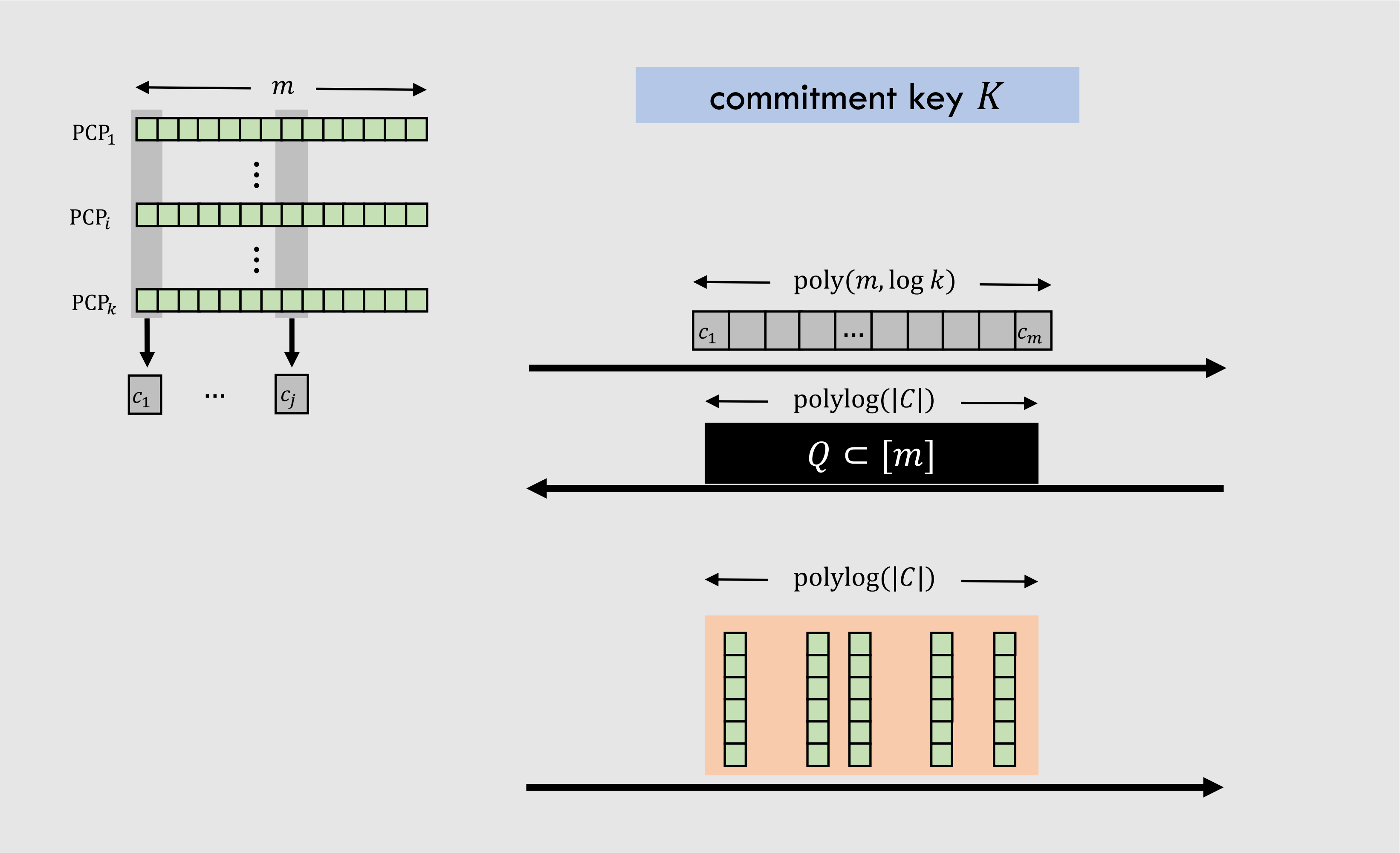
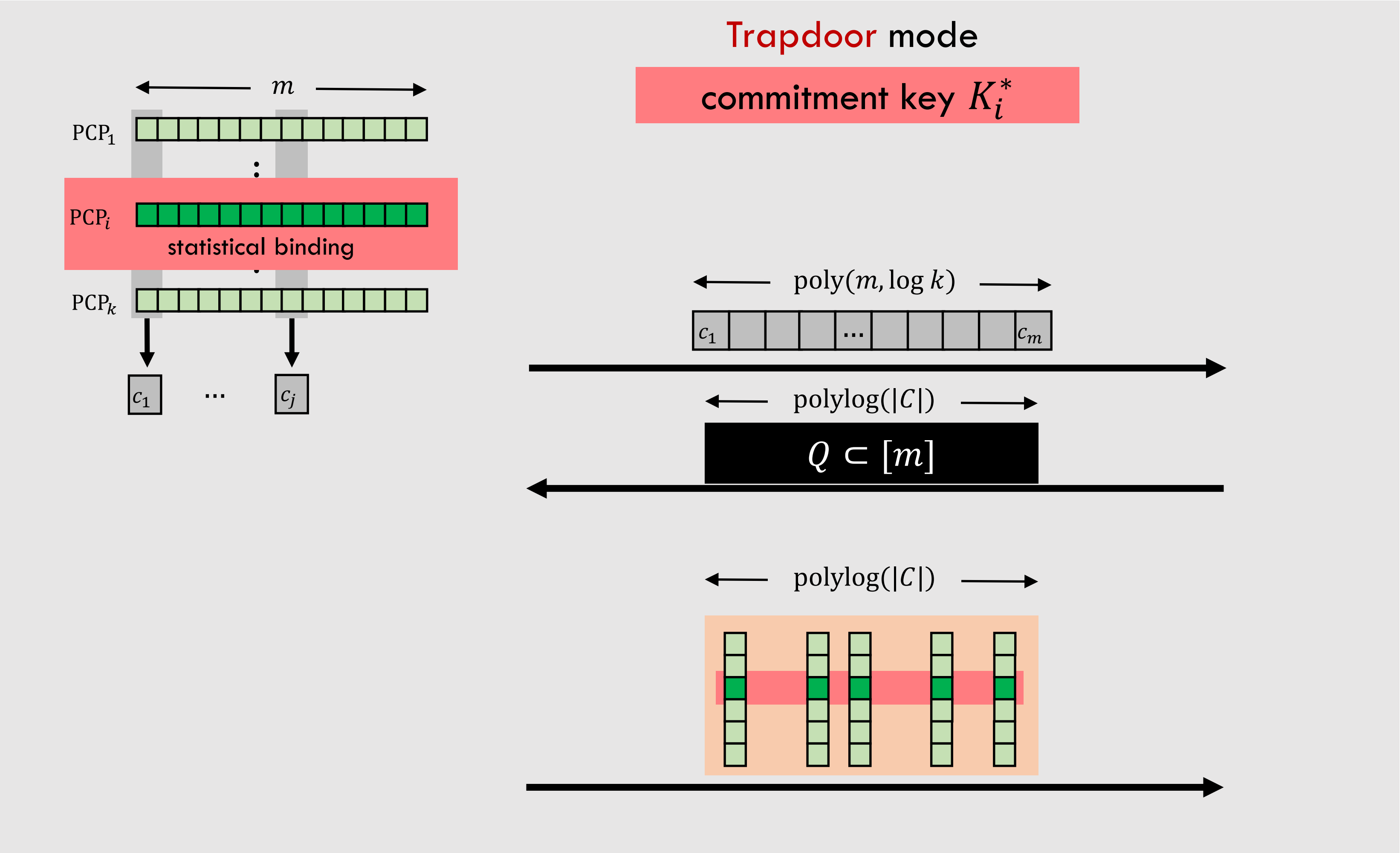


Verify:

1. Commitment openings are valid.
2. PCP responses verify on Q

Dual Mode Argument for Batch Index



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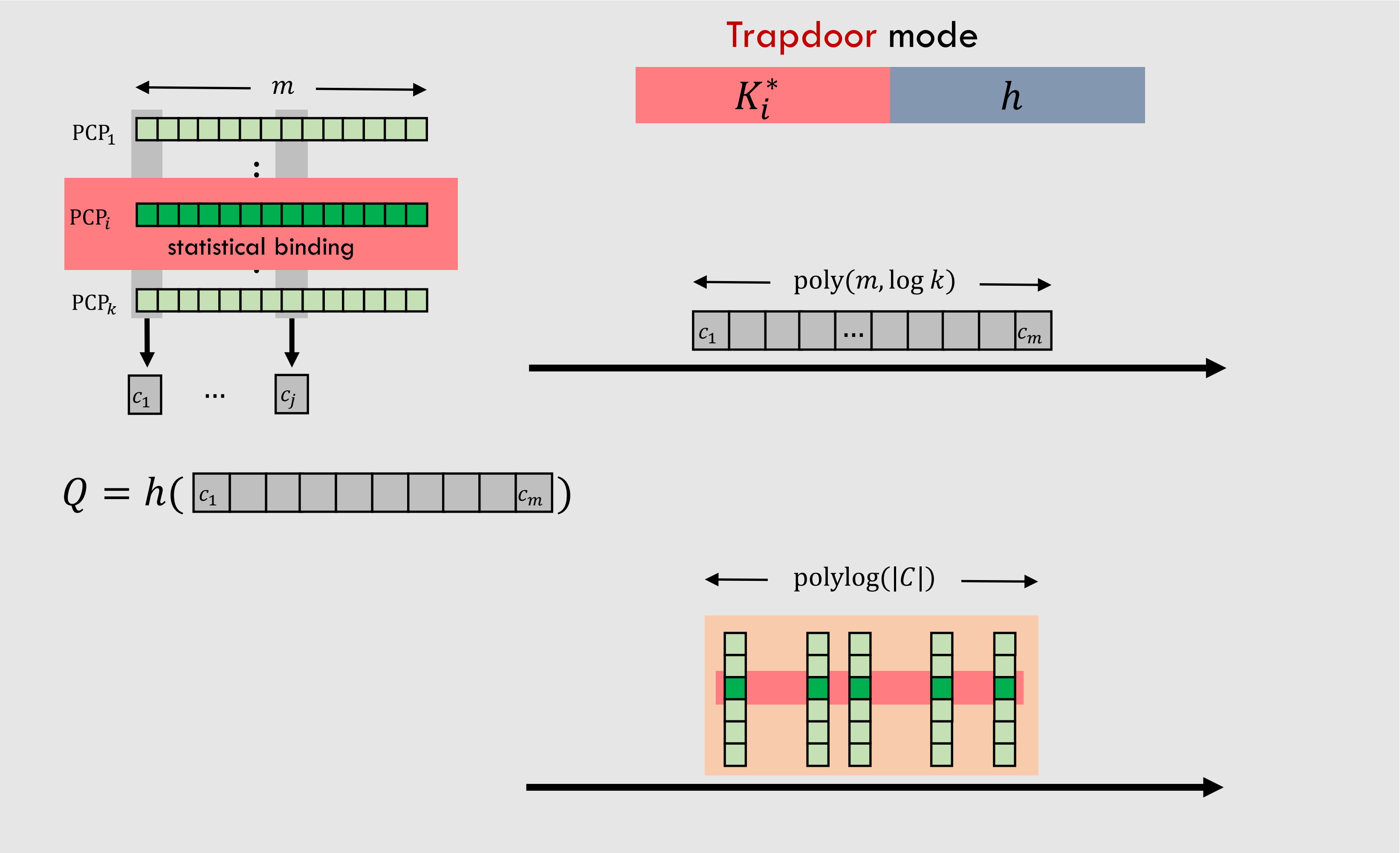

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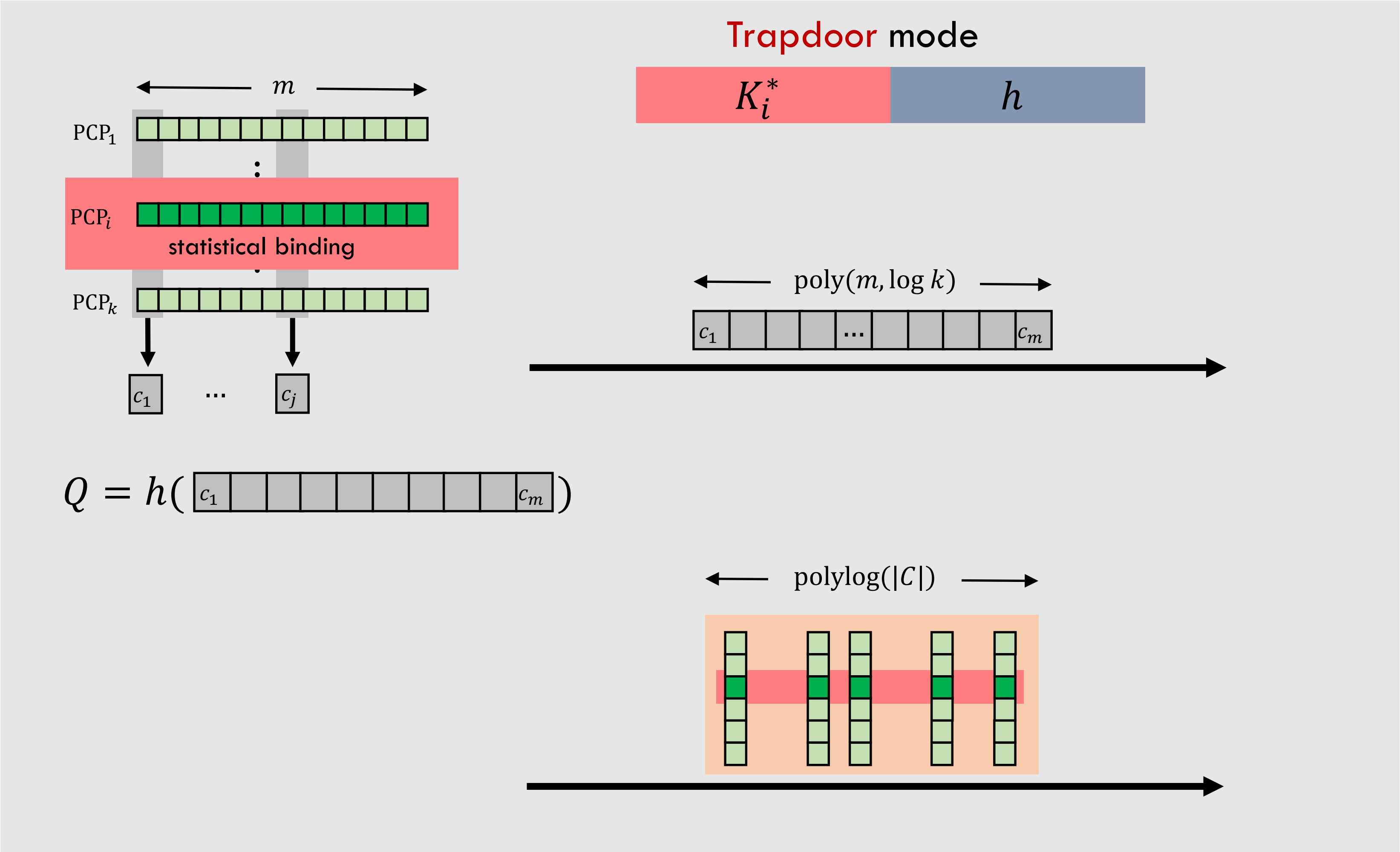


$L_C = \{i \mid \exists w \; s.t. \; C(i,w) = 1\}$

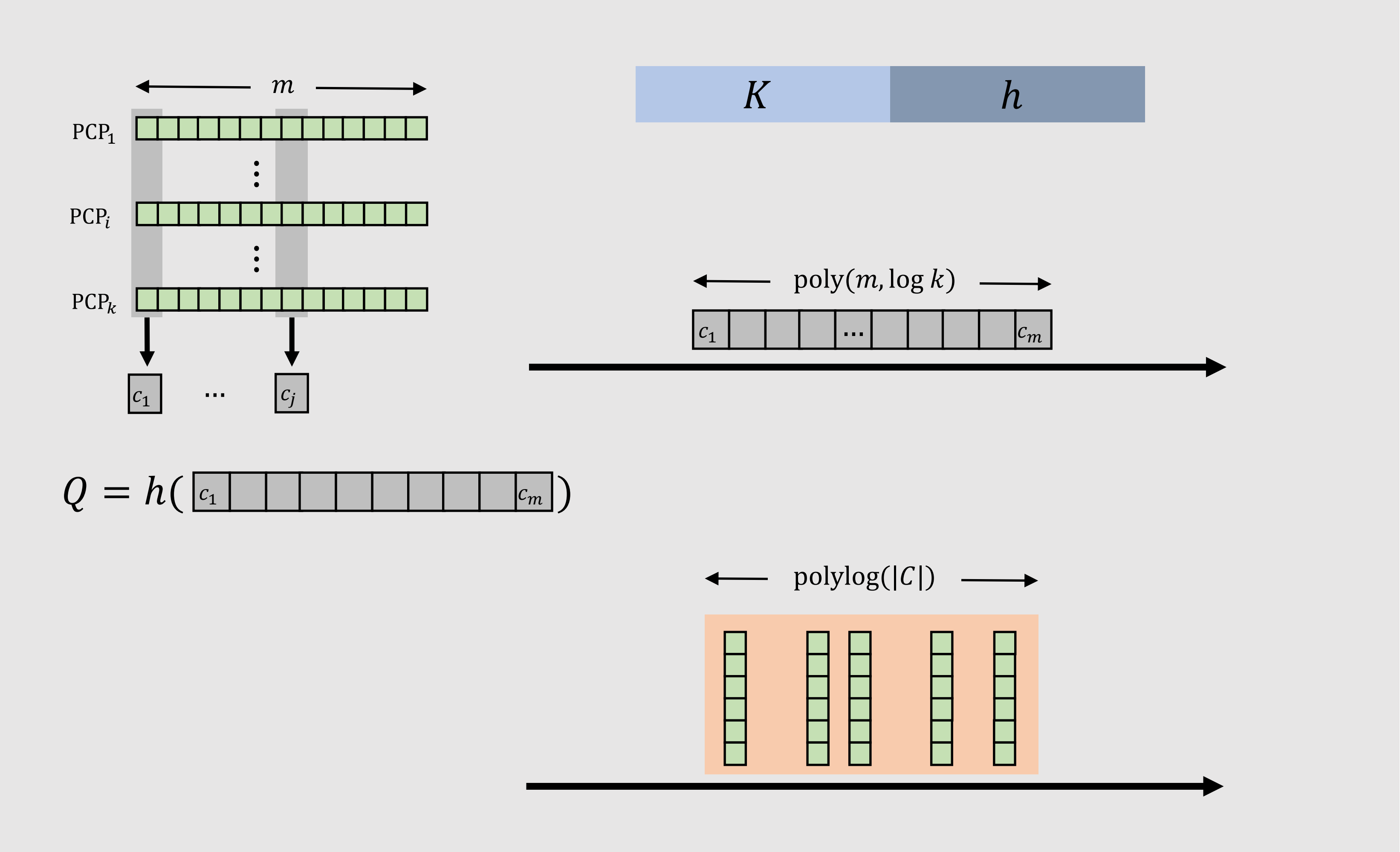
$\forall i \in [k], i \in L_C$

[Holmgren-Lombardi-Rothblum'21]
Assuming LWE, the transformation is sound.

Verify:

1. Commitment openings are valid.
2. PCP responses verify on Q

SNARG for Batch Index



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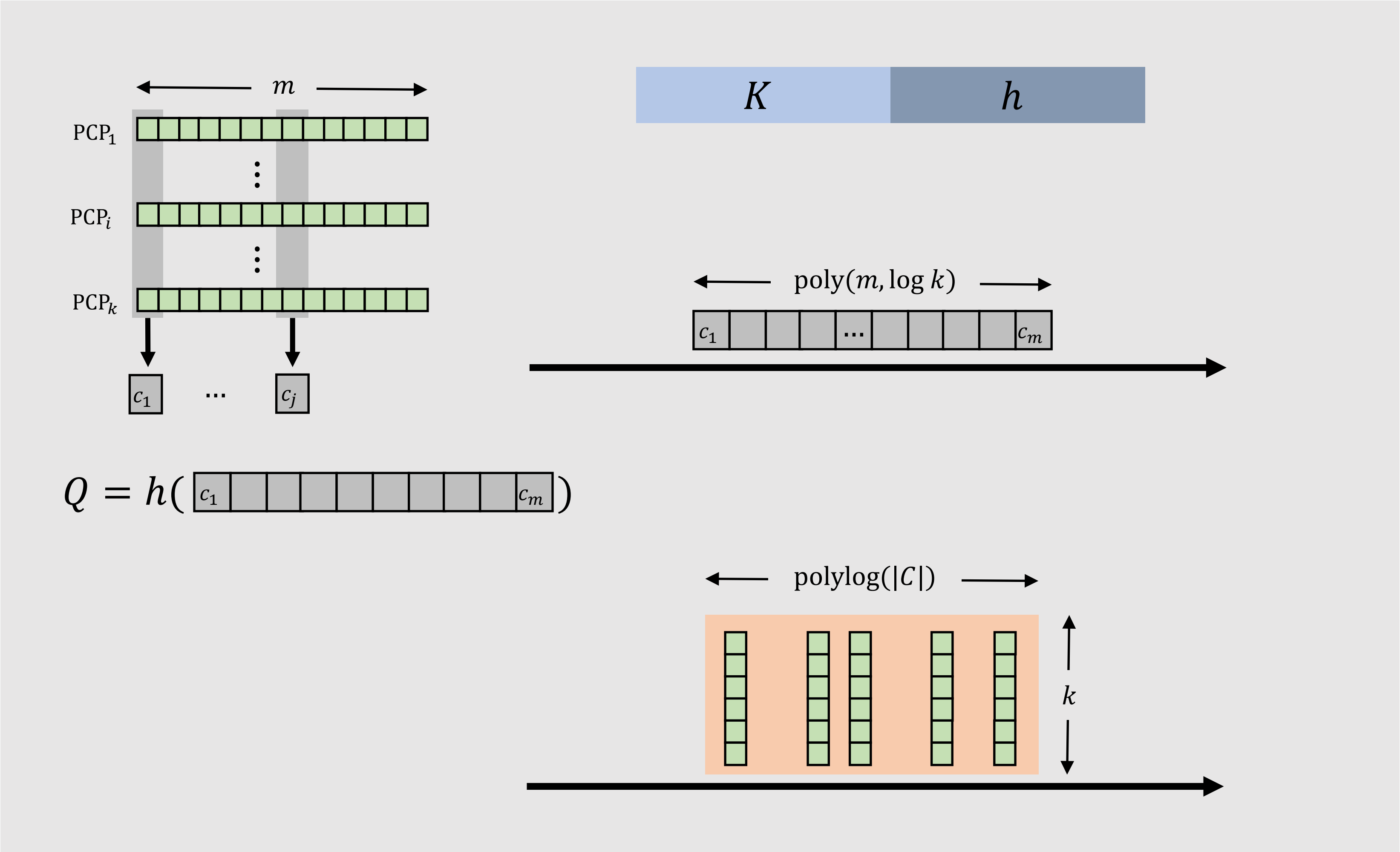
SNARG for Batch Index



$L_C = \{i \mid \exists w \; s.t. \; C(i,w) = 1\}$

$\forall i \in [k], i \in L_C$

|  | $= \Omega(k)$

Verify:

1. Commitment openings are valid.
2. PCP responses verify on Q

SNARG for Batch Index

Send proof of verification

$L_C = \{i \mid \exists w \text{ s.t. } C(i,w) = 1\}$

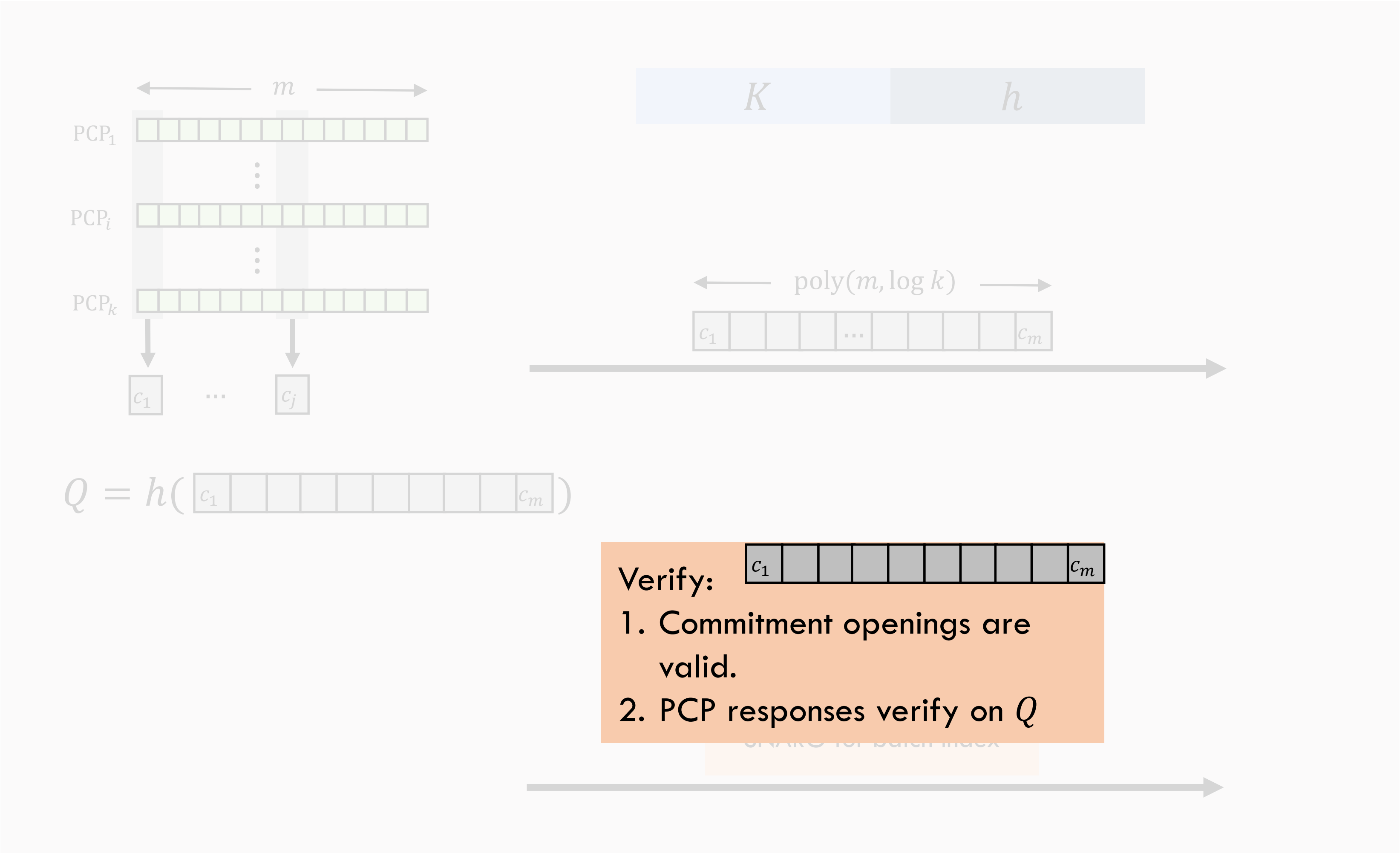
$\forall i \in [k], i \in L_C$

|  | $= \Omega(k)$

Verify:

1. Commitment openings are valid.
2. PCP responses verify on Q

SNARG for Batch Index



Verify: $c_1 \ \ldots \ c_m$

1. Commitment openings are valid.
2. PCP responses verify on Q

$L_C = \{i \mid \exists w \ s.t. \ C(i,w) = 1\}$

$\forall i \in [k], i \in L_C$

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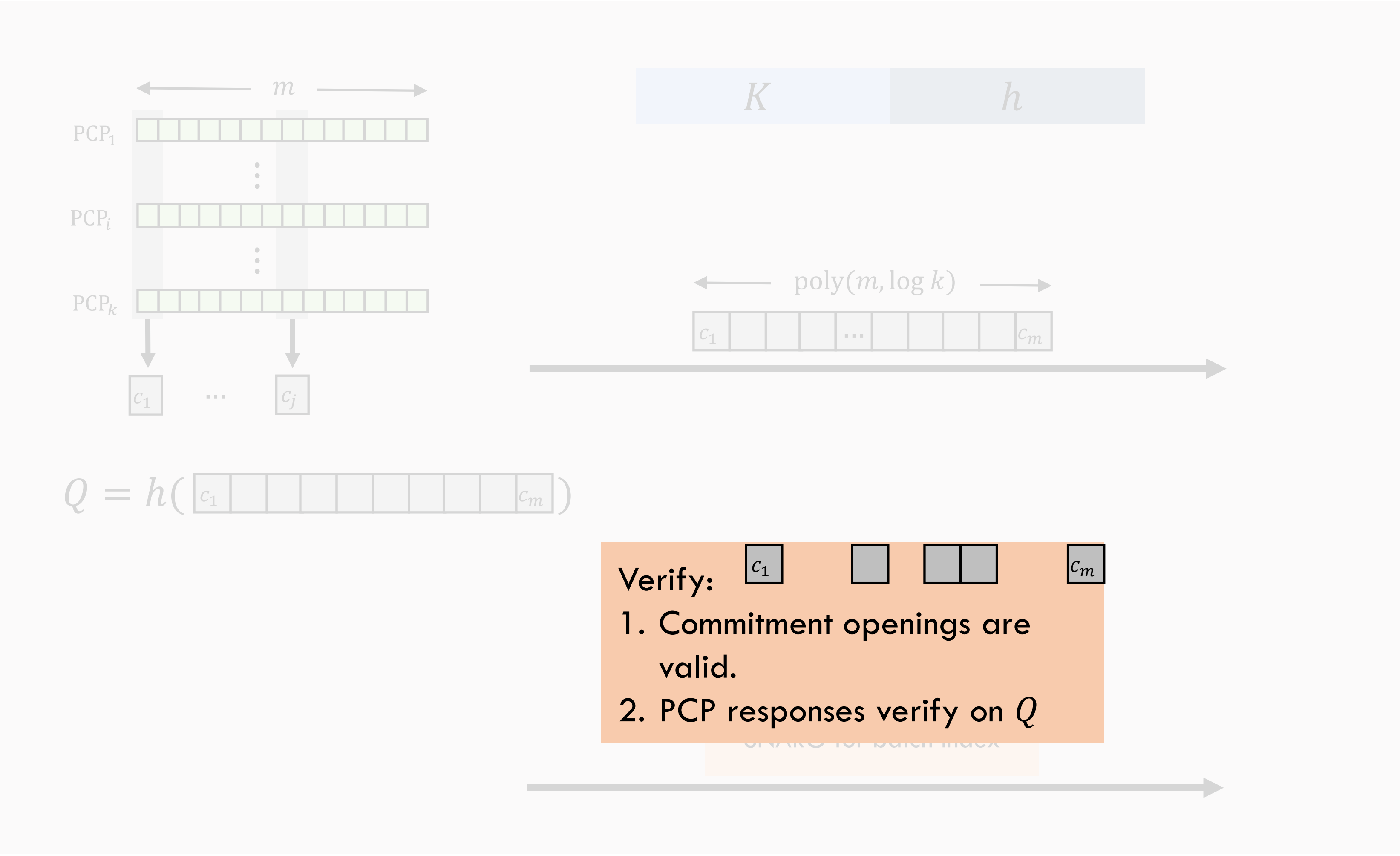
SNARG for Batch Index



Verify: c_1 … c_m

1. Commitment openings are valid.
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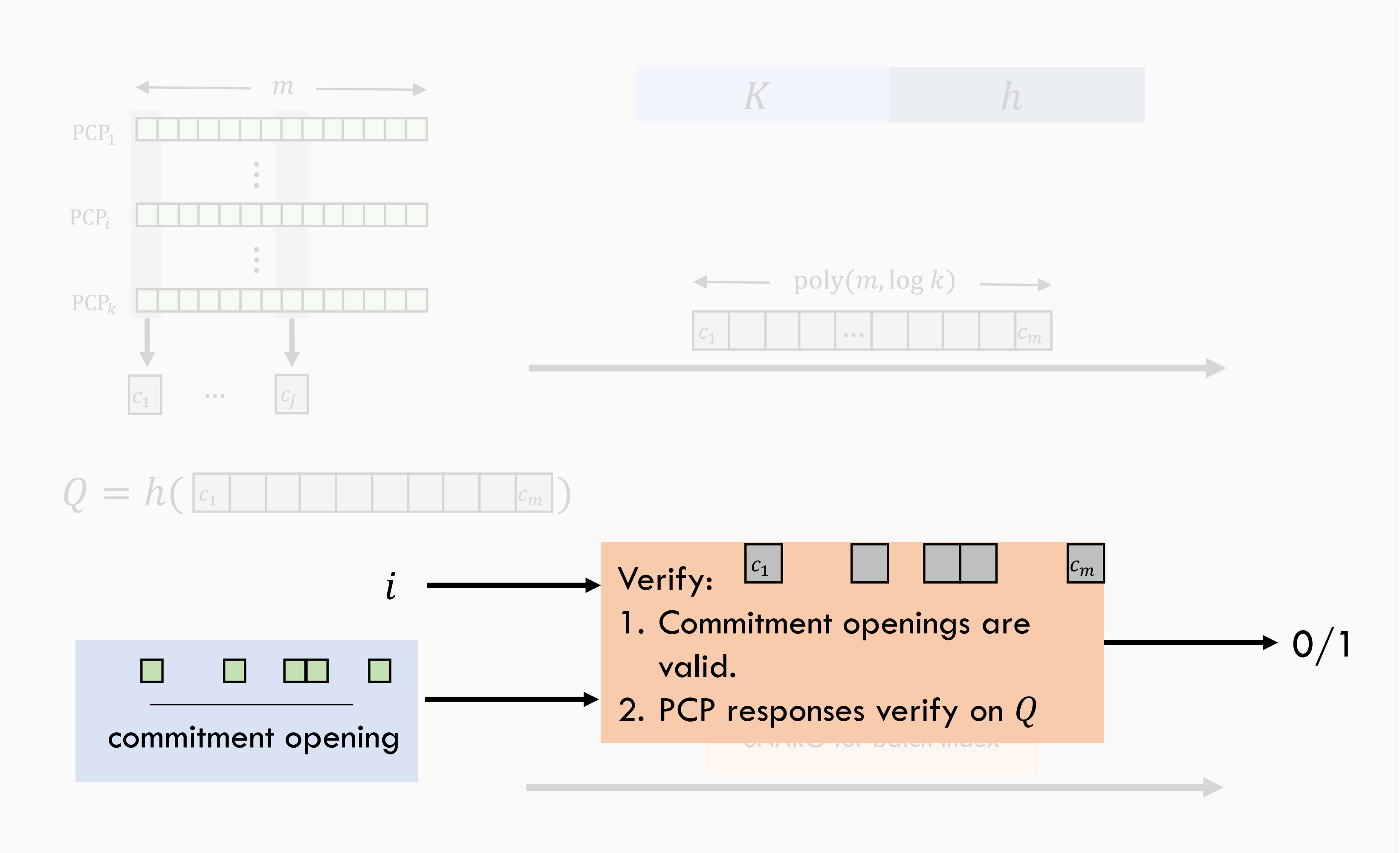
SNARG for Batch Index



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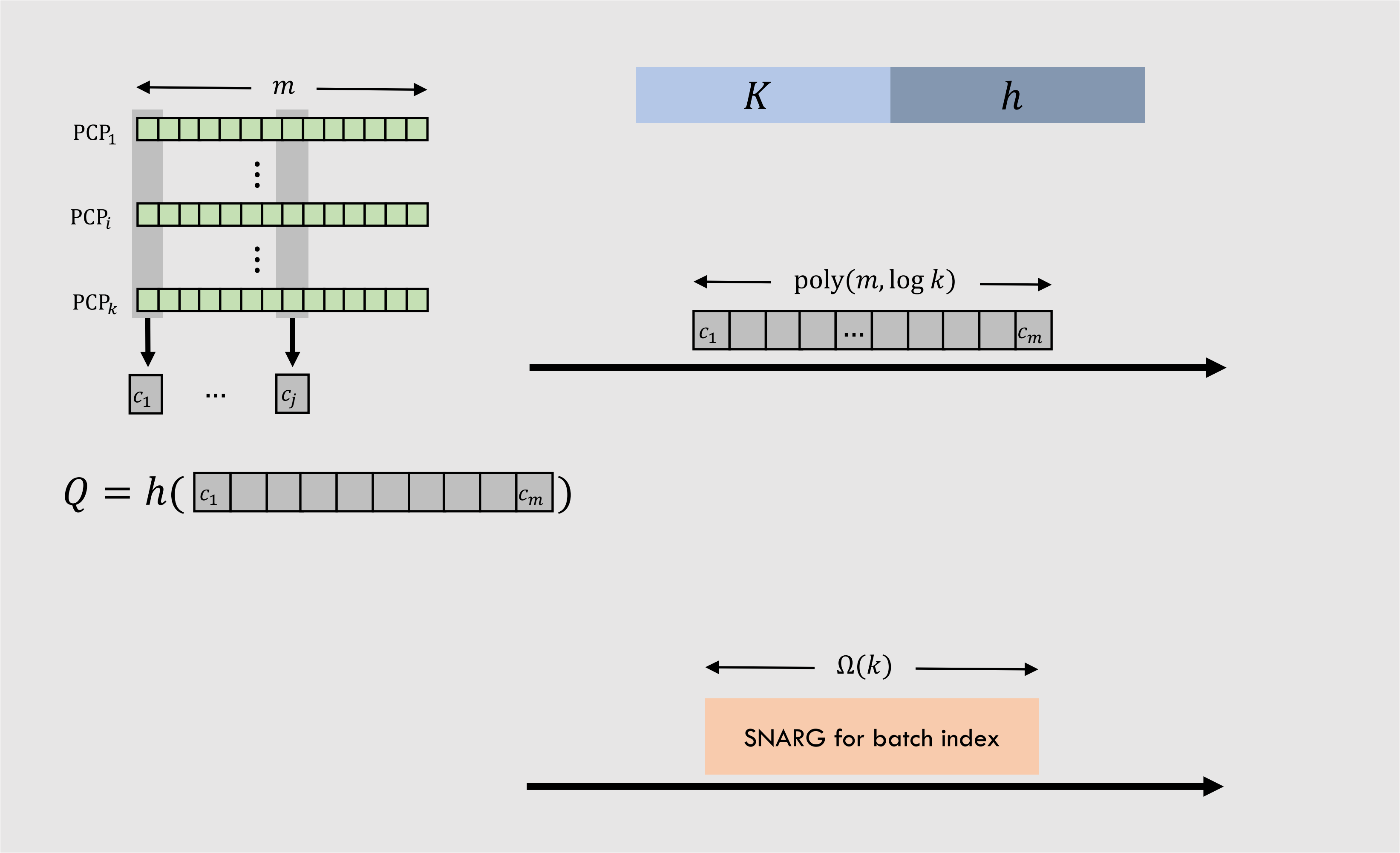
SNARG for Batch Index



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Verify: c_1 c_m

1. Commitment openings are valid.
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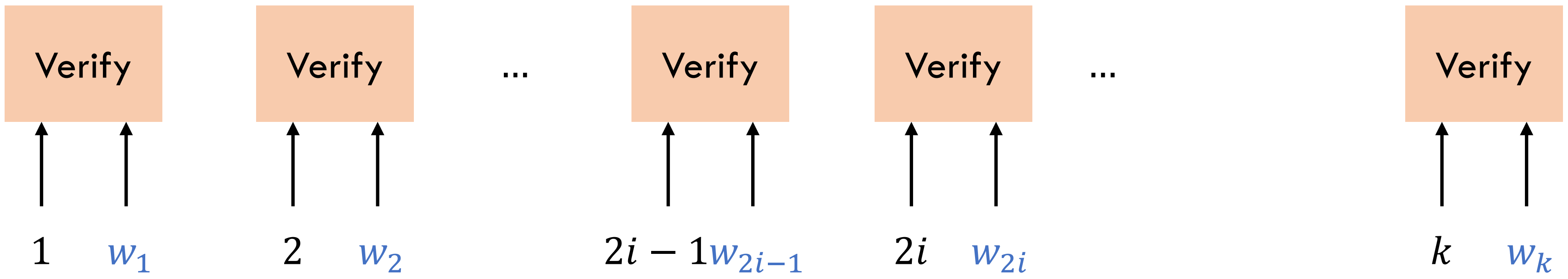
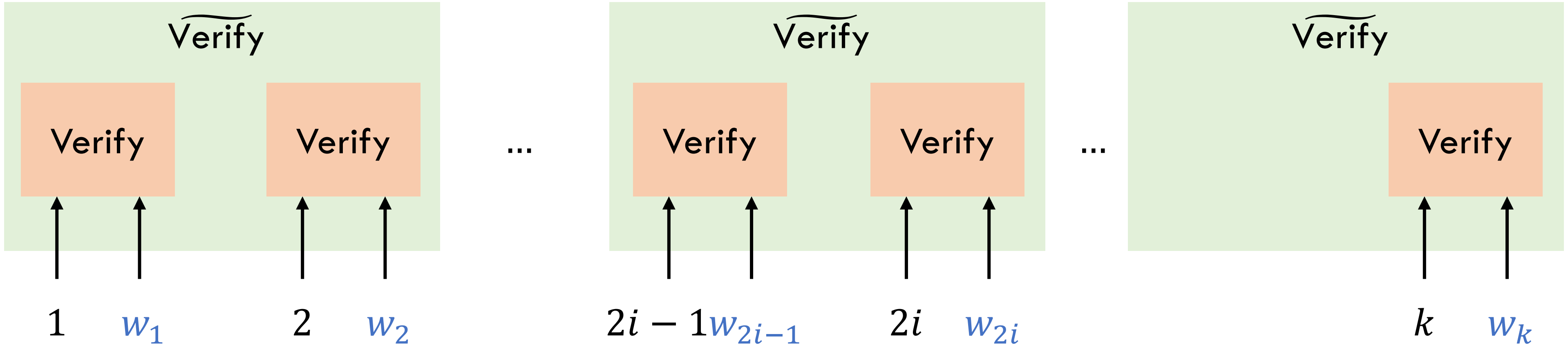
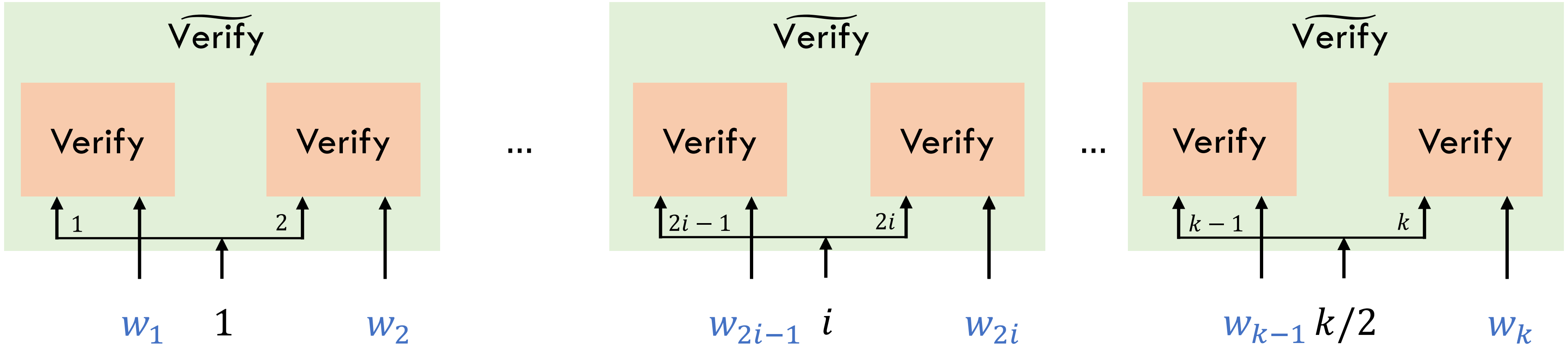
Grouping Instances for Recursion



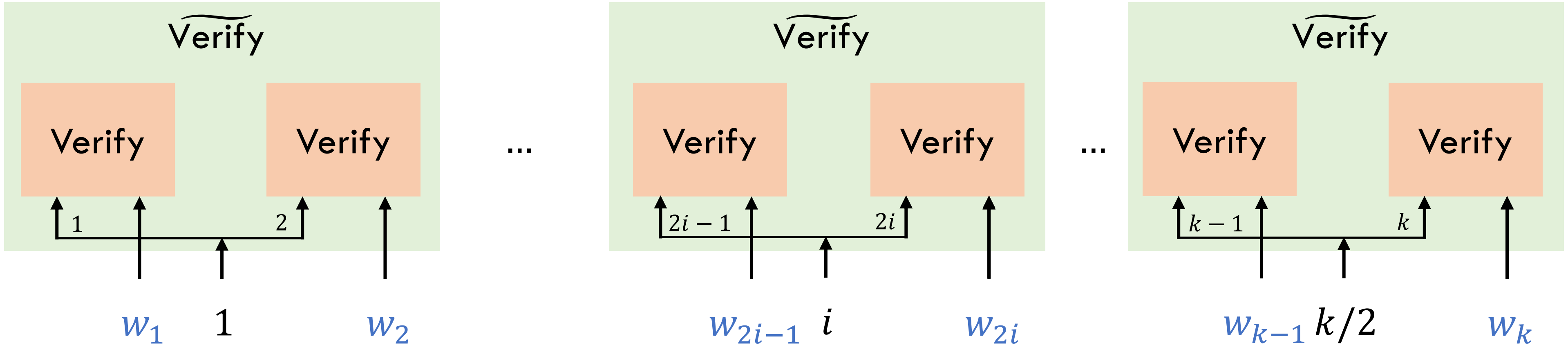
Grouping Instances for Recursion



Grouping Instances for Recursion



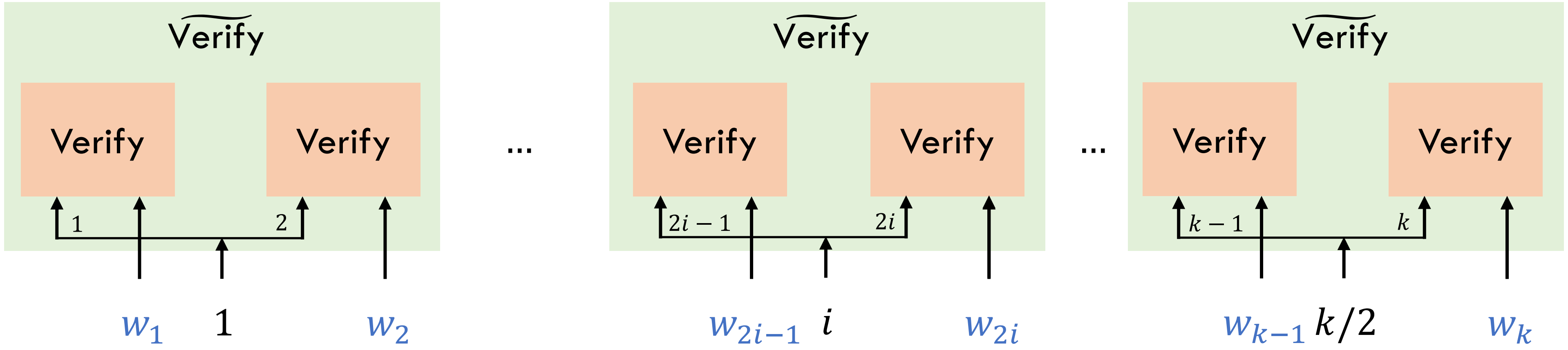
Grouping Instances for Recursion



$k/2$ instances for circuit $\widetilde{\text{Verify}}$

Grouping Instances for Recursion

$|\widetilde{\text{Verify}}| \geq 2|\text{Verify}|$



$k/2$ instances for circuit $\widetilde{\text{Verify}}$

Grouping Instances for Recursion

$L_C \; = \{i \mid \exists w \; s.t. \; C(i,w) = 1\}$

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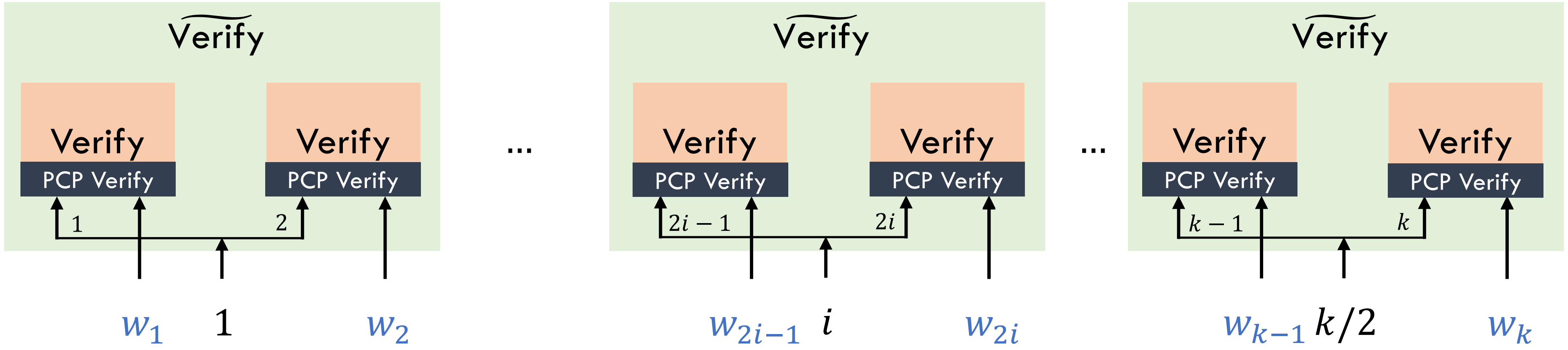

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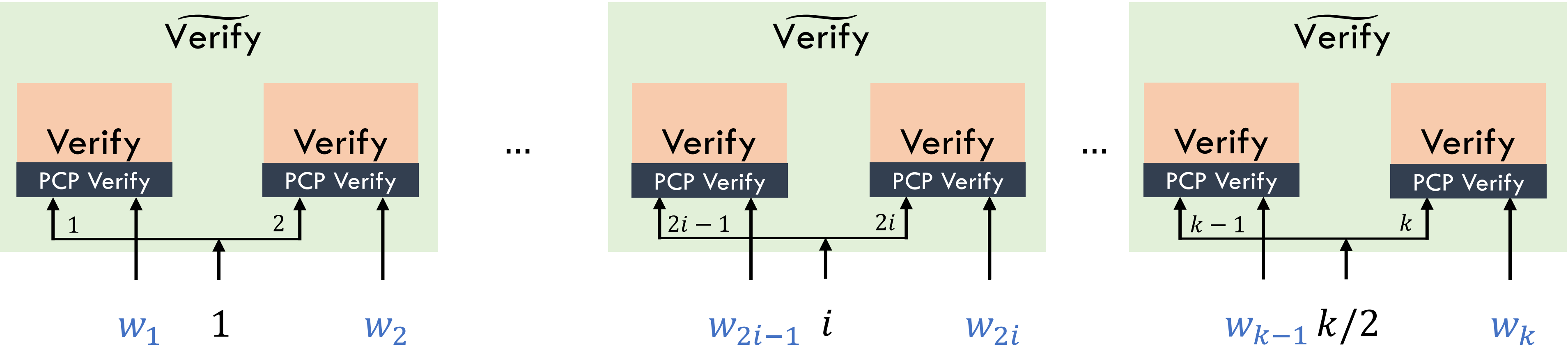
$\left|\widetilde{\text{Verify}}\right| \geq 2|\text{Verify}| \geq 2|C|$



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$\forall i \in [k], i \in L_C$

$|\widetilde{\text{Verify}}| \geq 2|\text{Verify}| \geq 2|C|$

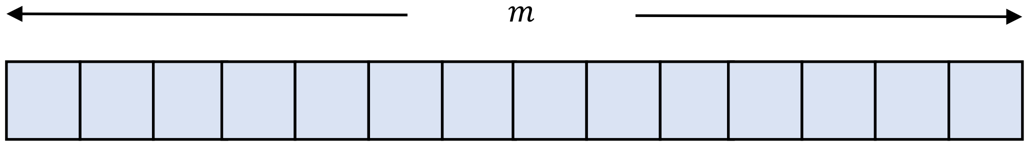
k instances for circuits of size $|C|$ → $k/2$ instances for circuits of size $2|C|$ → … → $O(1)$ instances for circuits of size $k|C|$

$\log k$ steps of recursion

Tool: PCP with Fast Online Verification

C, x, w

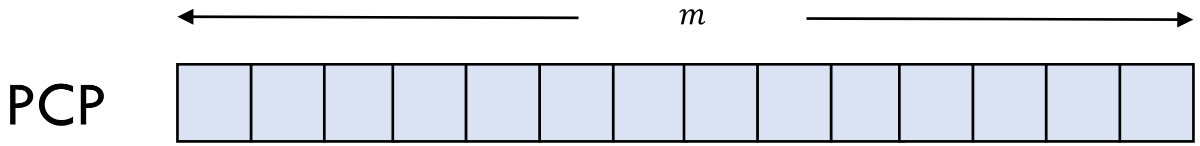
PCP



C, x

Tool: PCP with Fast Online Verification

C, x, w



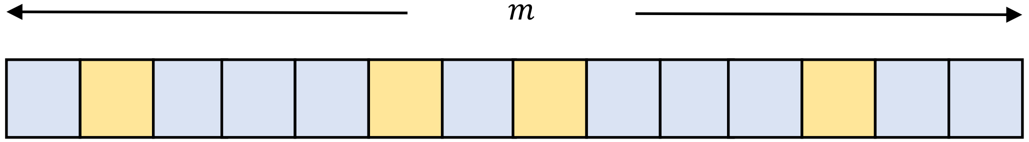
C, x

$Q \leftarrow [m]$

Tool: PCP with Fast Online Verification

C, x, w

PCP

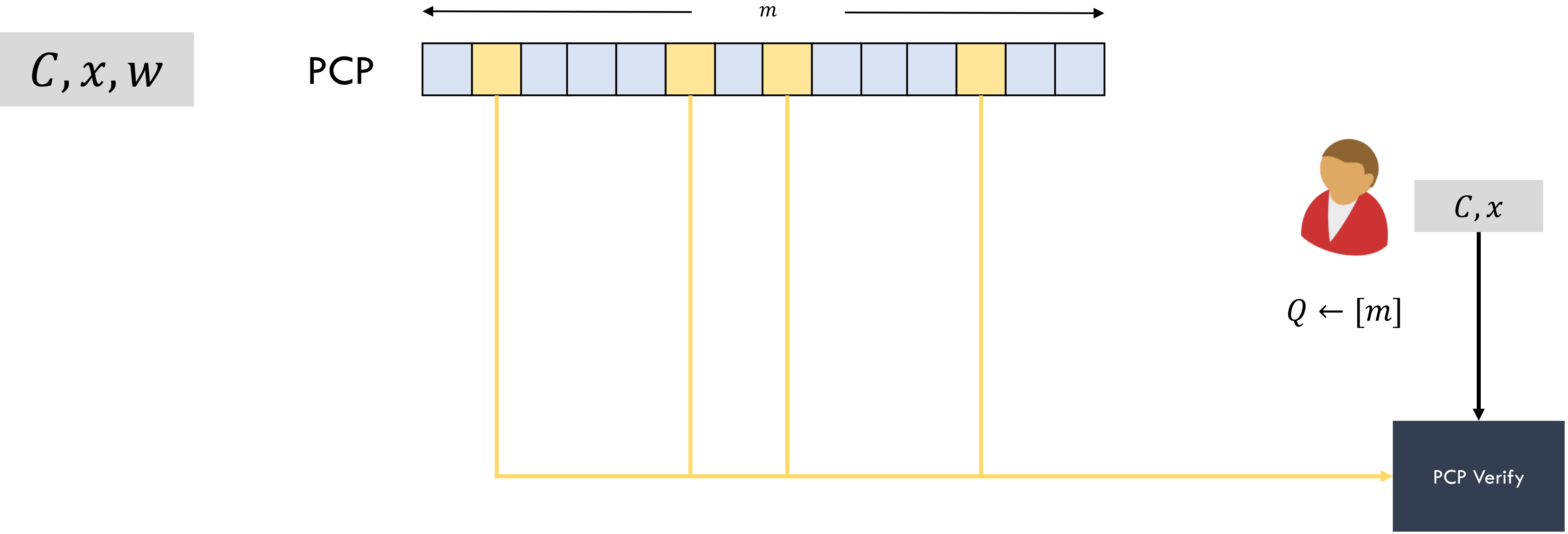


C, x

$Q \leftarrow [m]$

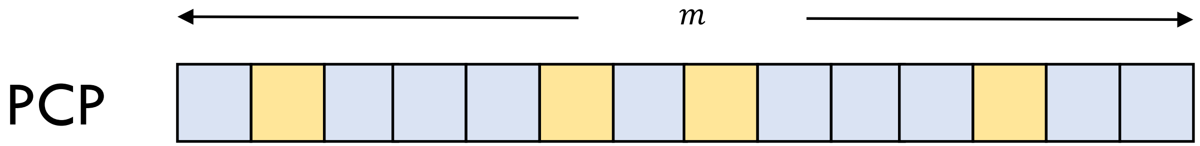
PCP Verify

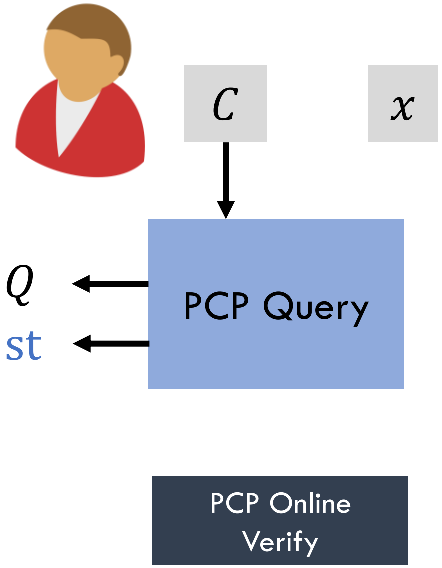
Tool: PCP with Fast Online Verification



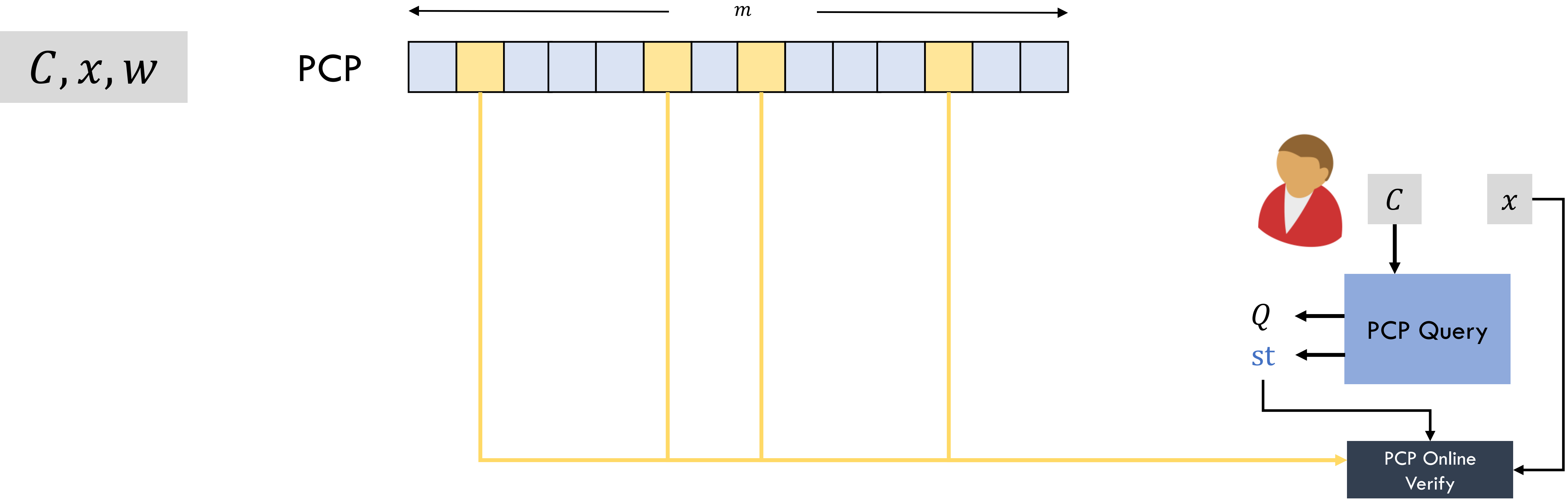
Tool: PCP with Fast Online Verification

C, x, w



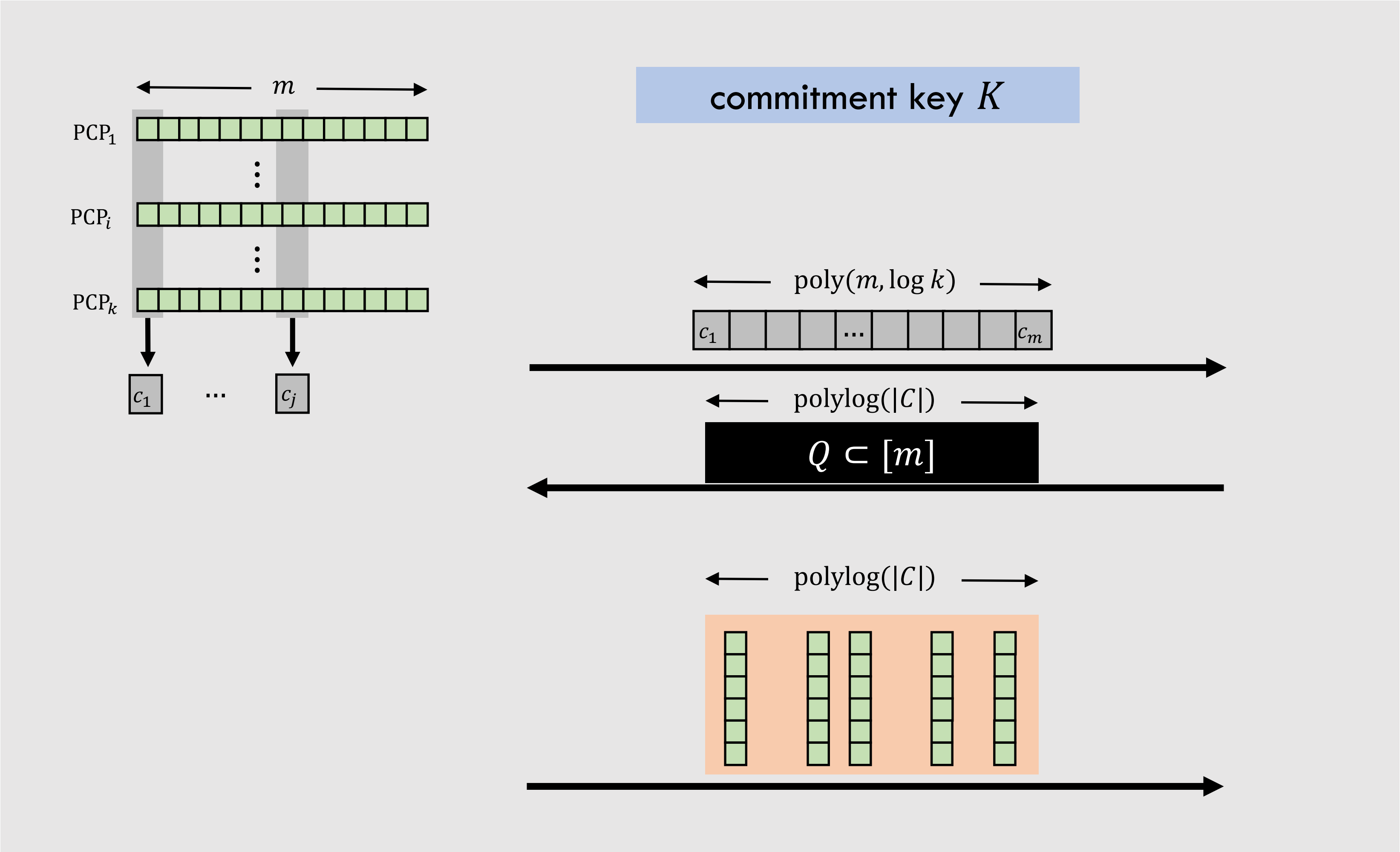


Tool: PCP with Fast Online Verification



$|\text{PCP Online Verify}| \approx O(|x|)$

Dual Mode Argument for Batch Index



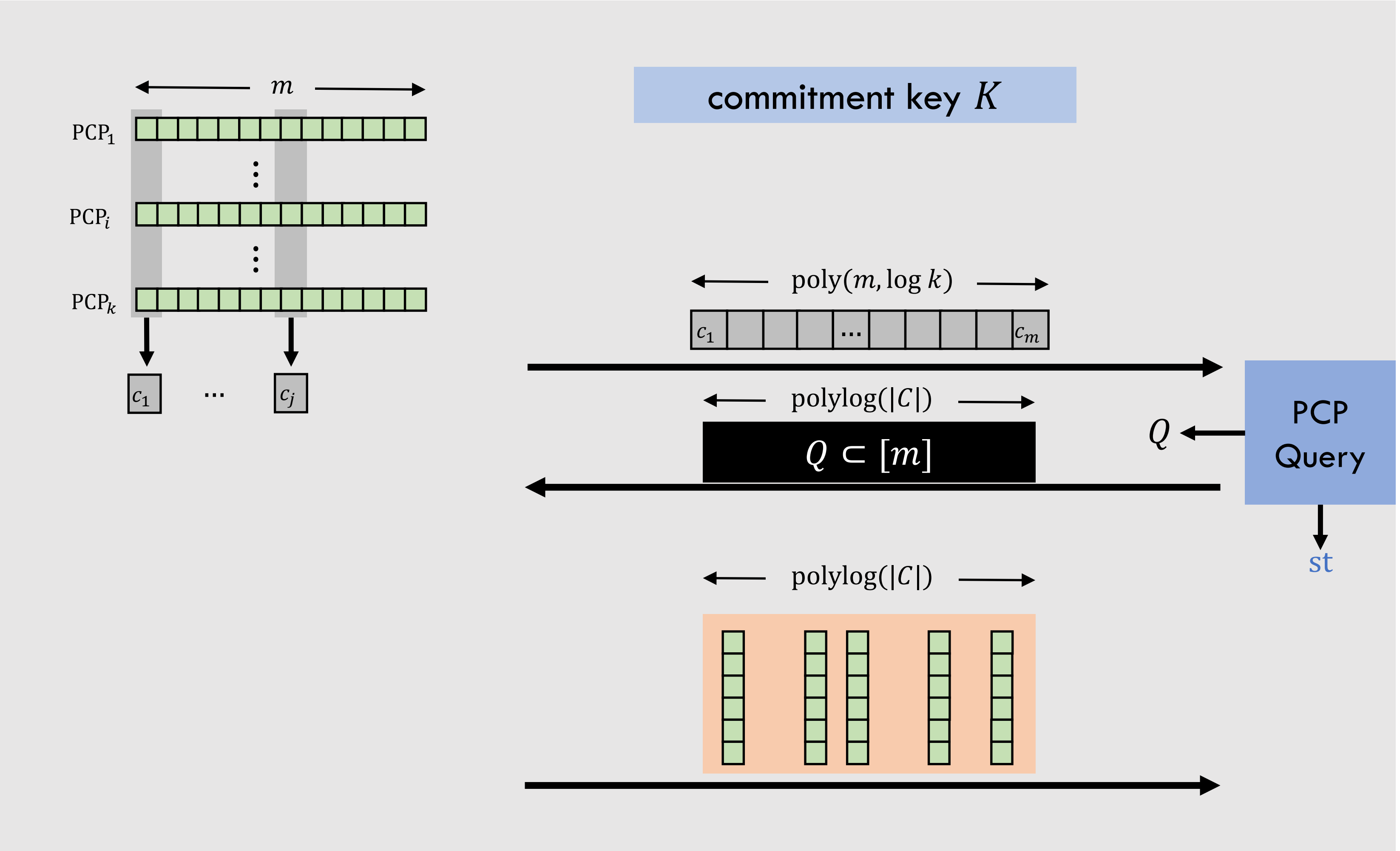
$L_C = \{i \mid \exists w \text{ s.t. } C(i,w) = 1\}$

$\forall i \in [k], i \in L_C$

Verify:

1. Commitment openings are valid.
2. PCP responses verify on Q

Dual Mode Argument for Batch Index



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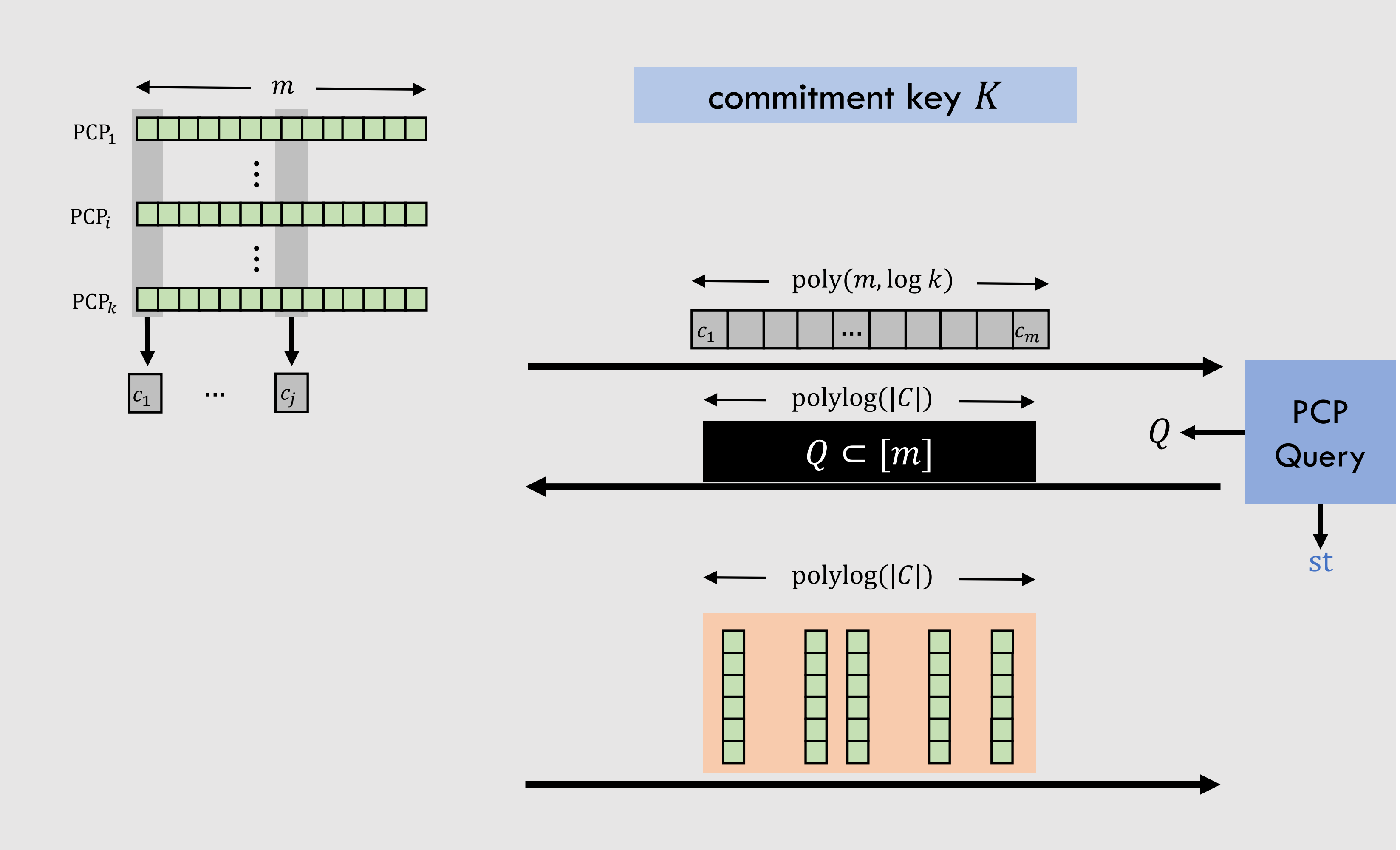
PCP Verify

Verify:

1. Commitment openings are valid.
2. PCP responses verify on Q, st

Dual Mode Argument for Batch Index



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$\forall i \in [k], i \in L_C$

PCP ○Verify

Verify:

1. Commitment openings are valid.
2. PCP responses verify on Q, st

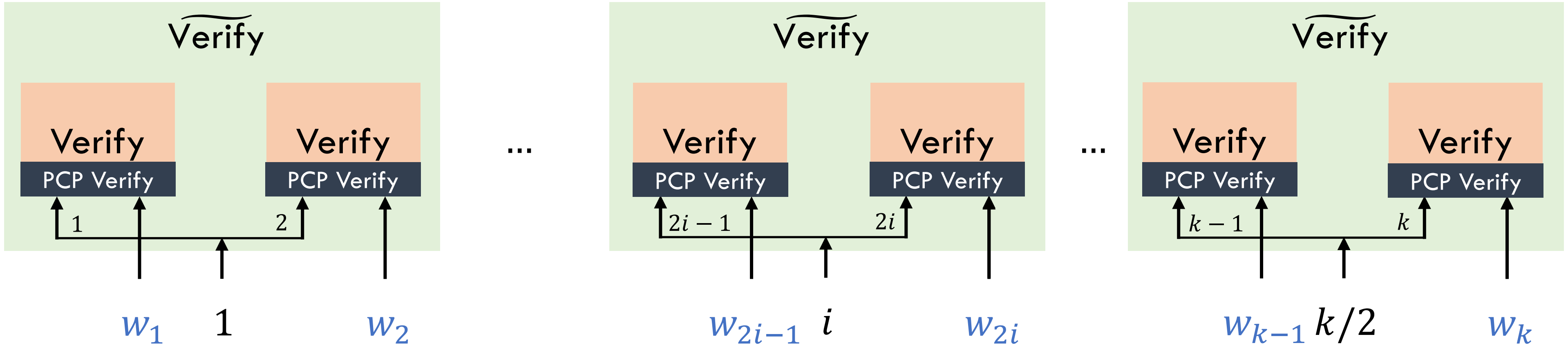
$|\mathrm{Verify}| \approx \mathrm{polylog}(k, |C|)$

Grouping Instances for Recursion

$L_C = \{i \mid \exists w \; s.t. \; C(i,w) = 1\}$

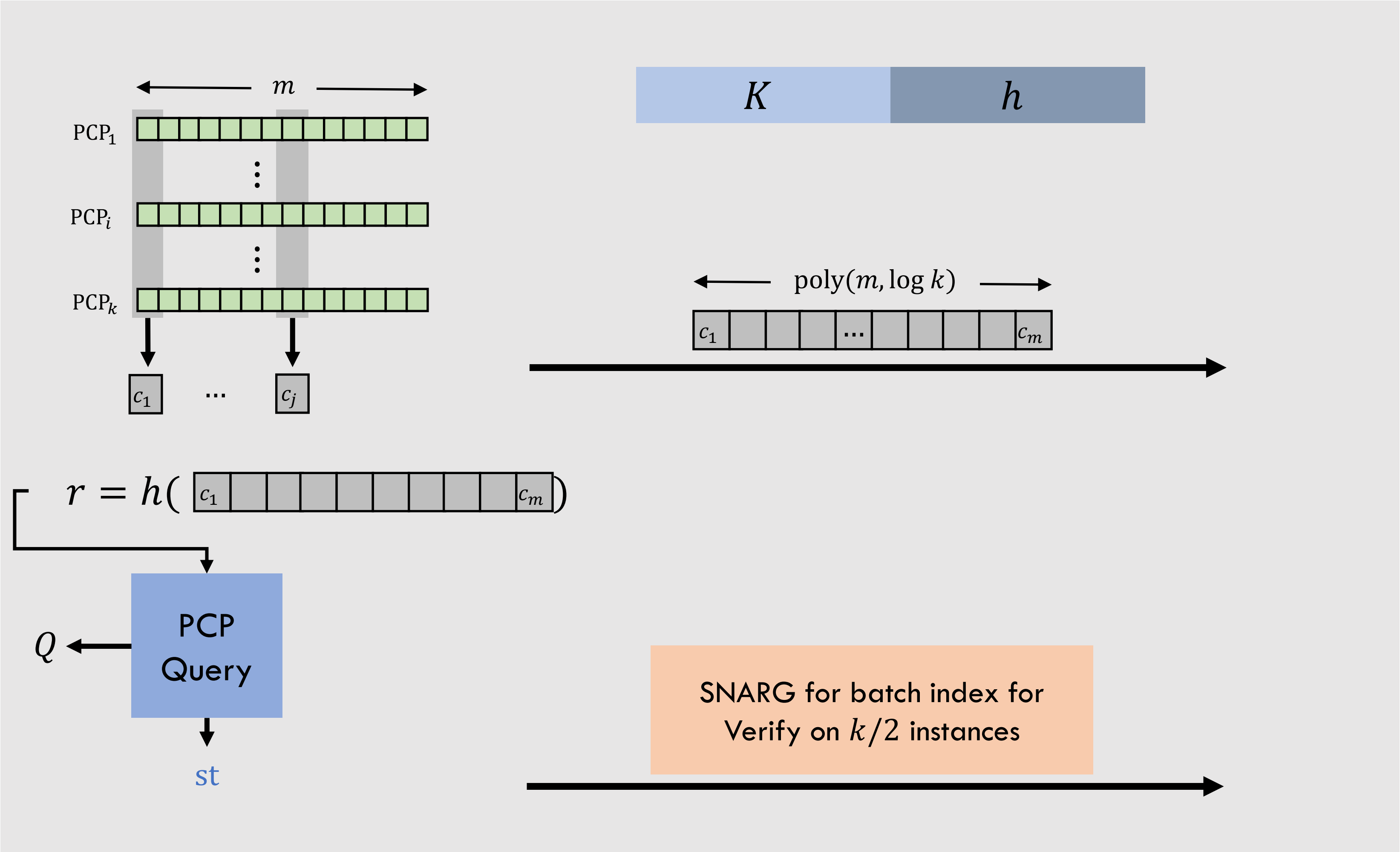
$\forall i \in [k], i \in L_C$

$|\widetilde{\text{Verify}}| \geq 2|\text{Verify}| \geq 2 \, \text{polylog}(k, |C|)$



$k/2$ instances for circuit $\widetilde{\text{Verify}}$

SNARG for Batch Index



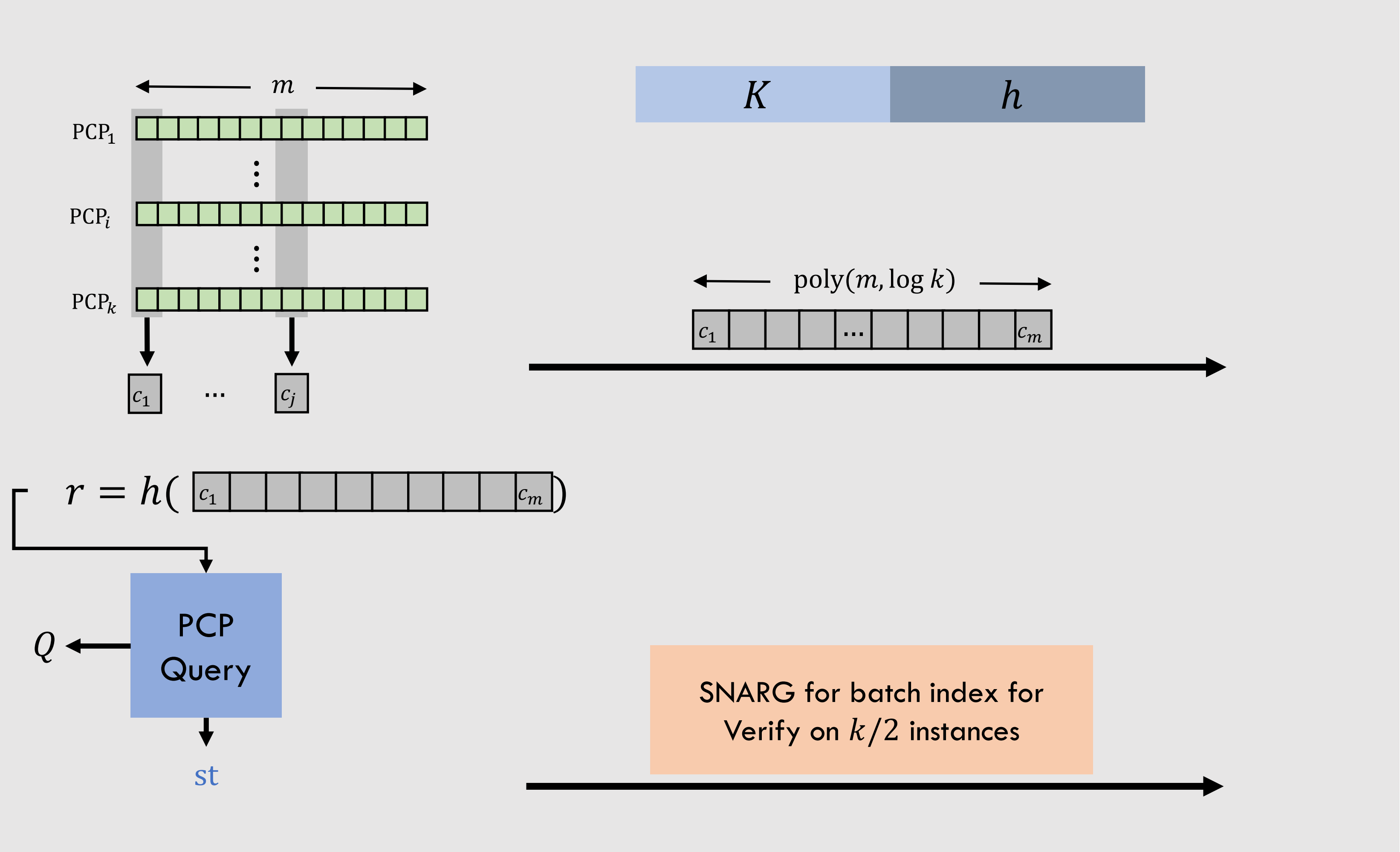
$L_C = \{i \mid \exists w \ s.t. \ C(i,w) = 1\}$

$\forall i \in [k], i \in L_C$

Verify: c_1 … c_m

1. Commitment openings are valid.
2. PCP responses verify on Q, st

SNARG for Batch Index



$L_C = \{i \mid \exists w \ s.t.\ C(i,w) = 1\}$

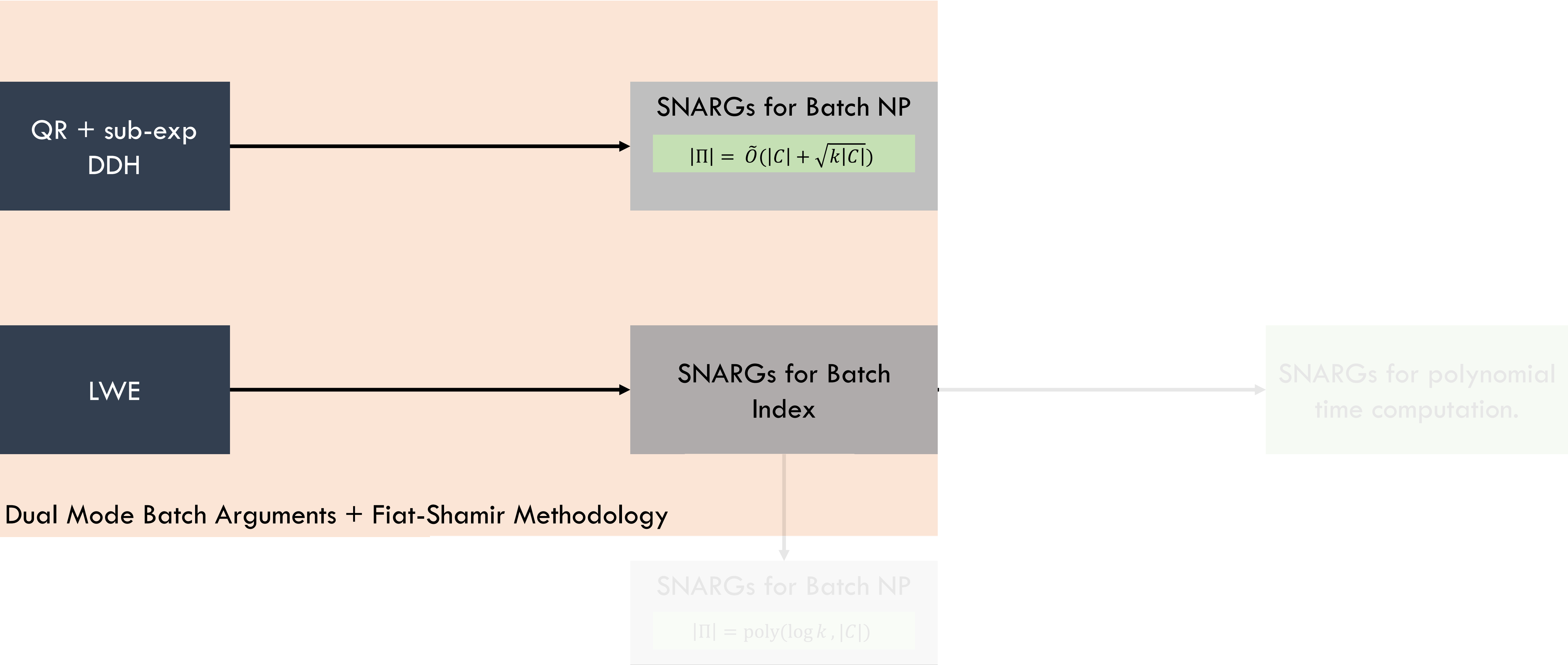
$\forall i \in [k], i \in L_C$

Recurse $\log k$ times

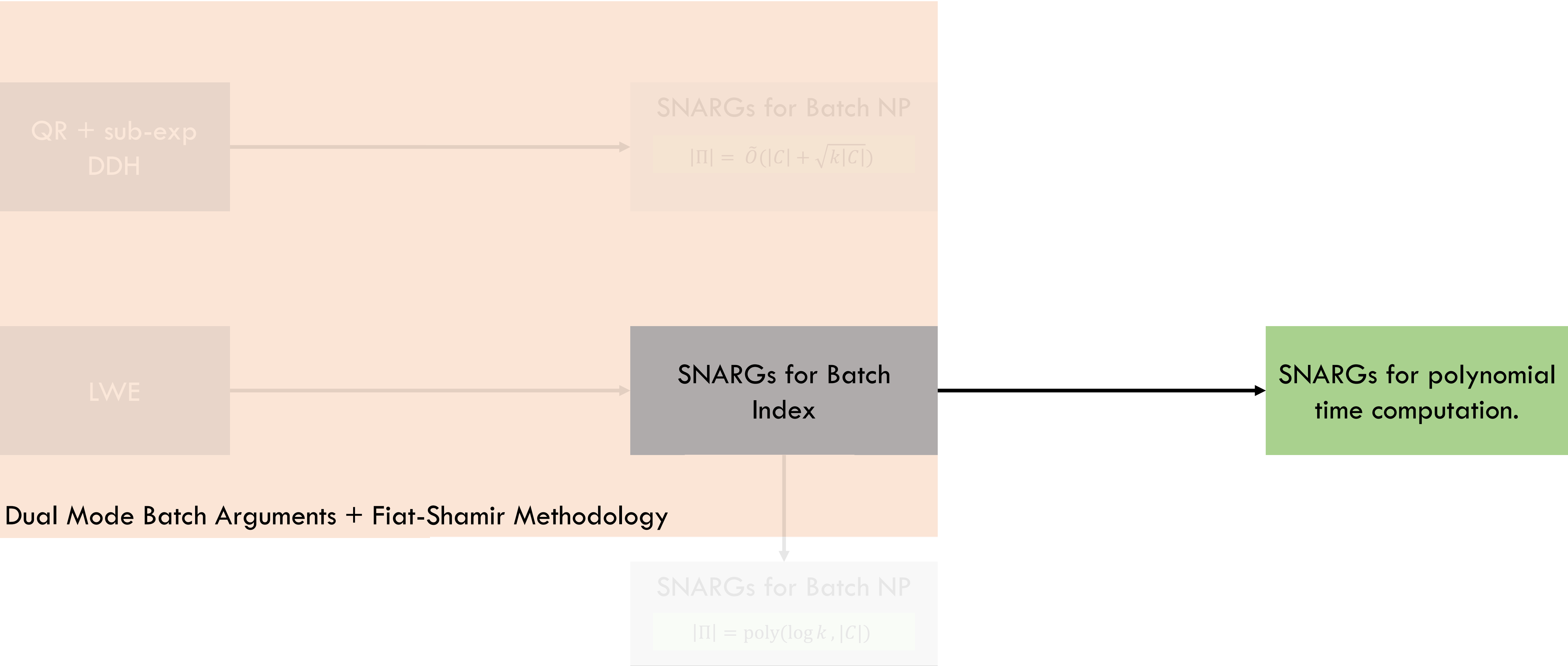
Verify: c_1 c_m

1. Commitment openings are valid.
2. PCP responses verify on Q, st

Results Overview

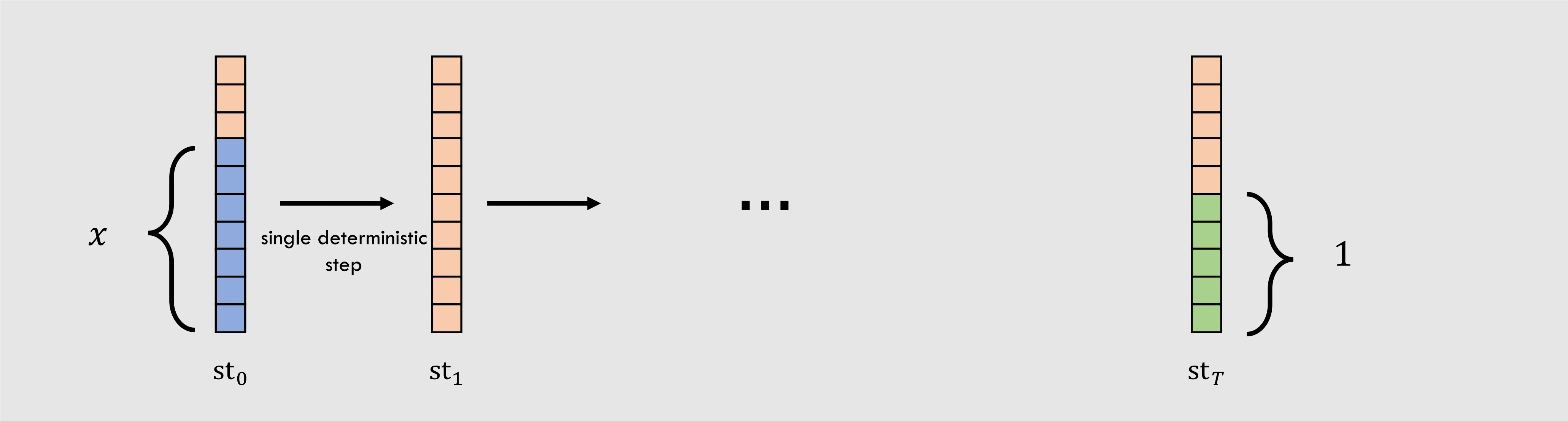


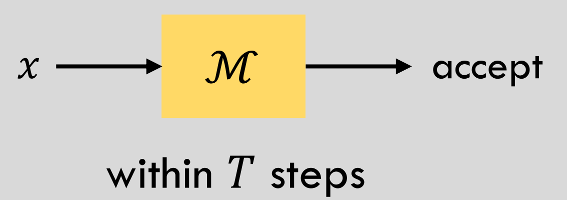
Results Overview



SNARGs for Batch Index → SNARGs for P

Delegation via Batching [Reingold-Rothblum-Rothblum'16]



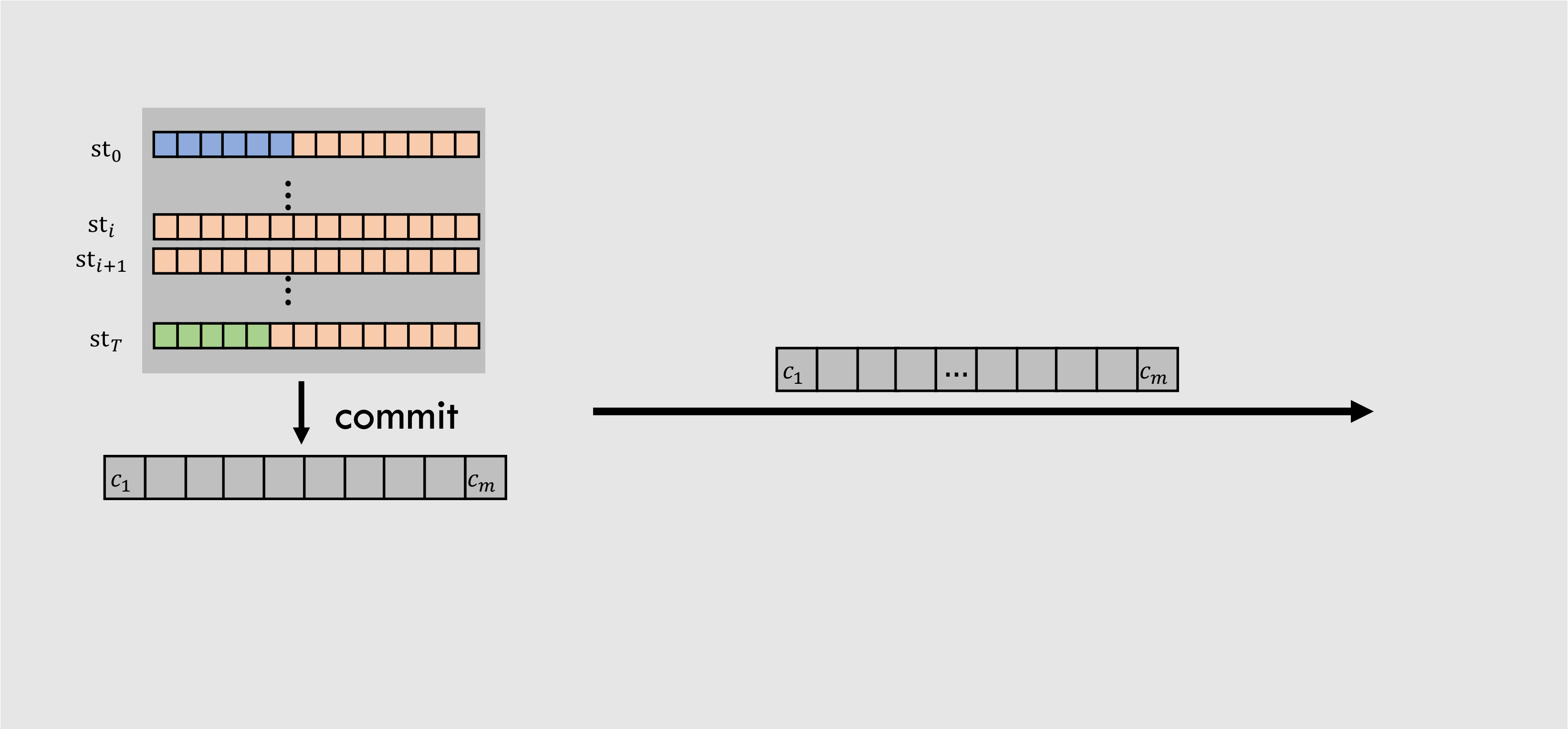


Prove for every $i \in [0, \dots, T-1]$

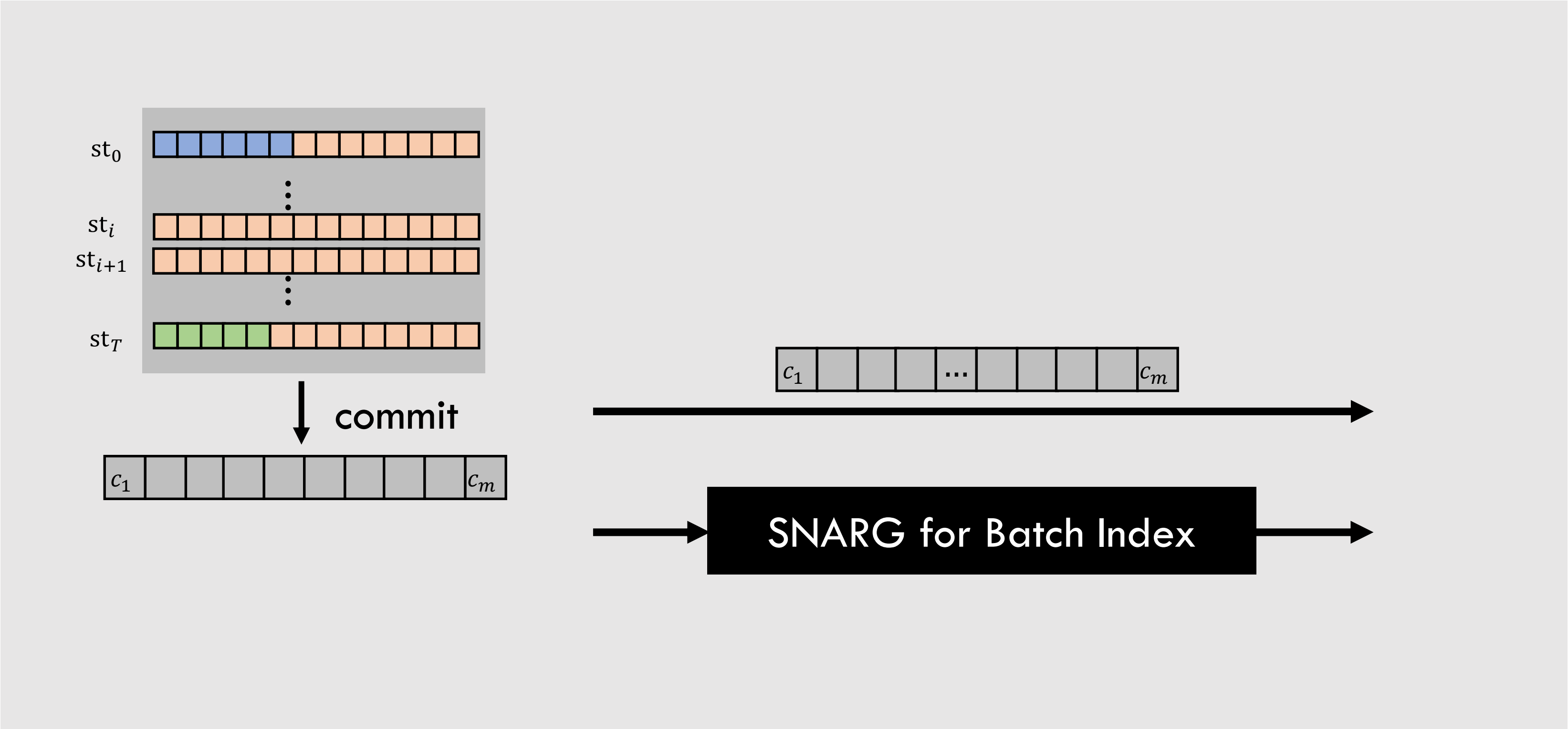
$$\mathsf{st}_i \to \mathsf{st}_{i+1}$$

is the correct transition.

SNARGs for Polynomial-time Computation



SNARGs for Polynomial-time Computation

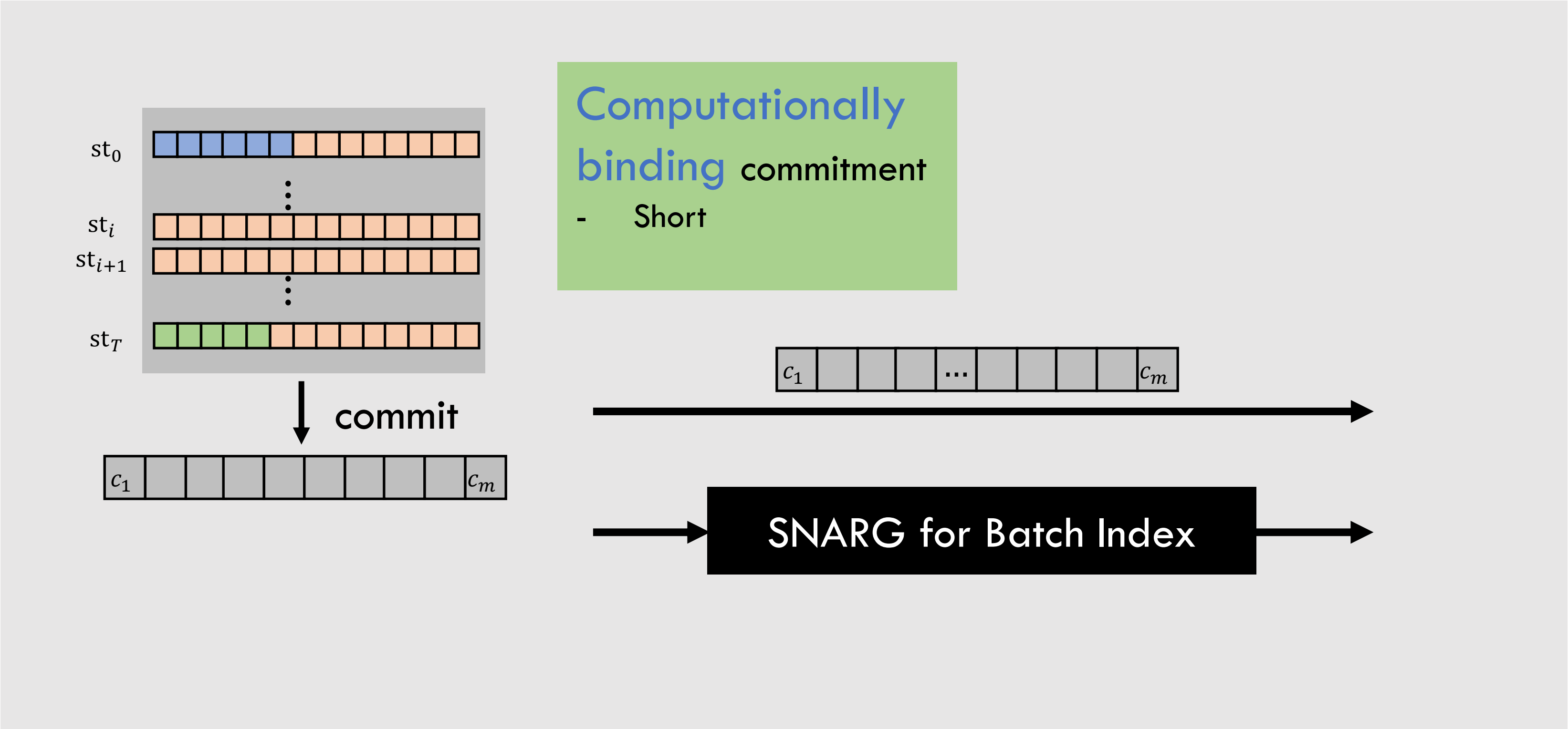


SNARG for Batch Index

For every $i \in [0, \ldots, T-1]$

1. Commitment contains st_i and st_{i+1}
2. Valid transition $\mathrm{st}_i \to \mathrm{st}_{i+1}$

SNARGs for Polynomial-time Computation

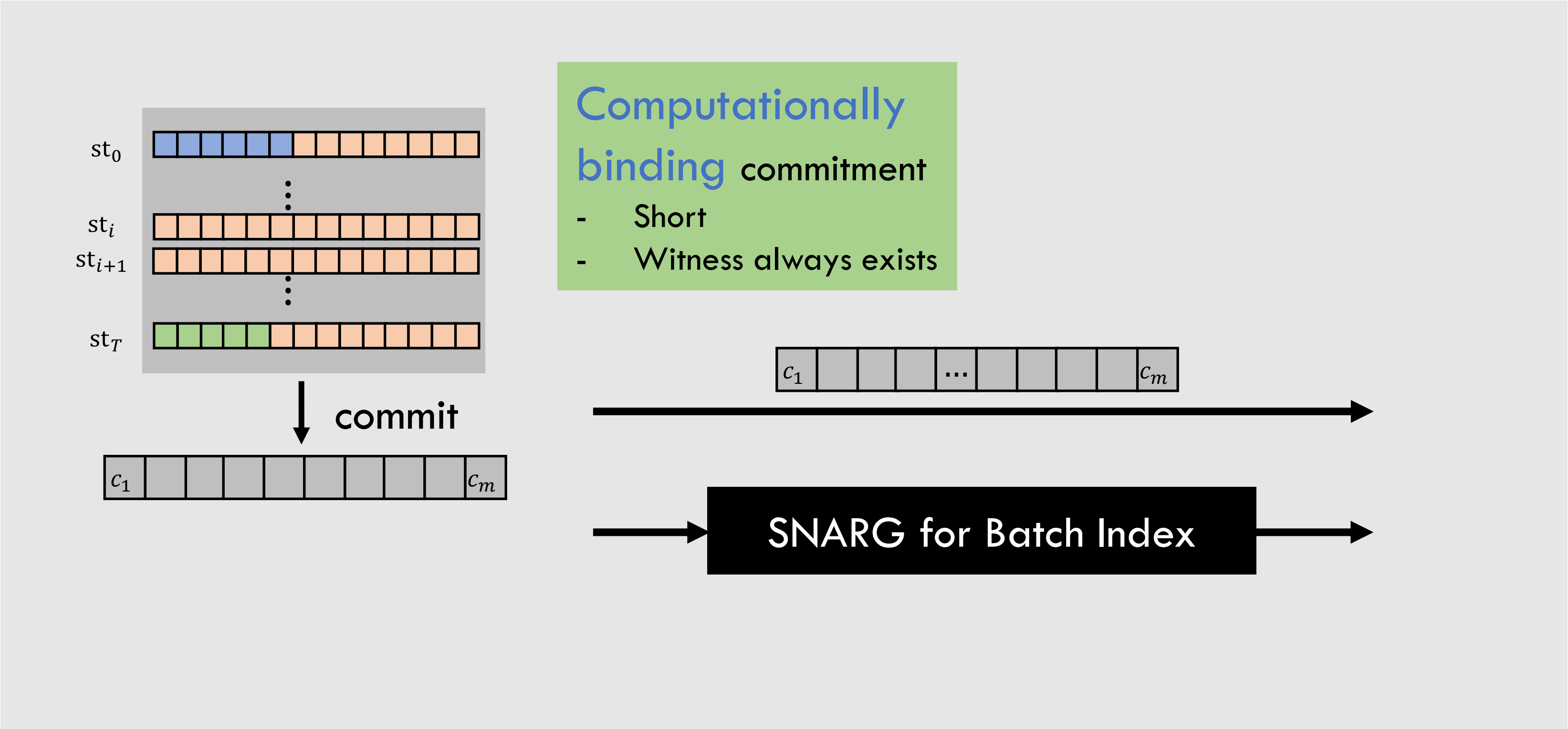


SNARG for Batch Index

For every $i \in [0, \dots, T-1]$

1. Commitment contains st_i and st_{i+1}
2. Valid transition $\mathsf{st}_i \rightarrow \mathsf{st}_{i+1}$

SNARGs for Polynomial-time Computation

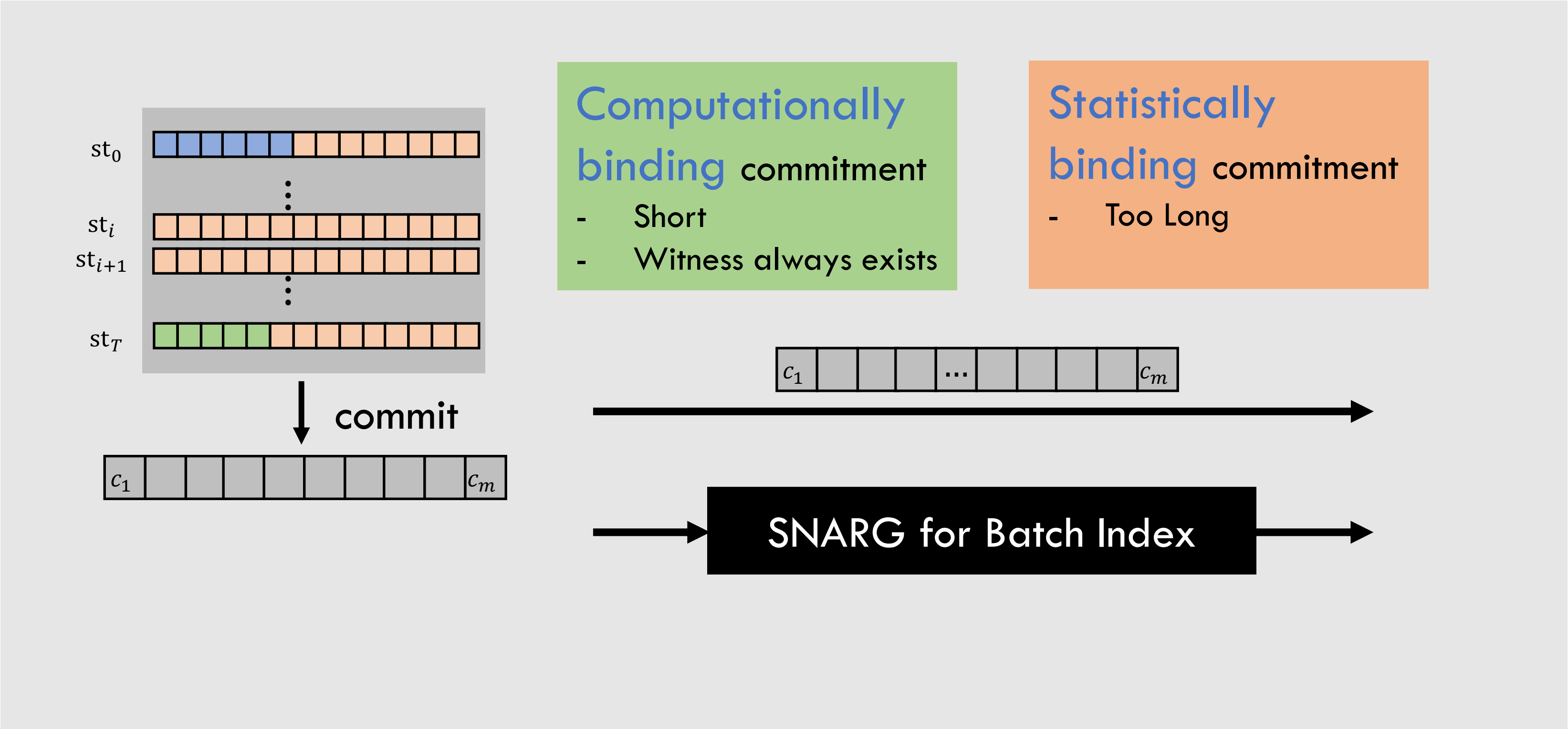


SNARG for Batch Index

For every $i \in [0, \dots, T-1]$

1. Commitment opening to st_i and st_{i+1}
2. Valid transition $\mathsf{st}_i \to \mathsf{st}_{i+1}$

SNARGs for Polynomial-time Computation

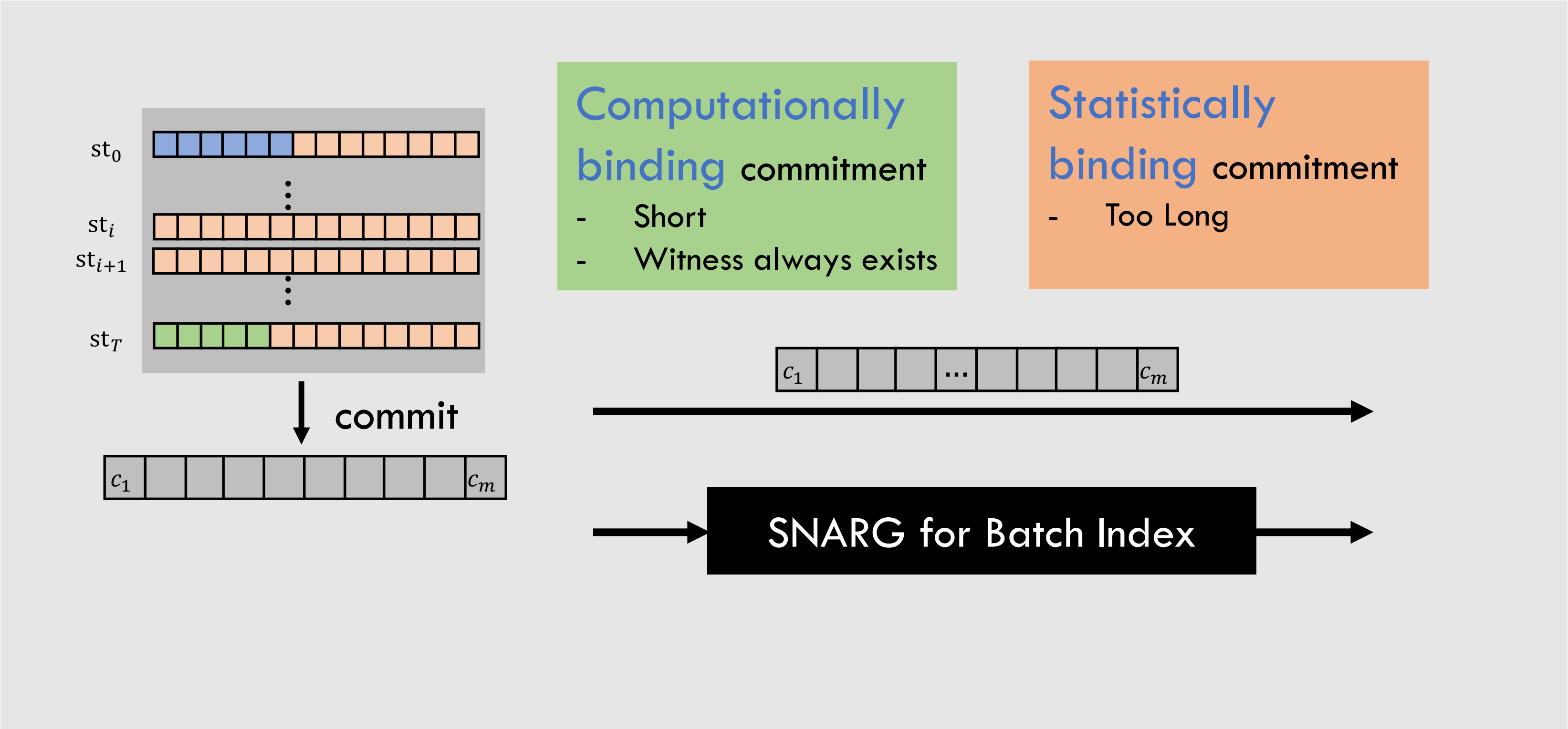


SNARG for Batch Index

For every $i \in [0, \ldots, T-1]$

1. Commitment opening to st_i and st_{i+1}
2. Valid transition $\mathsf{st}_i \to \mathsf{st}_{i+1}$

SNARGs for Polynomial-time Computation



Use SSB Commitments

SNARG for Batch Index

For every $i \in [0, \dots, T-1]$

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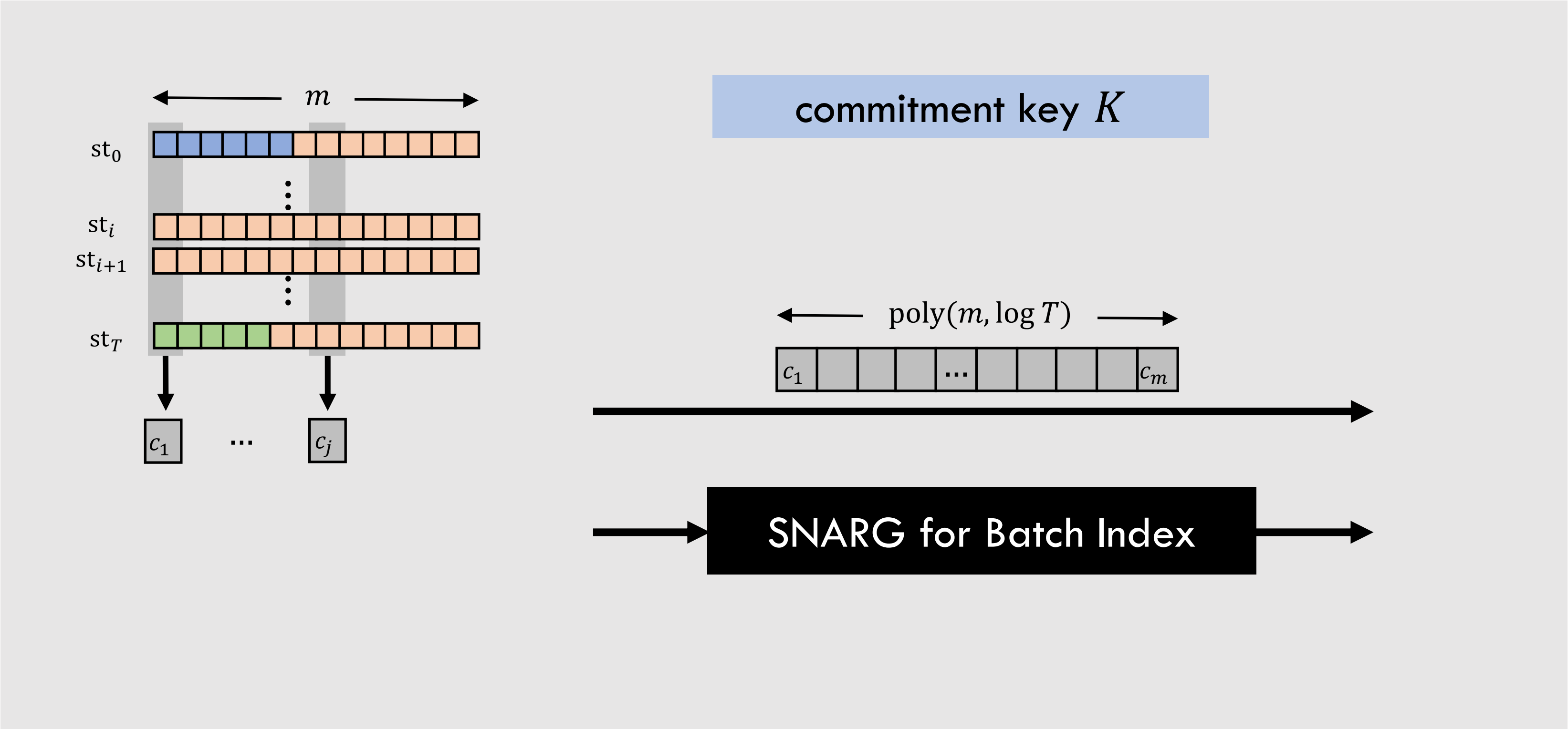
SNARGs for Polynomial-time Computation



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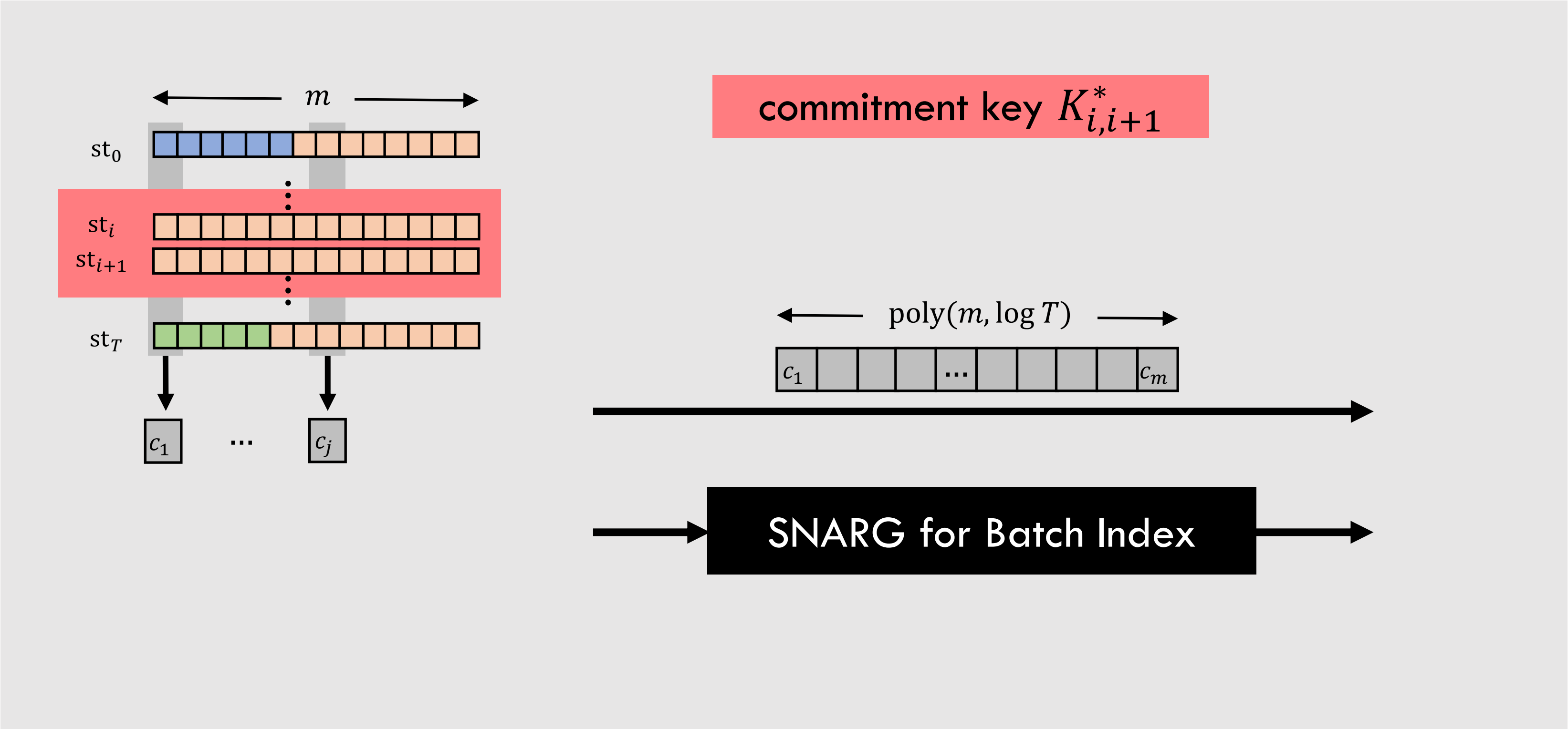

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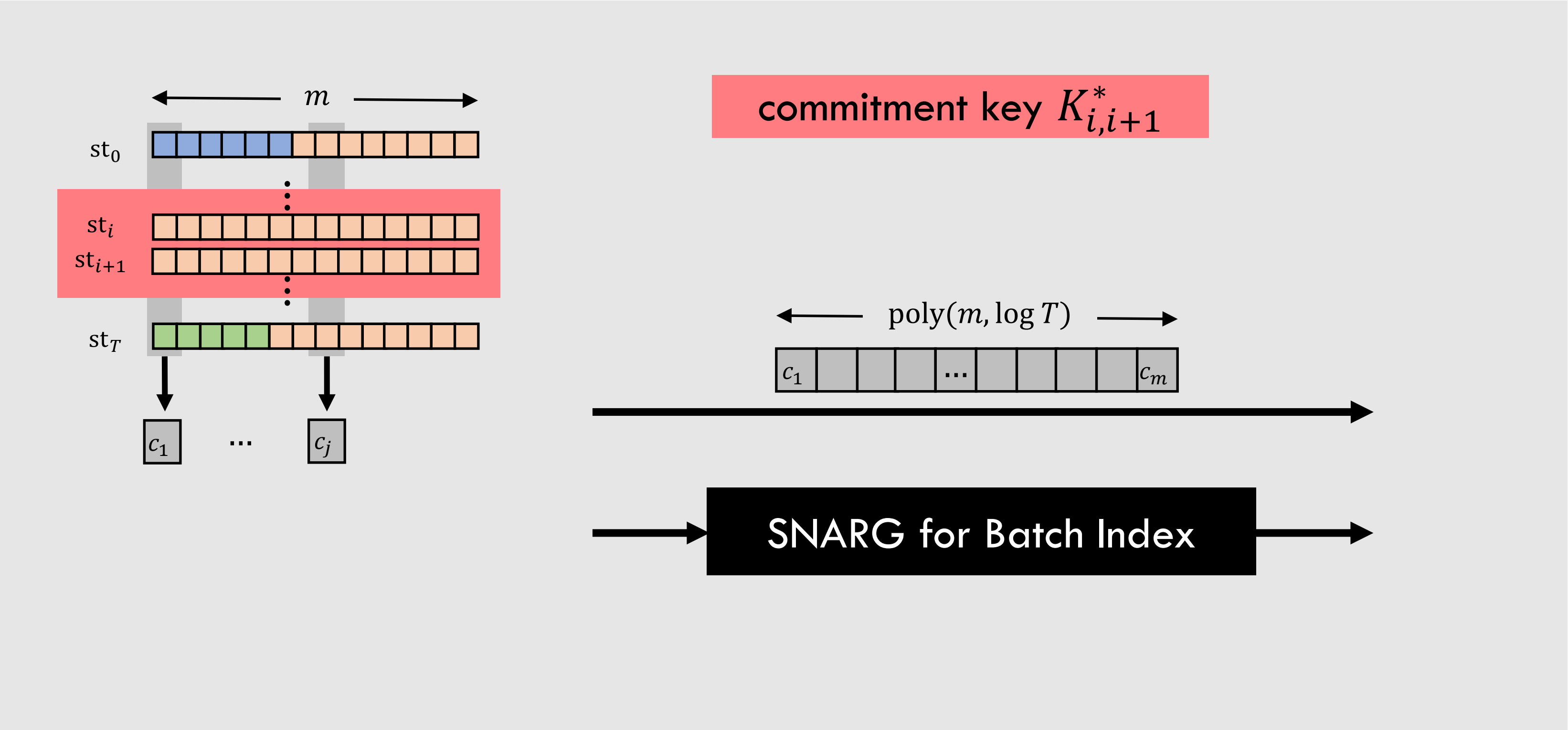
SNARGs for Polynomial-time Computation



Local Soundness

i-th state transition correct

SNARG for Batch Index

For every $i \in [0, \dots, T-1]$

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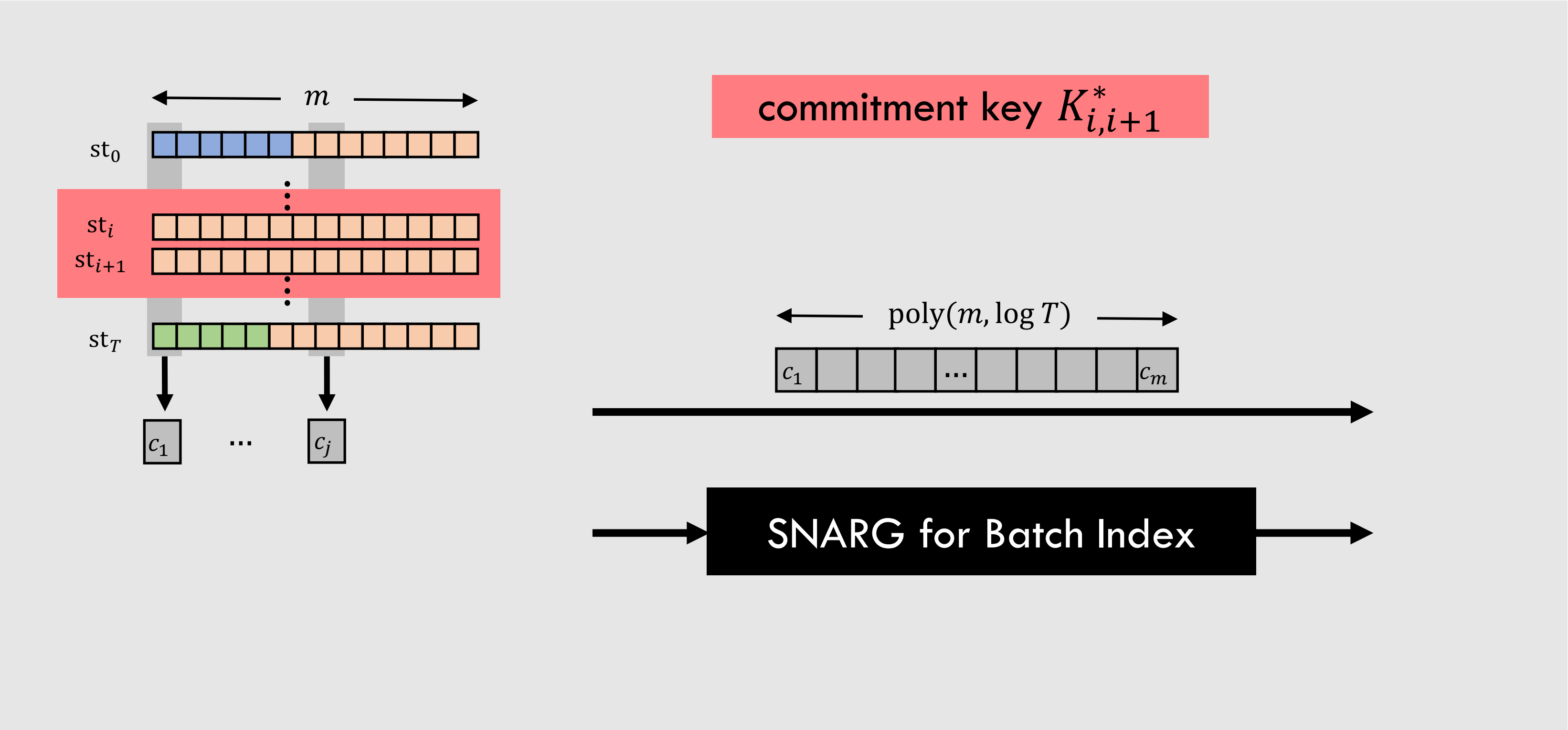
SNARGs for Polynomial-time Computation



Local Soundness

i-th state transition correct

→

Global Soundness

Local soundness at all i

SNARG for Batch Index

For every $i \in [0, \dots, T-1]$

1. Commitment opening to st_i and st_{i+1}
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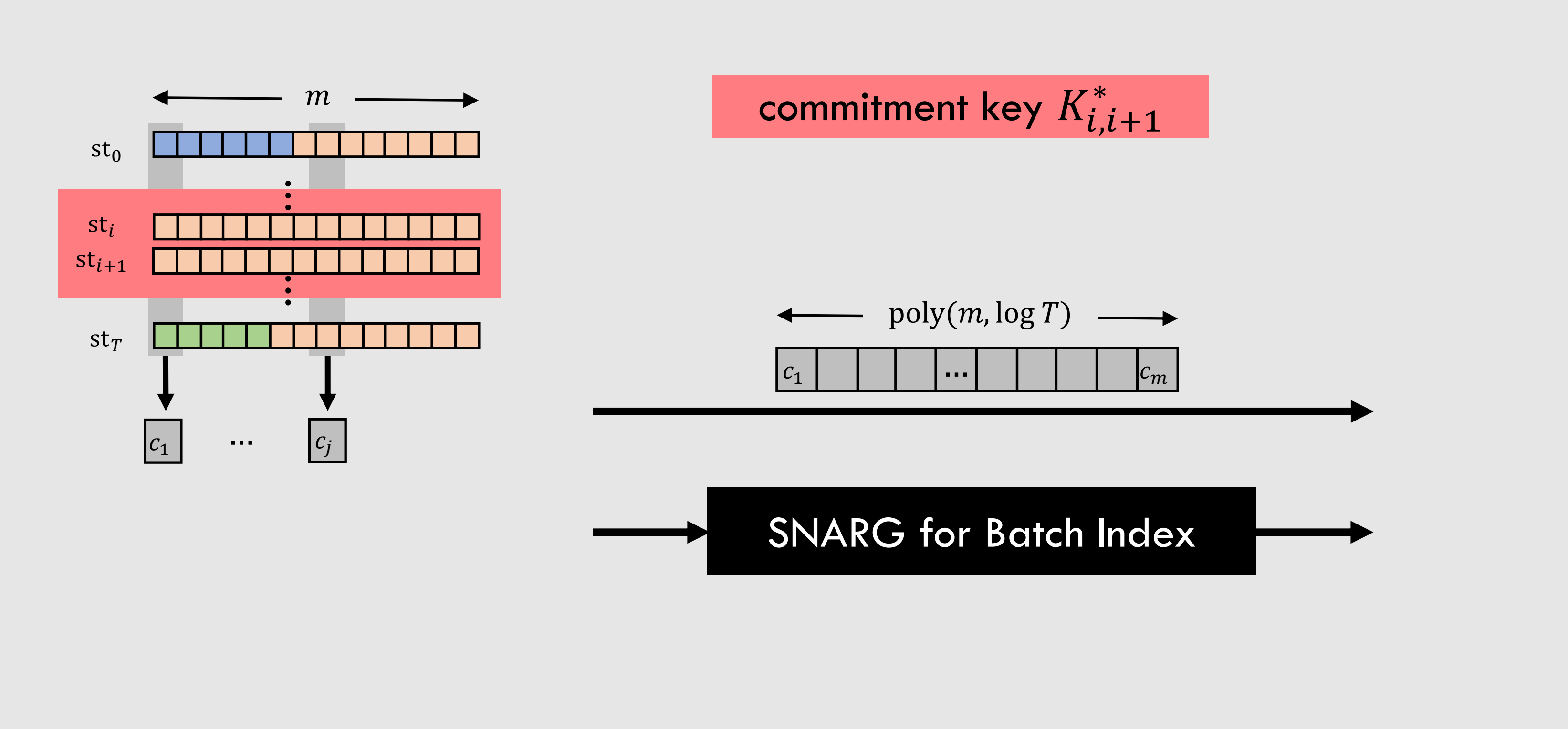
SNARGs for Polynomial-time Computation



No-Signaling SSB Commitment Scheme [González-Zacharakis'21]

Local Soundness

i-th state transition correct

Global Soundness

Local soundness at all i

SNARG for Batch Index

For every $i \in [0, \dots, T-1]$

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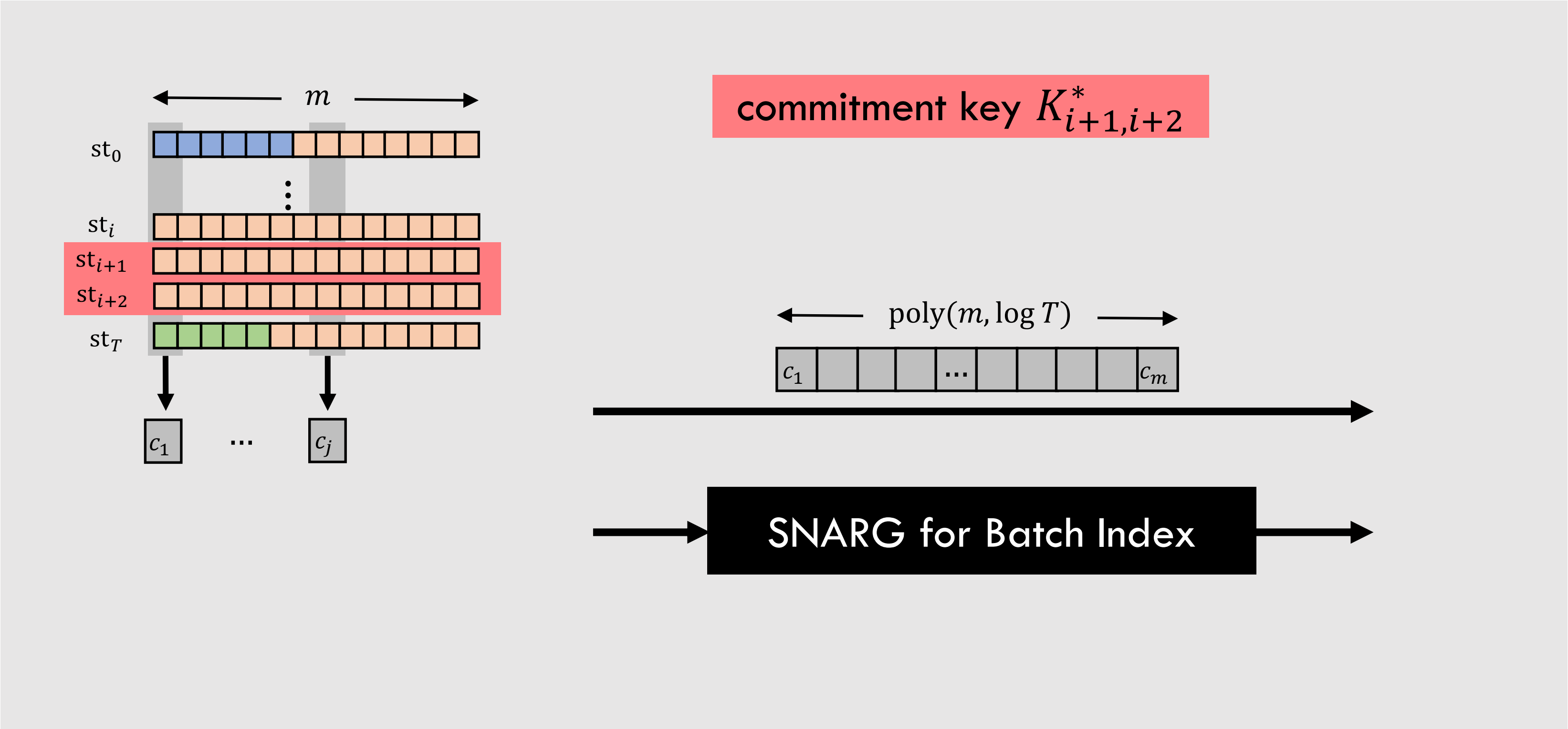
SNARGs for Polynomial-time Computation



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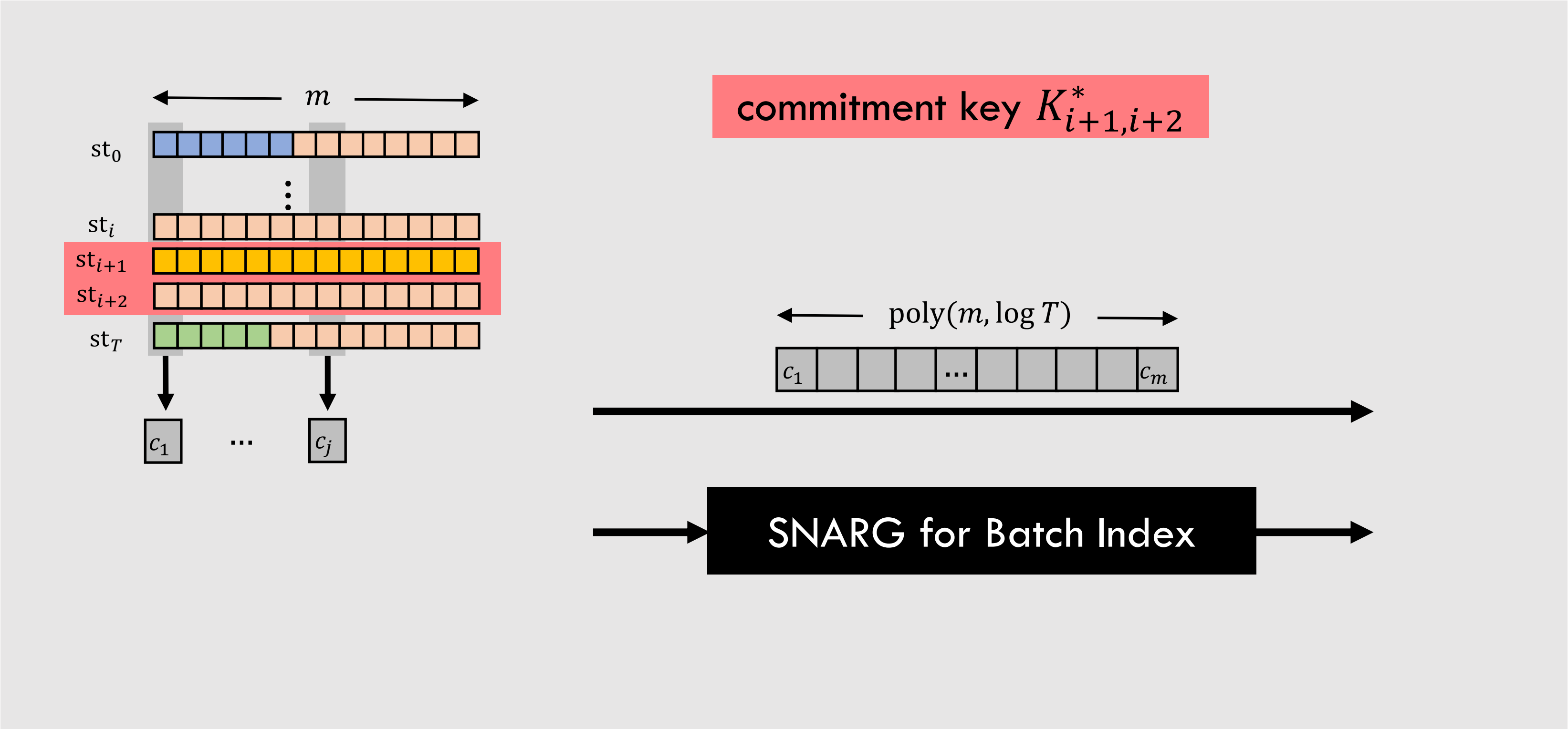
SNARGs for Polynomial-time Computation



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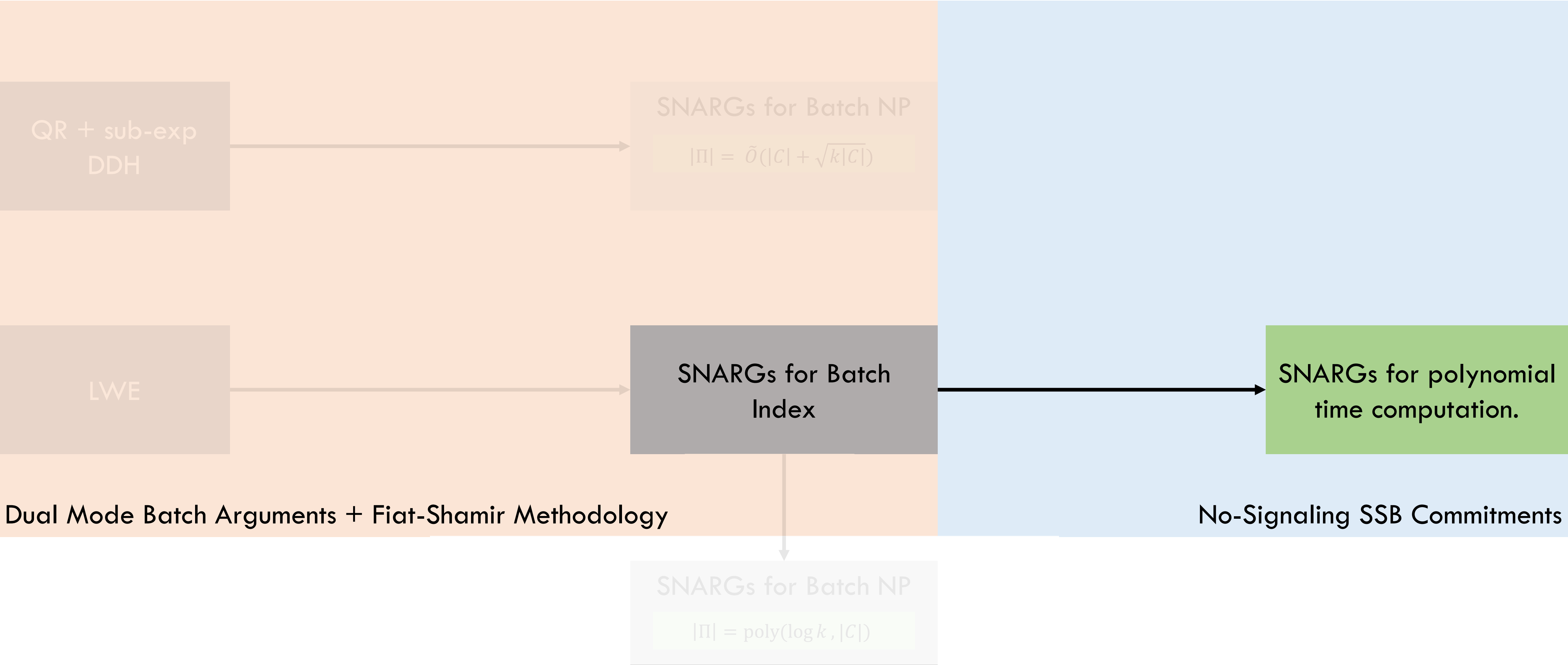
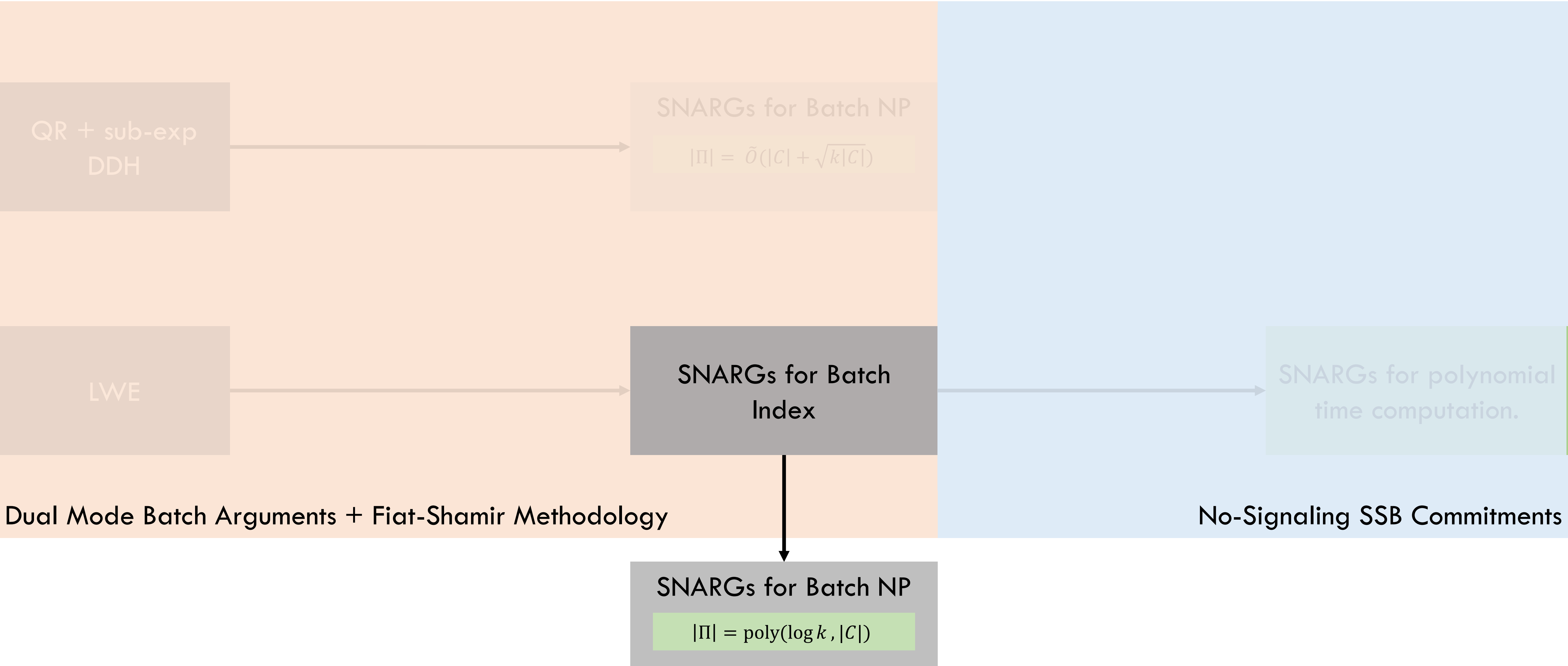
Results Overview

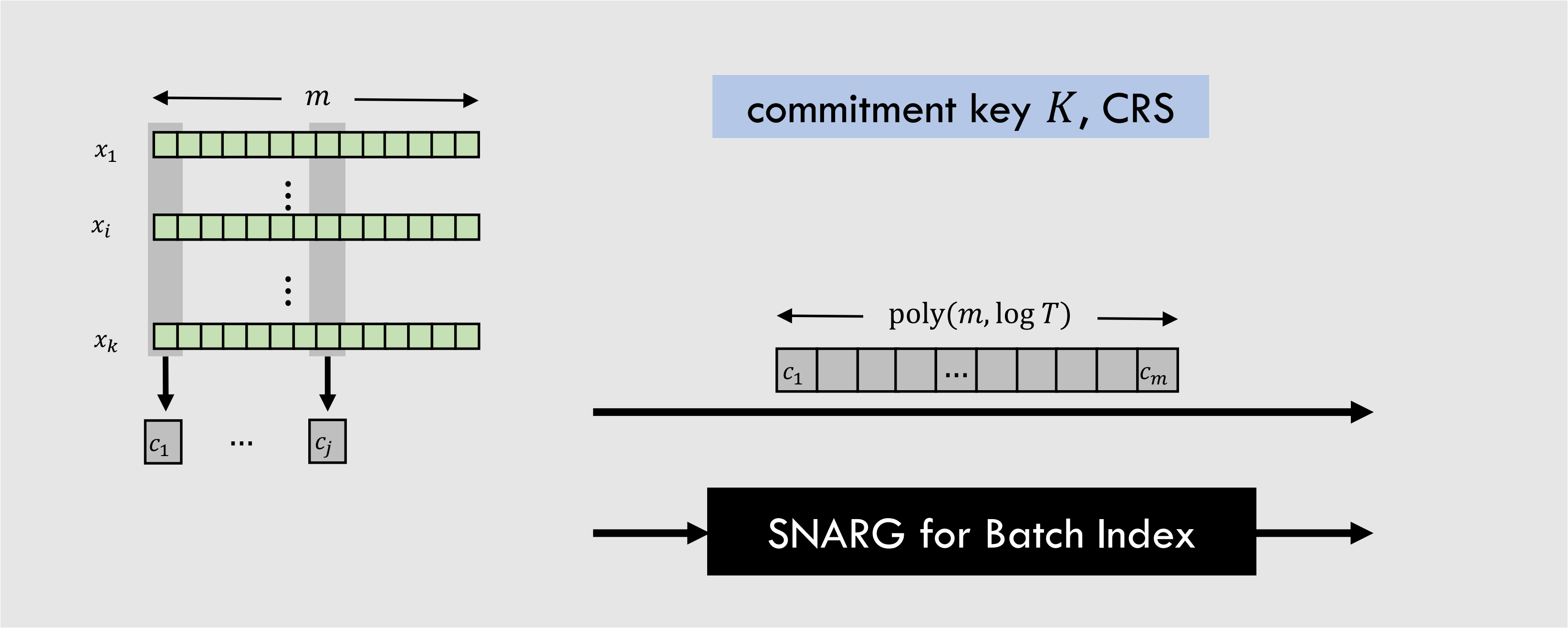


Results Overview



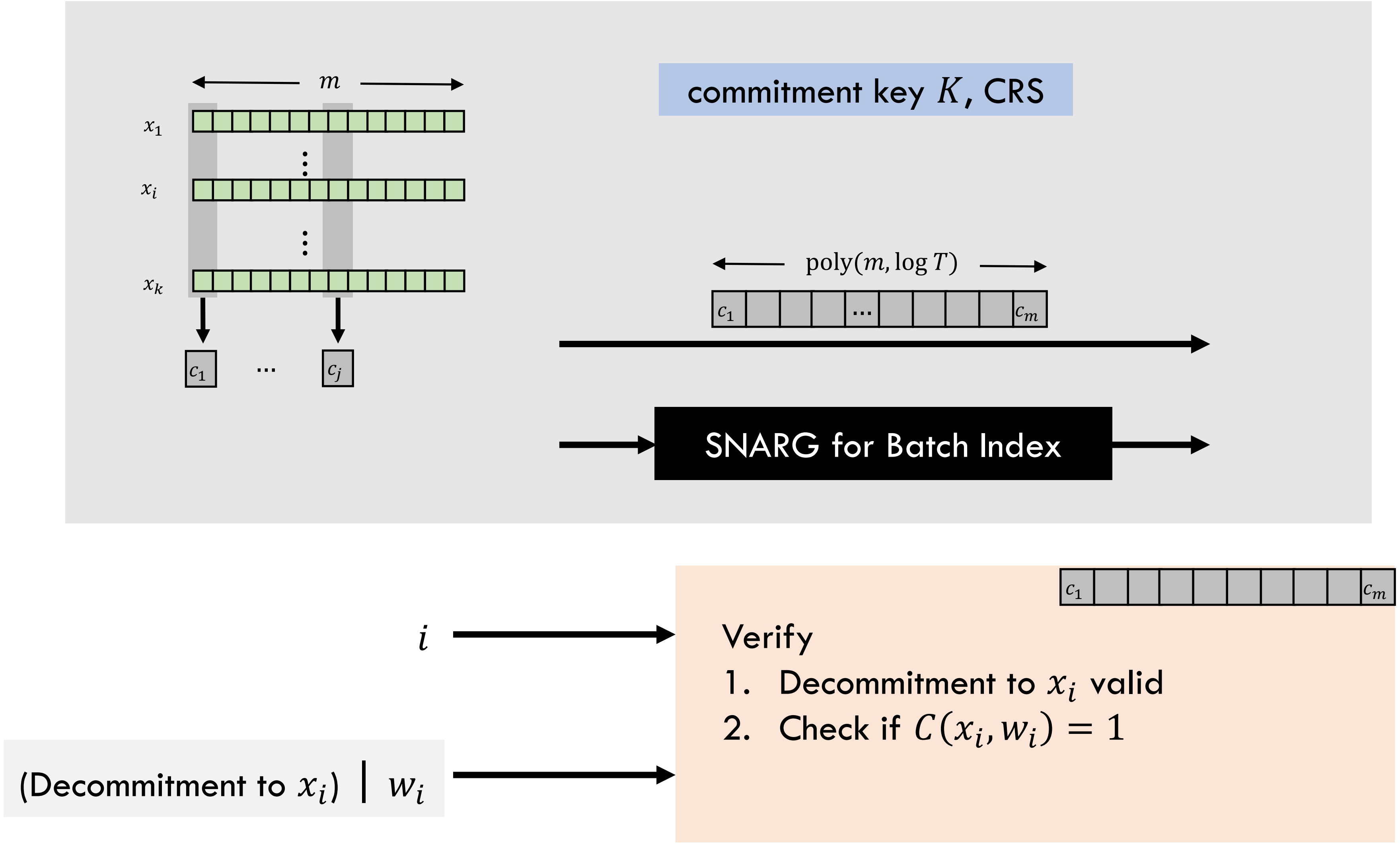
SNARGs for Batch Index $\rightarrow$ SNARGs for Batch NP

SNARGs for Batch NP



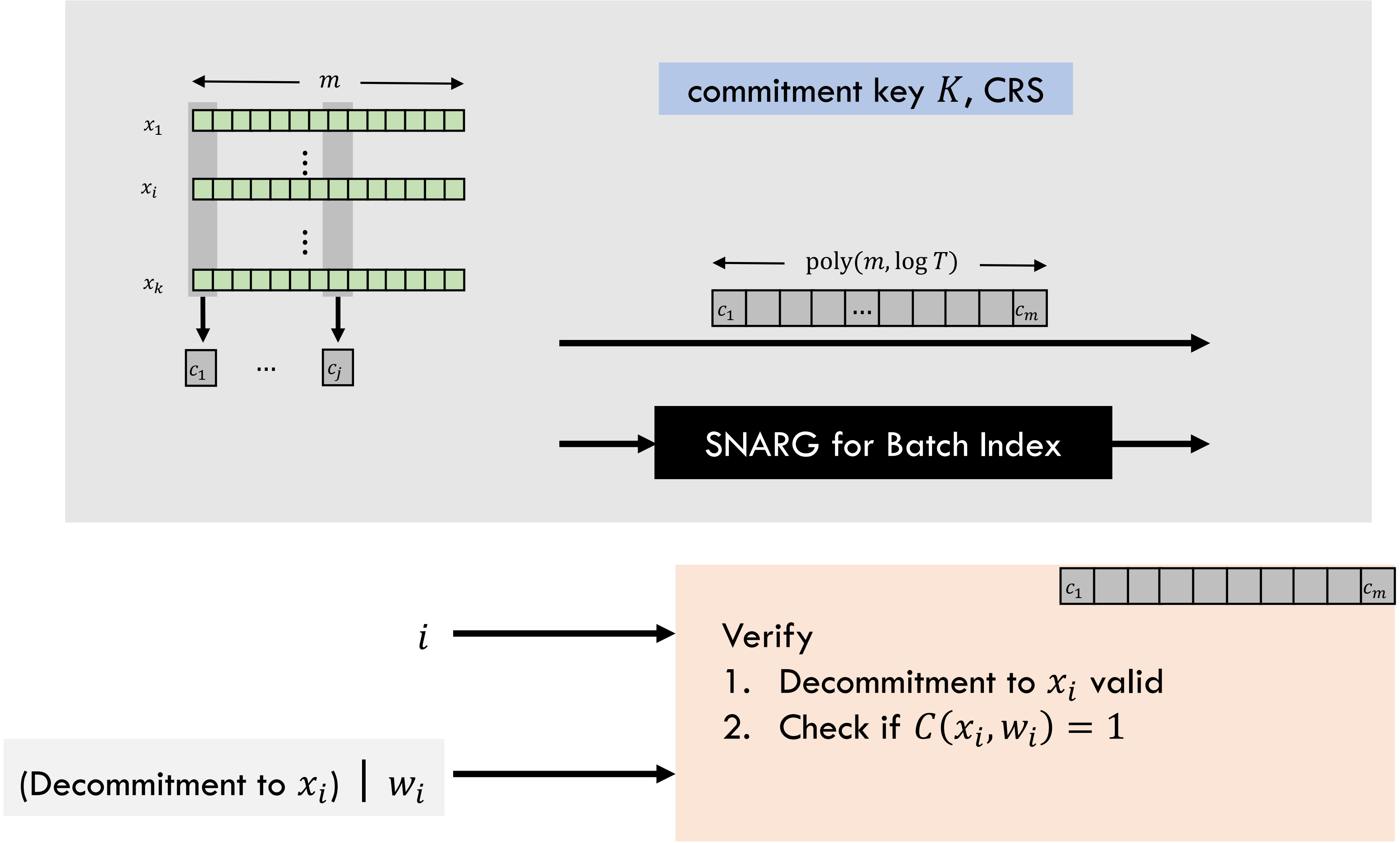
Use fixed randomness for the commitment (say 0)

SNARGs for Batch NP



Use fixed randomness for the commitment (say 0)

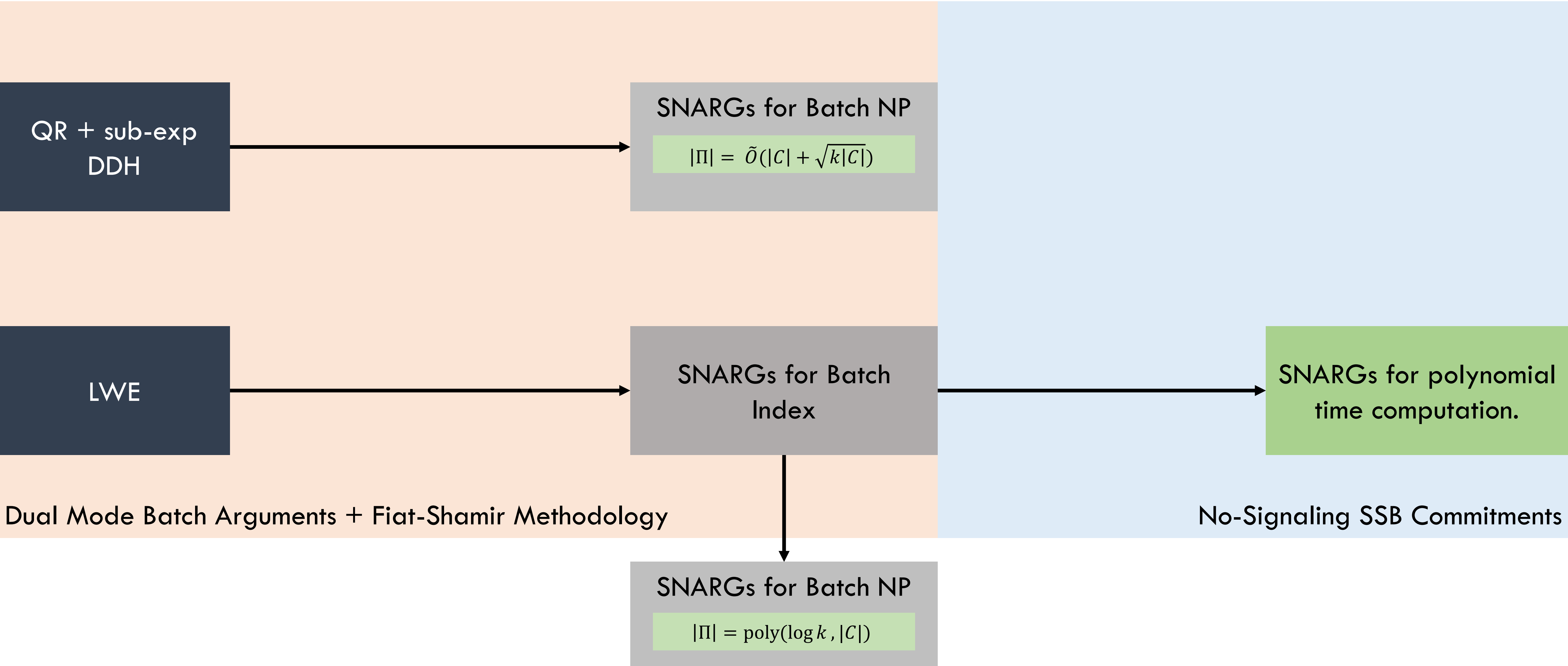
SNARGs for Batch NP



Use fixed randomness for the commitment (say 0)

Require SSB Commitments to rely on SNARG soundness.

Recap


QR + sub-exp DDH
SNARGs for Batch NP
$|\Pi| = \tilde{O}(|C| + \sqrt{k|C|})$
LWE
SNARGs for Batch Index
SNARGs for polynomial time computation.
Dual Mode Batch Arguments + Fiat-Shamir Methodology
No-Signaling SSB Commitments
SNARGs for Batch NP
$|\Pi| = \mathrm{poly}(\log k, |C|)$

Open Questions

Achieving succinct delegation from DDH?

Incrementally verifiable computation from LWE?

Establishing hardness of complexity classes such as PPAD, PLS?

Thank you. Questions?

Arka Rai Choudhuri

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ia.cr/2021/807, ia.cr/2021/808