Correlation Intractability and SNARGs from Sub-exponential DDH

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MIT

Jiaheng Zhang
UC Berkeley

Succinct Non-Interactive Arguments (SNARGs)

$\mathcal{M}$, x

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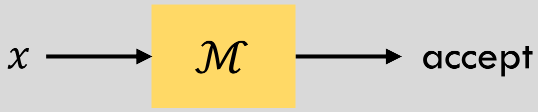

Succinct Non-Interactive Arguments (SNARGs)

$\mathcal{M}$, x

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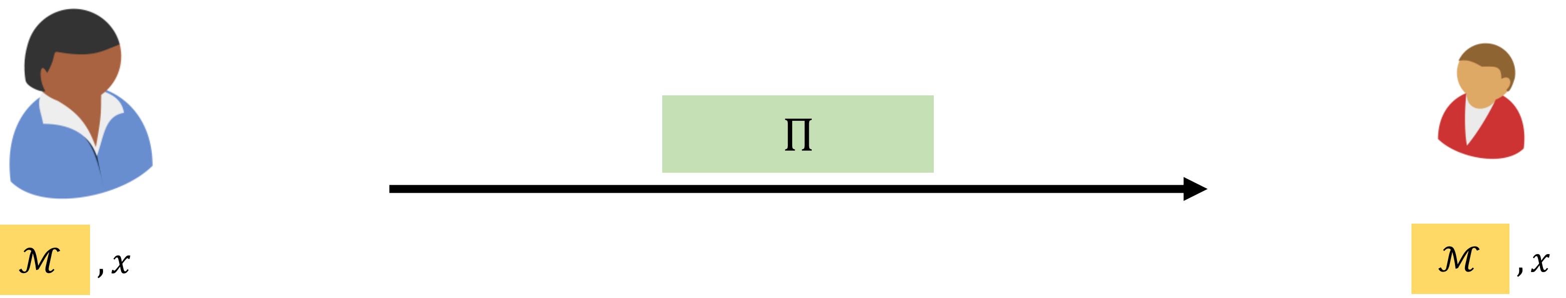
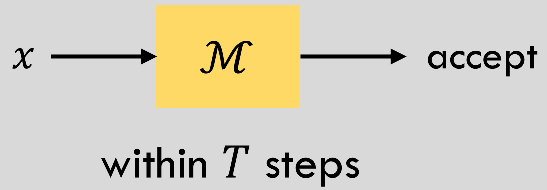
$x \longrightarrow \mathcal{M} \longrightarrow$ accept

within T steps

 wants to delegate computation to 

Succinct Non-Interactive Arguments (SNARGs)





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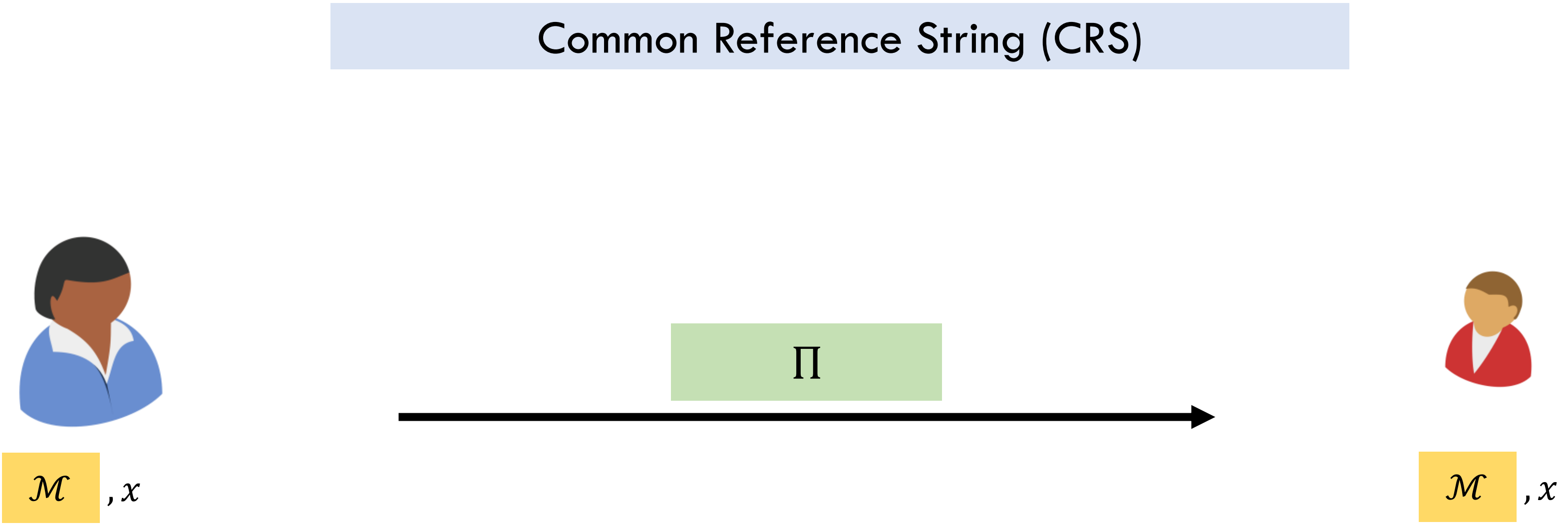
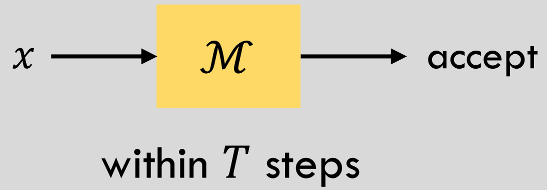
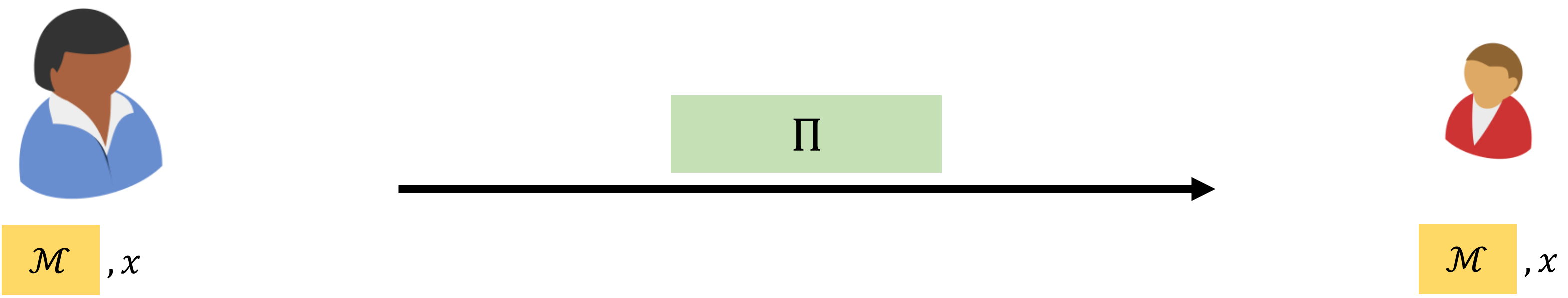




Succinct Non-Interactive Arguments (SNARGs)

Common Reference String (CRS)



Π is publicly verifiable

$x \rightarrow \mathcal{M} \rightarrow$ accept

within T steps

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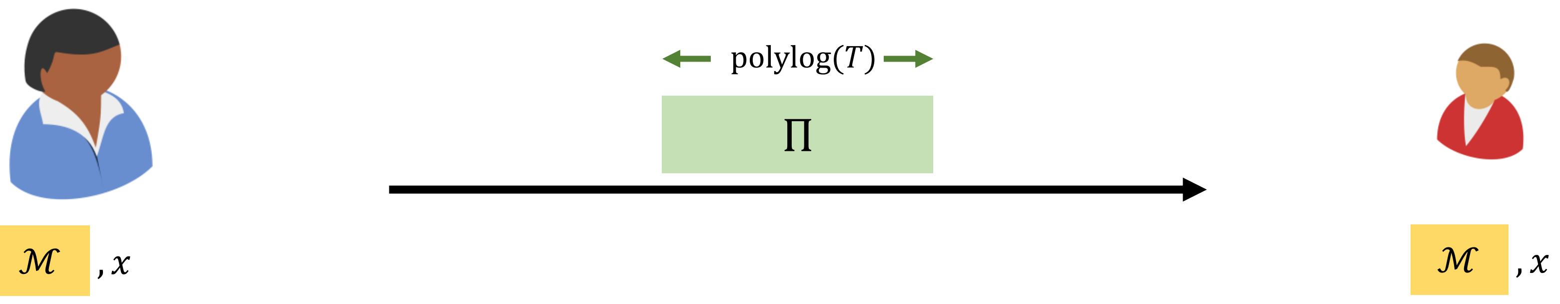
Common Reference String (CRS)



Verifier running time:
polylog(T)

Π is publicly verifiable

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within T steps

Succinct Non-Interactive Arguments (SNARGs)

Common Reference String (CRS)

$\mathcal{M}$, x

← polylog(T) →

Π

$\mathcal{M}$, x

Verifier running time:
polylog(T)

Π is publicly verifiable

x → $\mathcal{M}$ → accept

within T steps

No PPT can produce accepting Π if

x → $\mathcal{M}$ ✗→ accept

within T steps

Succinct Non-Interactive Arguments (SNARGs)

Common Reference String (CRS)

$\leftarrow$ polylog(T) $\rightarrow$

Π

$\mathcal{M}$, x

$\mathcal{M}$, x

Verifier running time: polylog(T)

Π is publicly verifiable

$x \longrightarrow \mathcal{M} \longrightarrow$ accept

within T steps

No PPT can produce accepting x, Π if

$x \longrightarrow \mathcal{M} \not\longrightarrow$ accept

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Succinct Non-Interactive Arguments (SNARGs)

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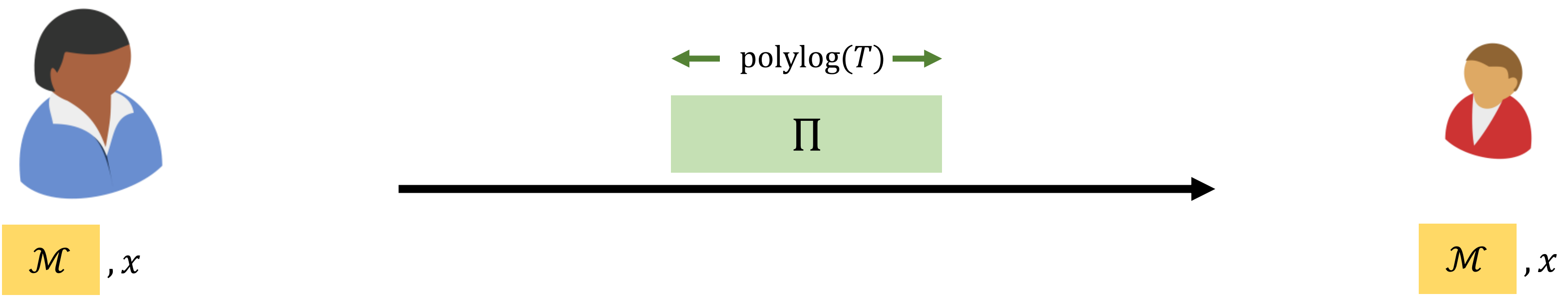

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What kind of computation can we hope to delegate based on standard assumptions?

Succinct Non-Interactive Arguments (SNARGs)

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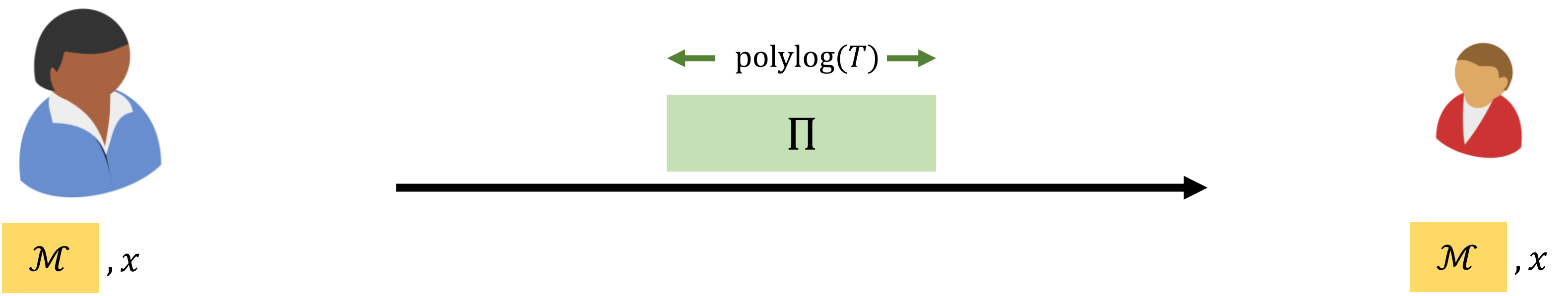

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What kind of computation can we hope to delegate based on standard assumptions?

Nondeterministic polynomial-time computation (NP)? Unlikely! [Gentry-Wichs'11]

SNARGs for Batch NP (or BARGs)

CRS

$C, x_1, \cdots, x_k$

$C, x_1, \cdots, x_k$

$\text{SAT} = \{(C, x) \mid \exists w \ s.t. \ C(x, w) = 1\}$

$\forall i \in [k], (C, x_i) \in \text{SAT}$

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No PPT  can produce accepting Π if

$\exists i^* \in [k], (C, x_{i^*}) \notin \mathrm{SAT}$

$\mathrm{SAT} = \{(C, x) \mid \exists w \ s.t. \ C(x, w) = 1\}$

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SNARGs for Batch NP (or BARGs)

CRS

$C, x_1, \cdots, x_k$

$w_1, \dots, w_k$ →

$C, x_1, \cdots, x_k$

Π is publicly verifiable

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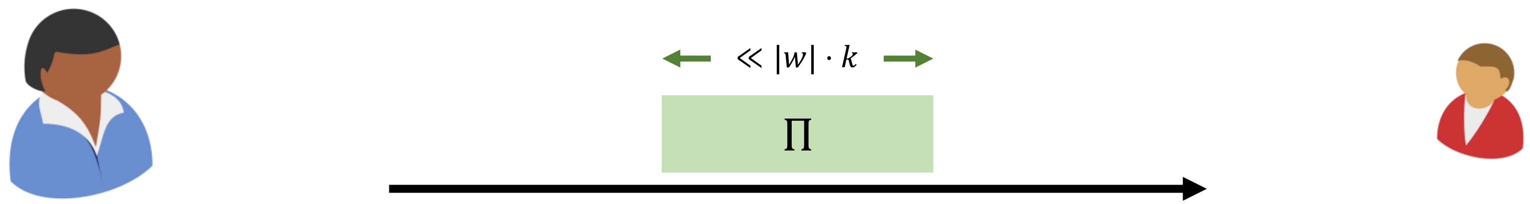

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SNARGs for Batch NP (or BARGs)

CRS

$C, x_1, \cdots, x_k$

$\ll |w| \cdot k$

Π

$C, x_1, \cdots, x_k$

Verifier running time:
$k \cdot |x| + |\Pi|$

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Usefulness of BARGs

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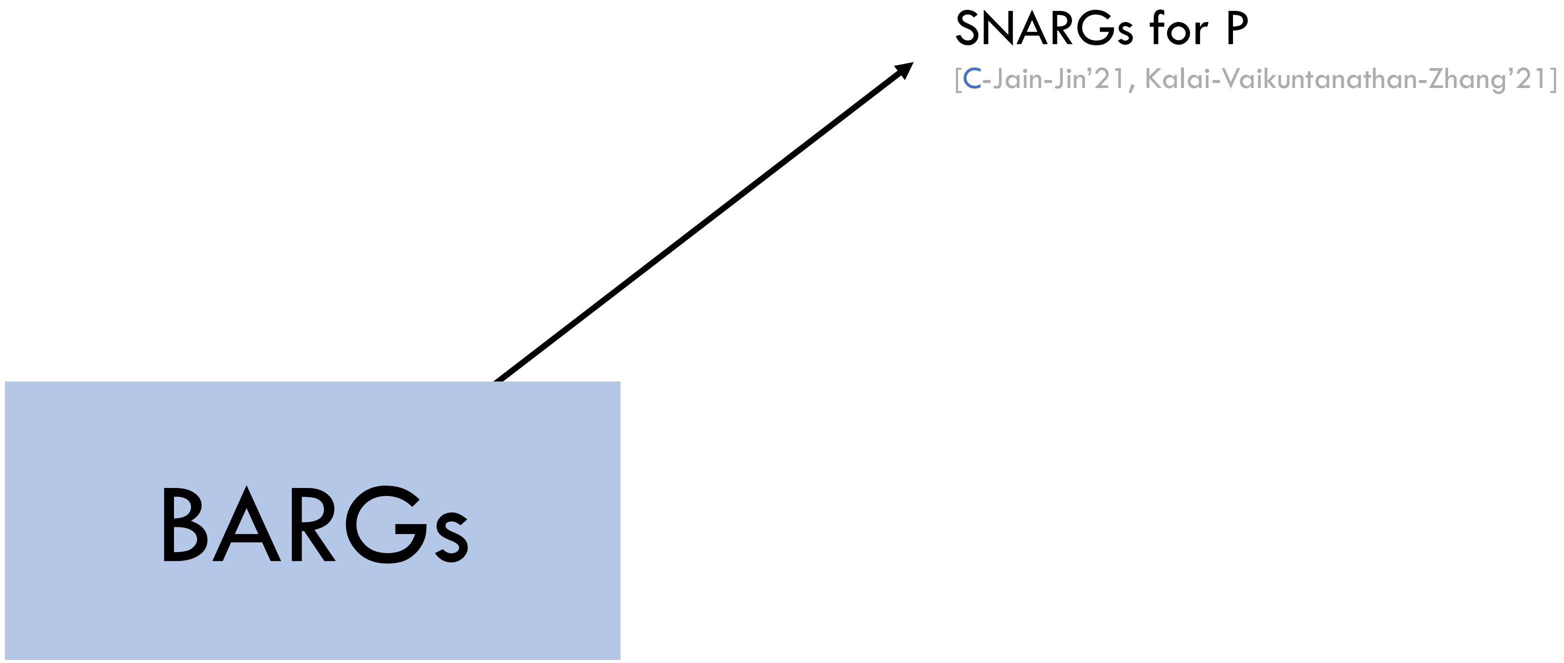

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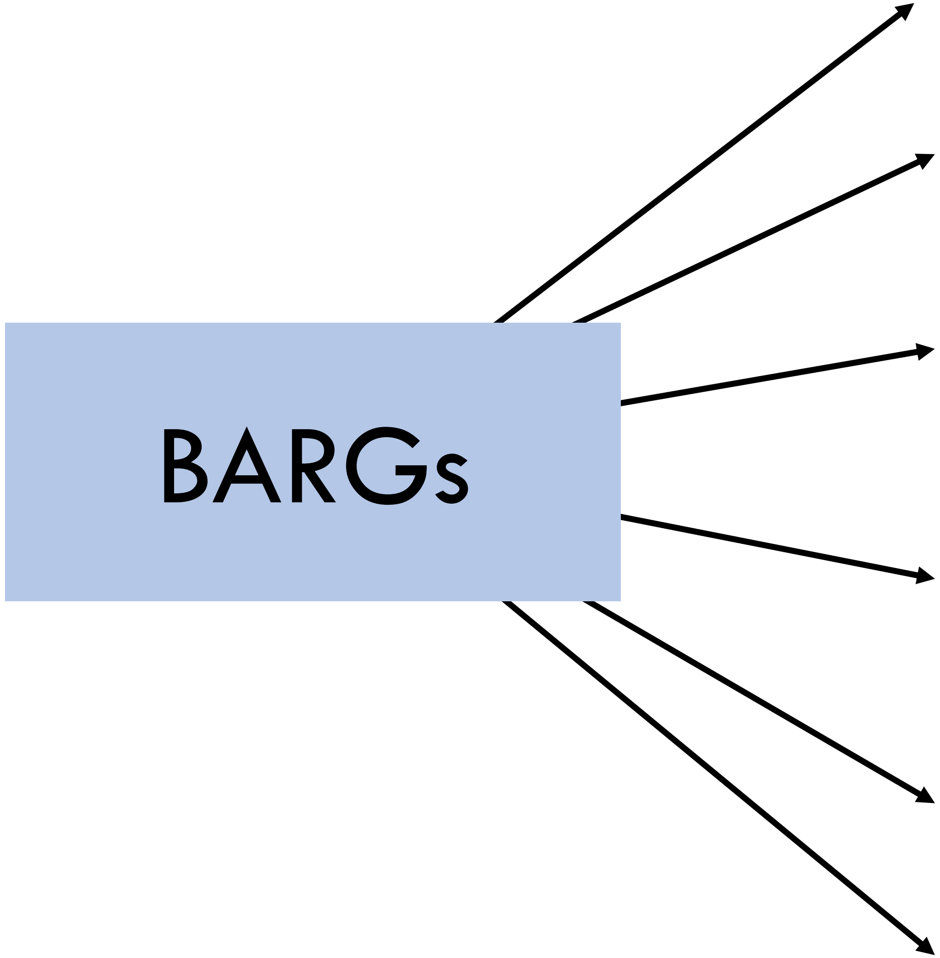
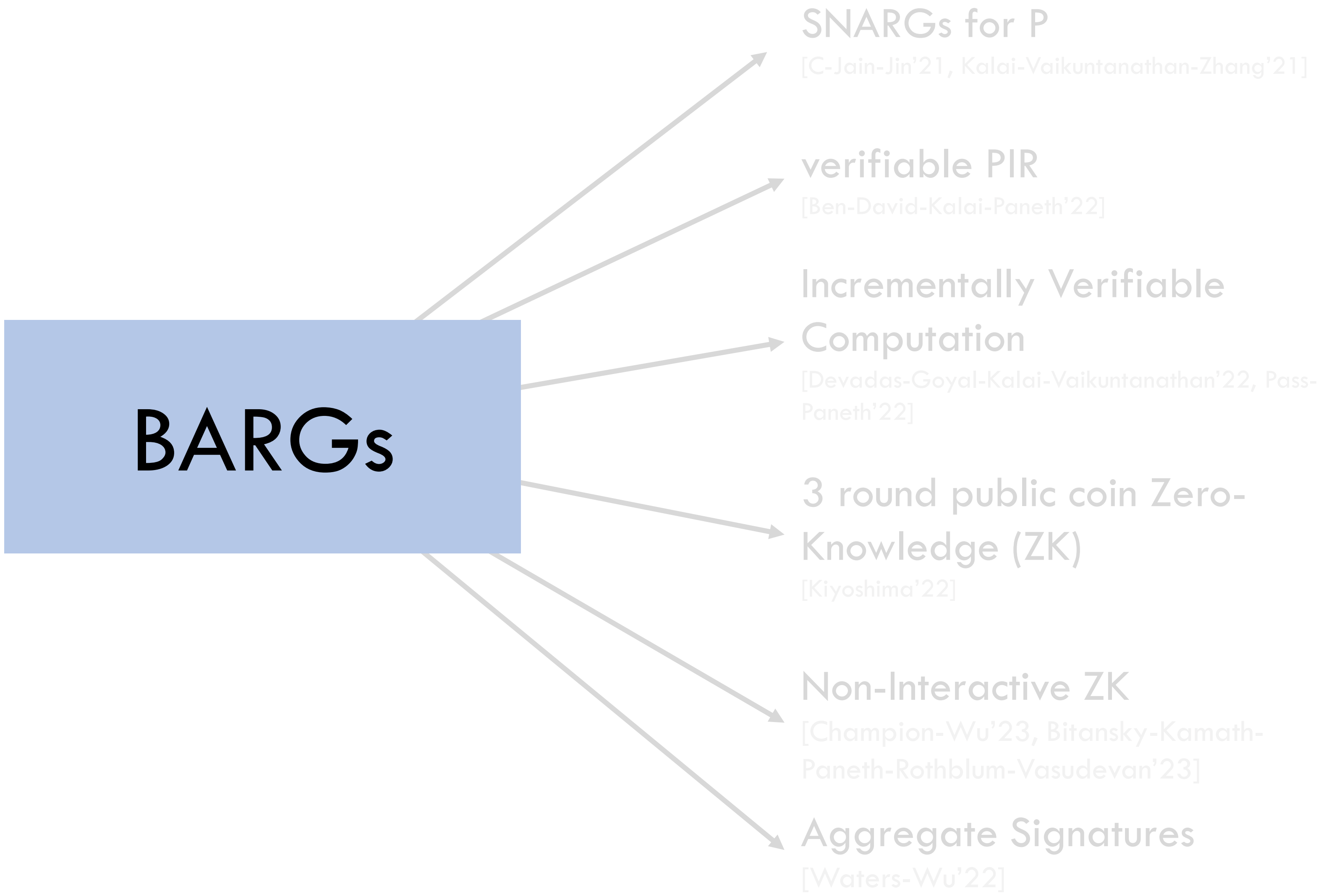


- SNARGs for P
  [C-Jain-Jin'21, Kalai-Vaikuntanathan-Zhang'21]
- verifiable PIR
  [Ben-David-Kalai-Paneth'22]
- Incrementally Verifiable Computation
  [Devadas-Goyal-Kalai-Vaikuntanathan'22, Pass-Paneth'22]
- 3 round public coin Zero-Knowledge (ZK)
  [Kiyoshima'22]
- Non-Interactive ZK
  [Champion-Wu'23, Bitansky-Kamath-Paneth-Rothblum-Vasudevan'23]
- Aggregate Signatures
  [Waters-Wu'22]

Construction of BARGs



Construction of BARGs

LWE
[C-Jain-Jin'21]

DLIN
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Sub-exp DDH + QR
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QR – Quadratic residuosity, LWE – Learning with Error, DDH – Decisional Diffie-Hellman, DLIN – Decisional Linear Assumption over Bilinear Groups.

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BARGs

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Our Results

Theorem 1

Assuming sub-exponential hardness of DDH, there exists SNARGs for batch NP where

$$|\Pi| = \text{poly}(\log k, |C|)$$

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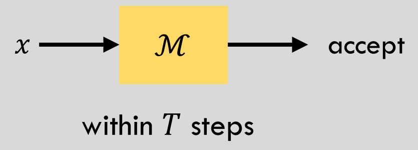


Theorem 2

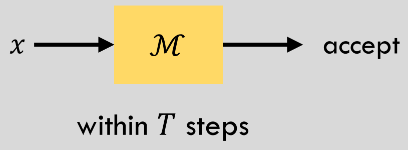
Assuming sub-exponential hardness of DDH, there exists SNARGs for P where

$|\mathsf{CRS}|, |\Pi|, |$$| = \mathrm{polylog}(T)$

Our Results

Recent concurrent work [Kalai-Lombardi-Vaikuntanathan'23]:

SNARGs for bounded depth circuits assuming sub-exponential hardness of DDH.



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Meta View: Advanced Primitives from DDH

DDH

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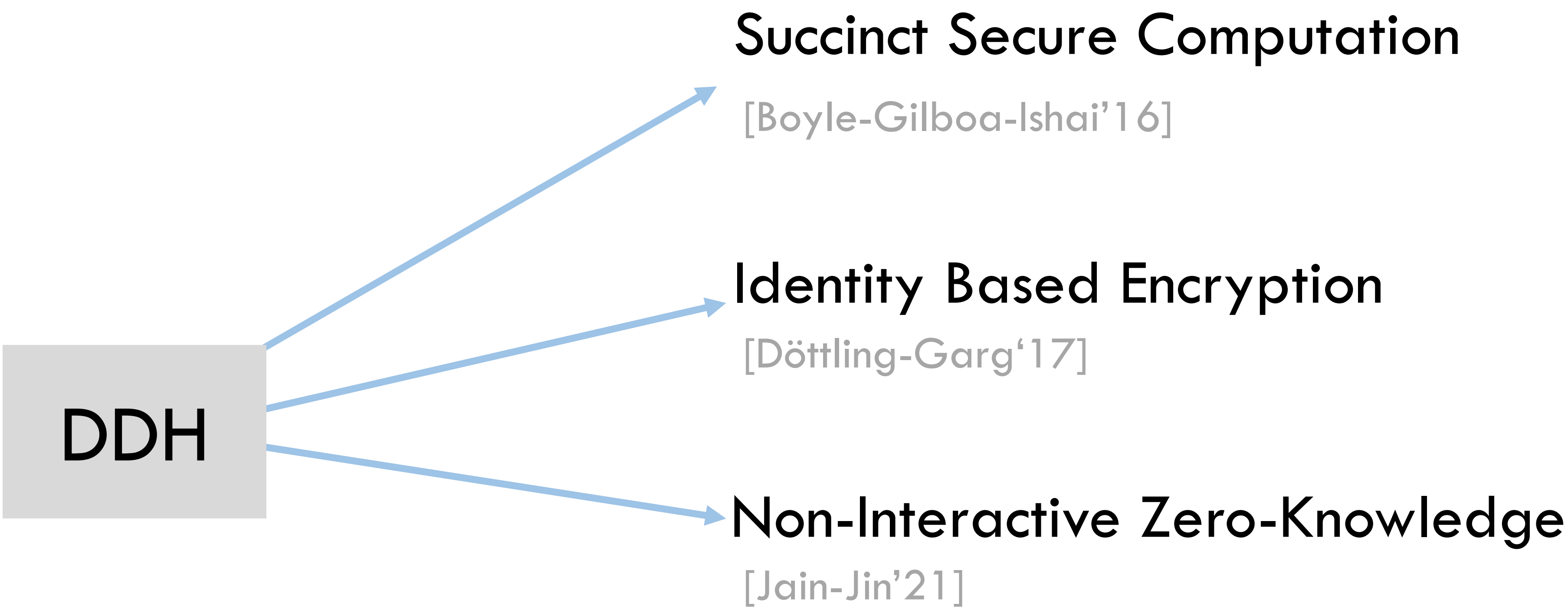

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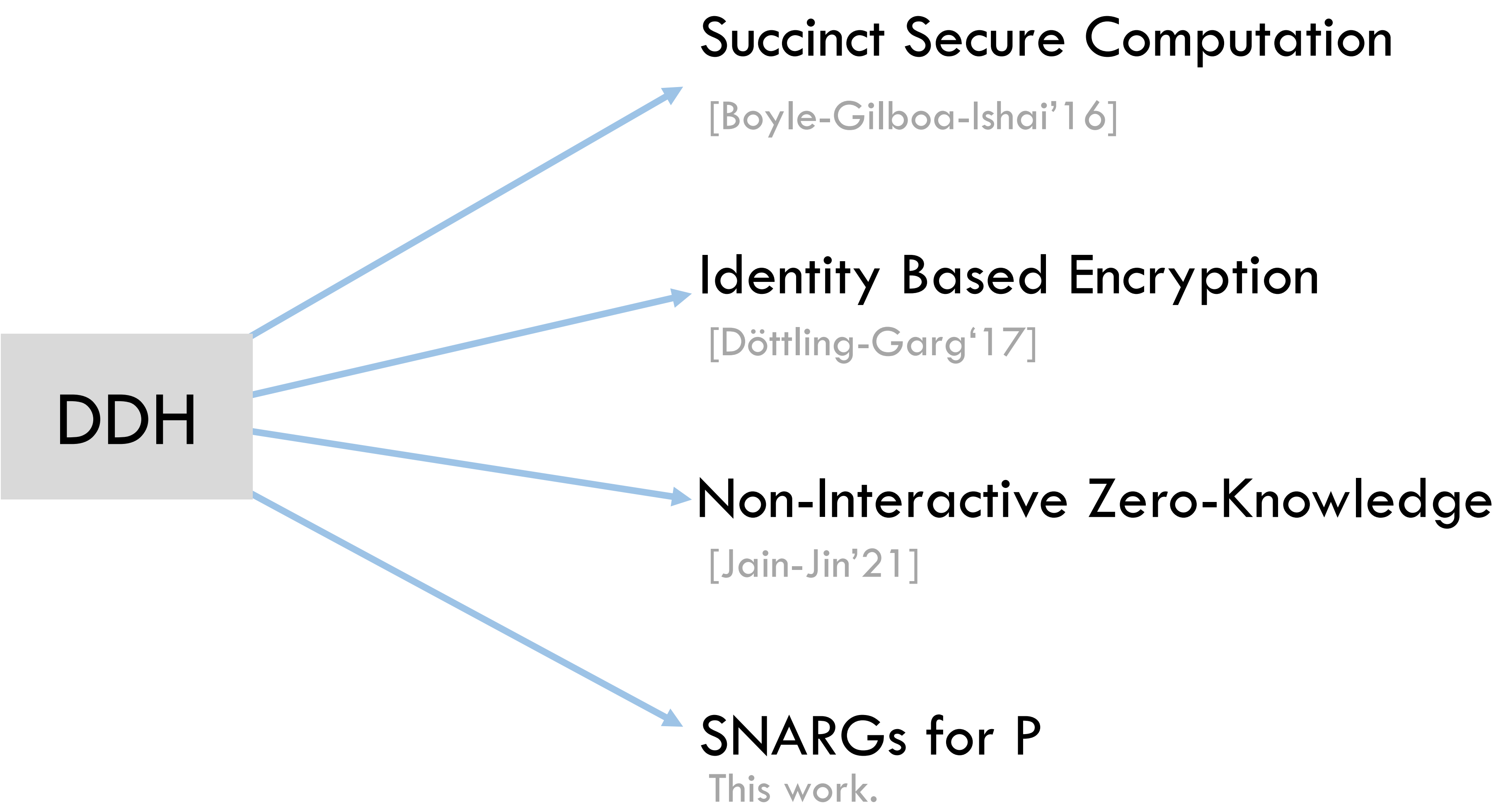
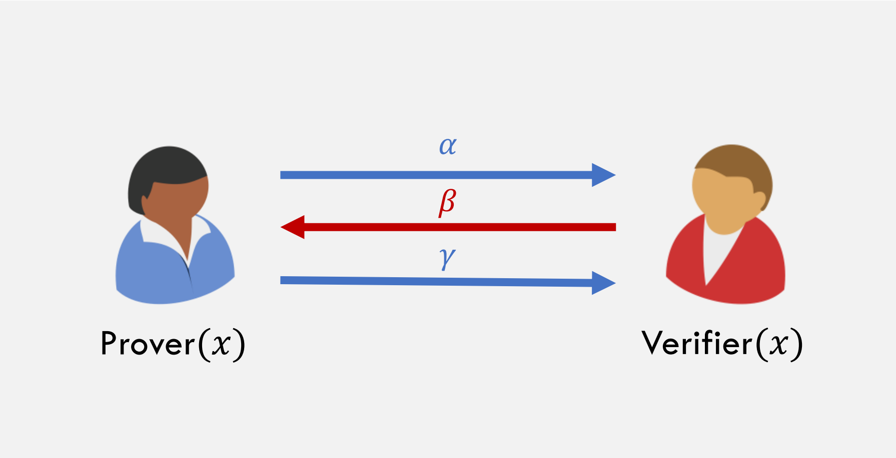


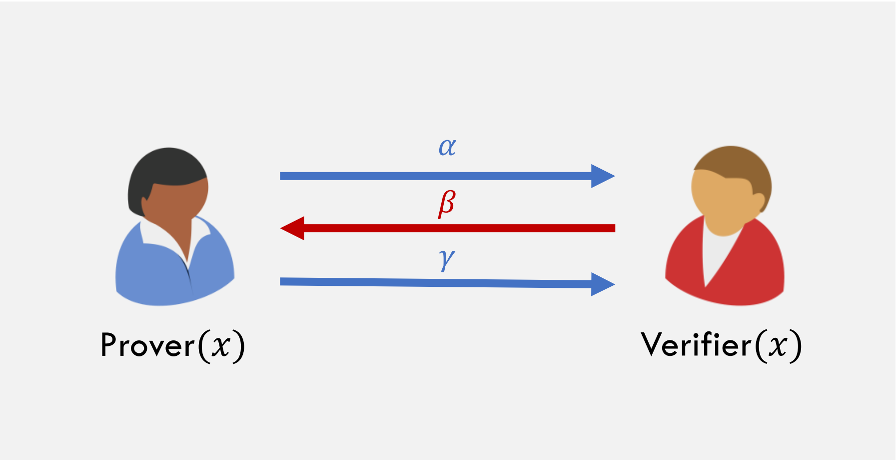
Tools and Techniques

Fiat-Shamir (FS) Methodology: Recipe for Success



β is a random string

Fiat-Shamir (FS) Methodology



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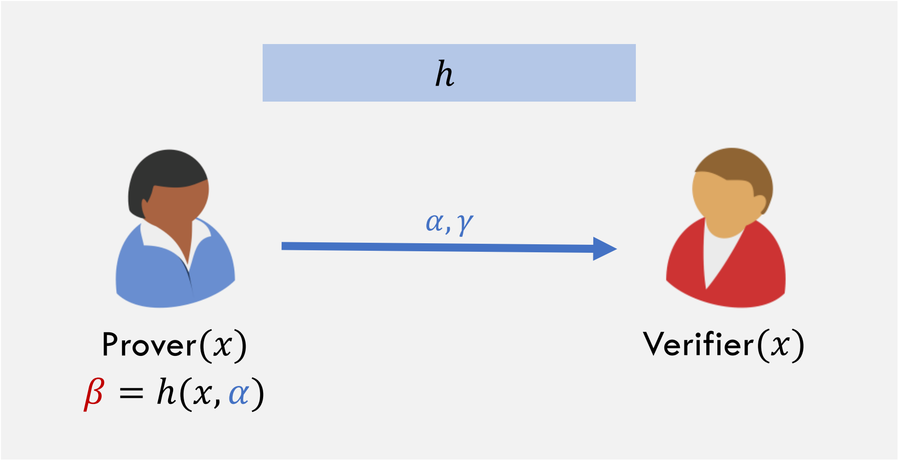
[Fiat-Shamir'86]



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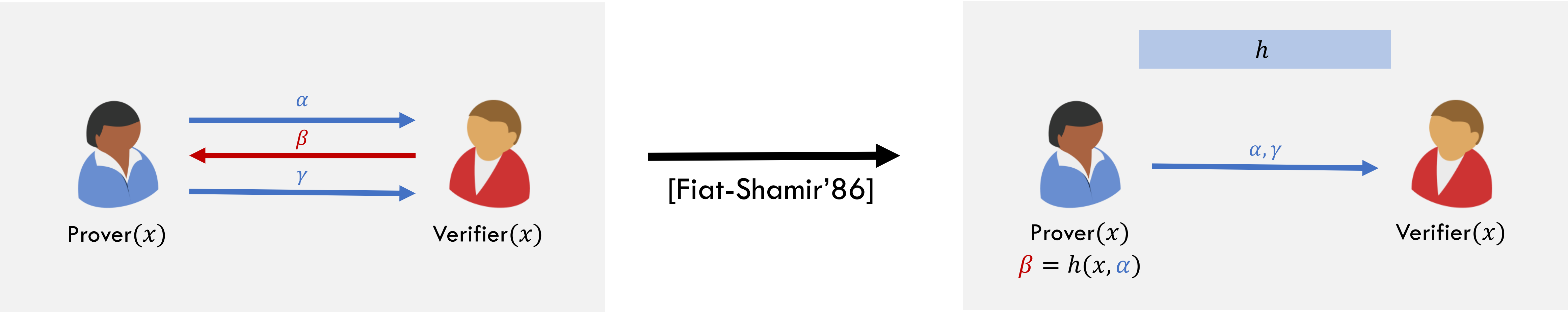

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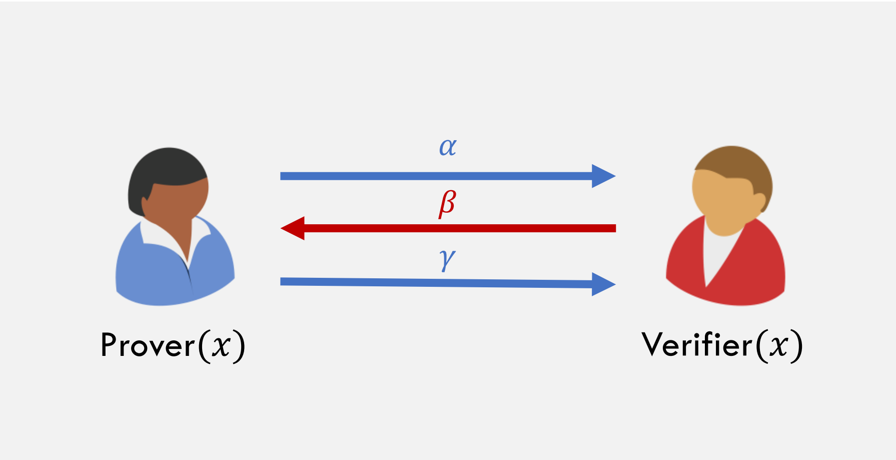

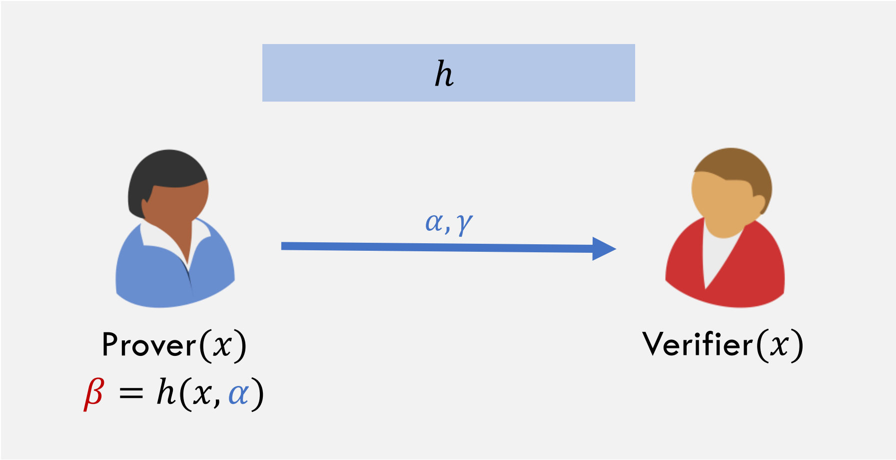

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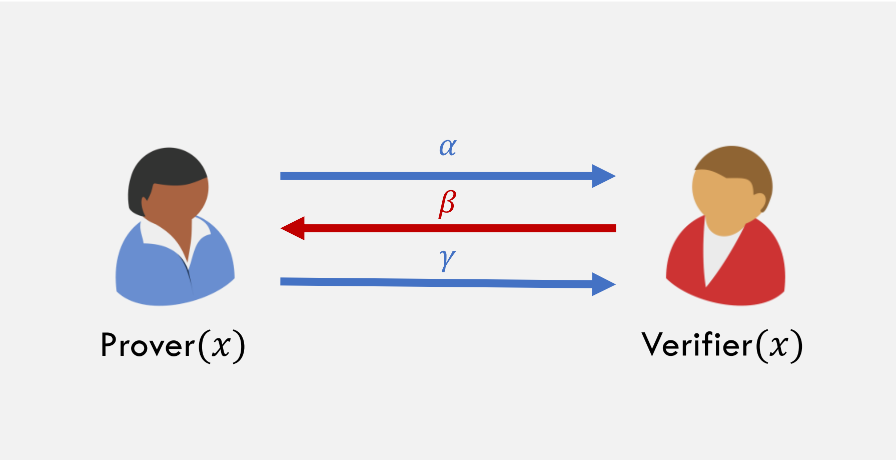
If $x \notin \mathcal{L}$, no PPT  can find α such that

$$h(x, \alpha) \in \mathrm{BAD}_{x,\alpha}$$

Correlation Intractability [Canetti-Goldreich-Halevi'98]



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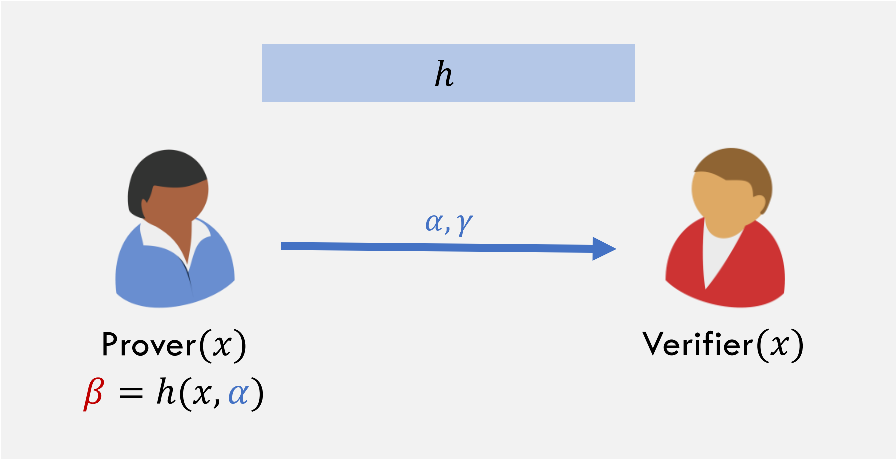

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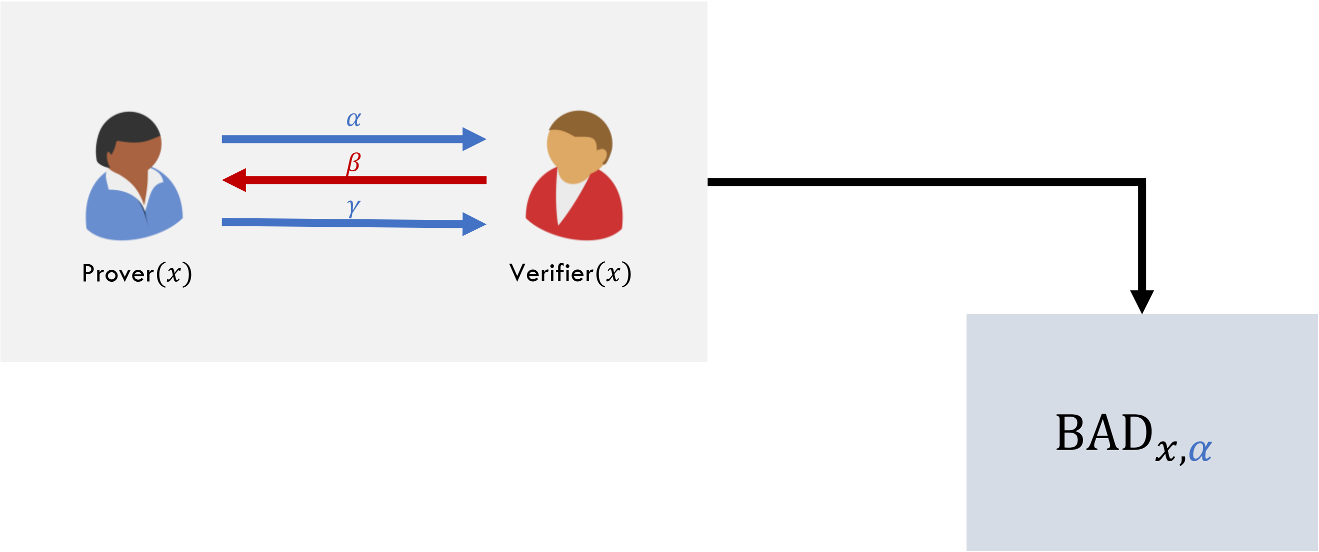
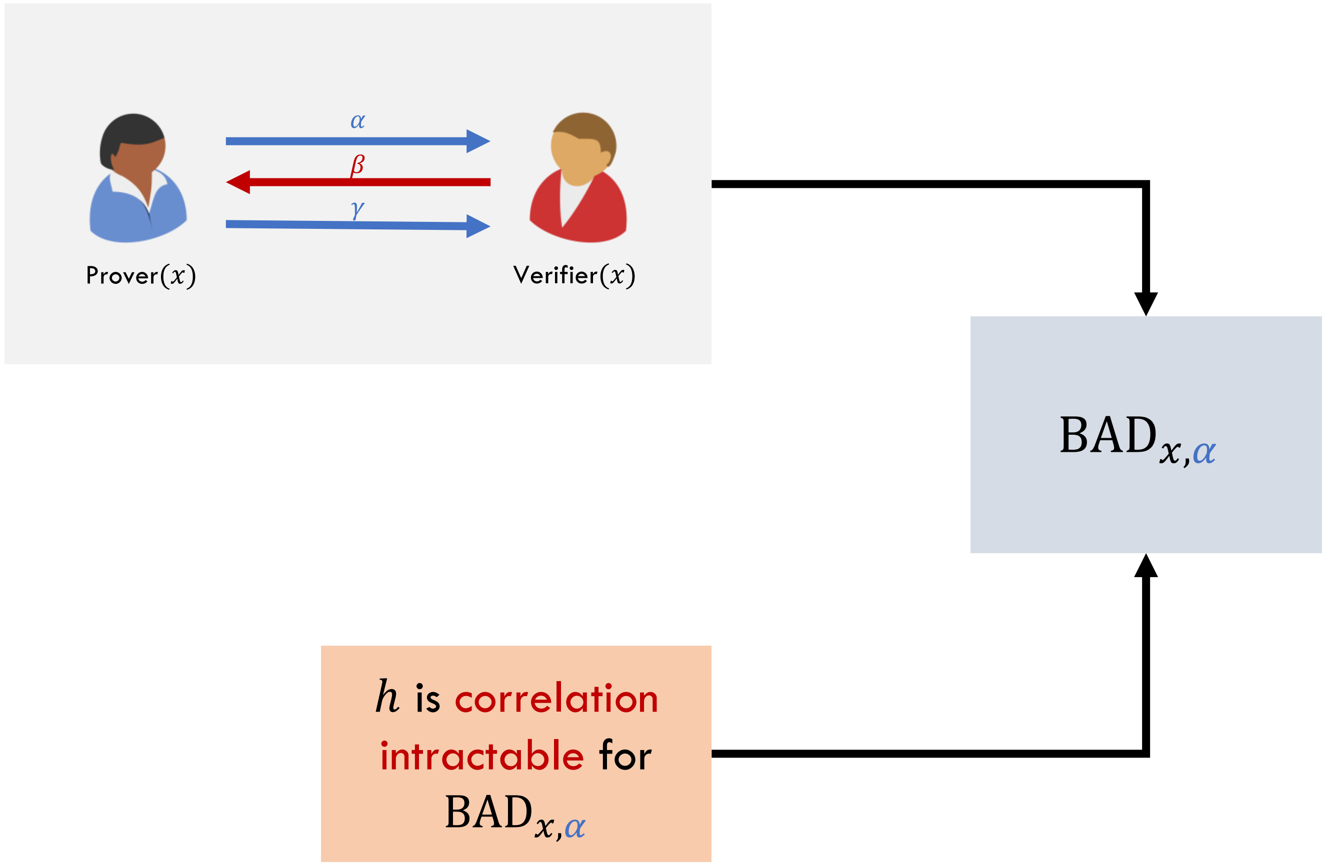
h is **correlation intractable (CI)** for $\mathrm{BAD}_{x,\alpha}$

Instantiating the FS Transform



Instantiating the FS Transform



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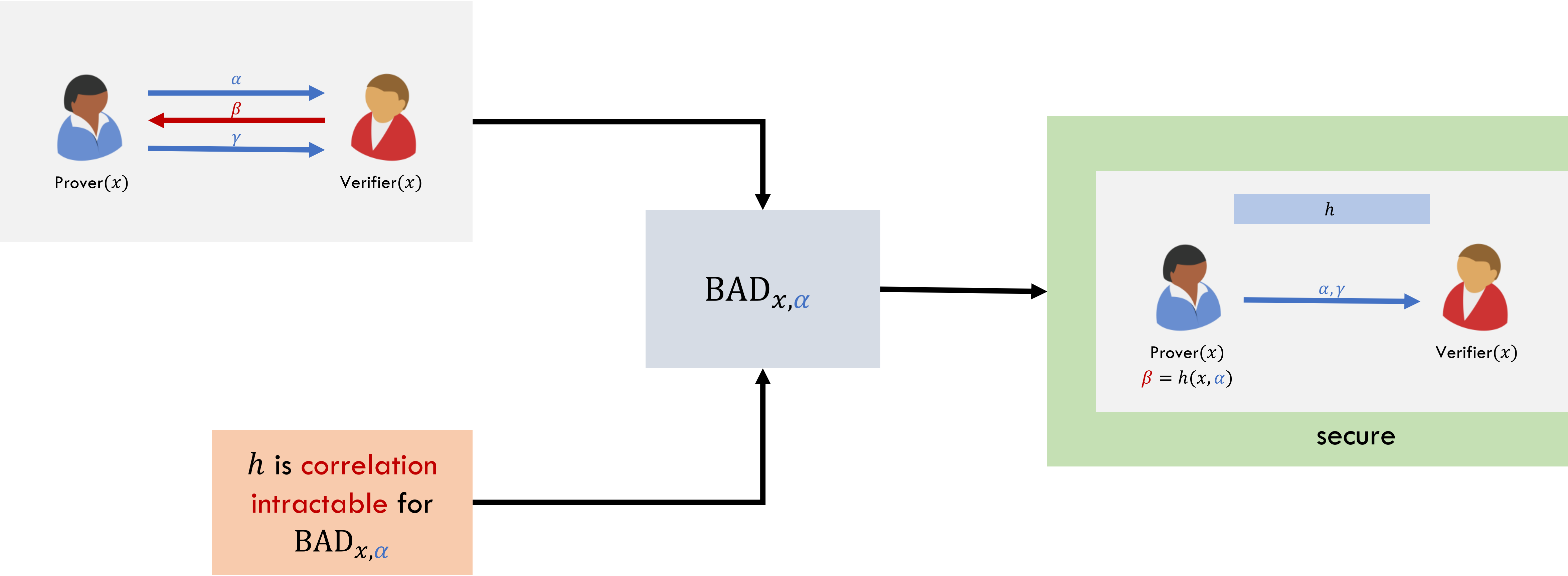
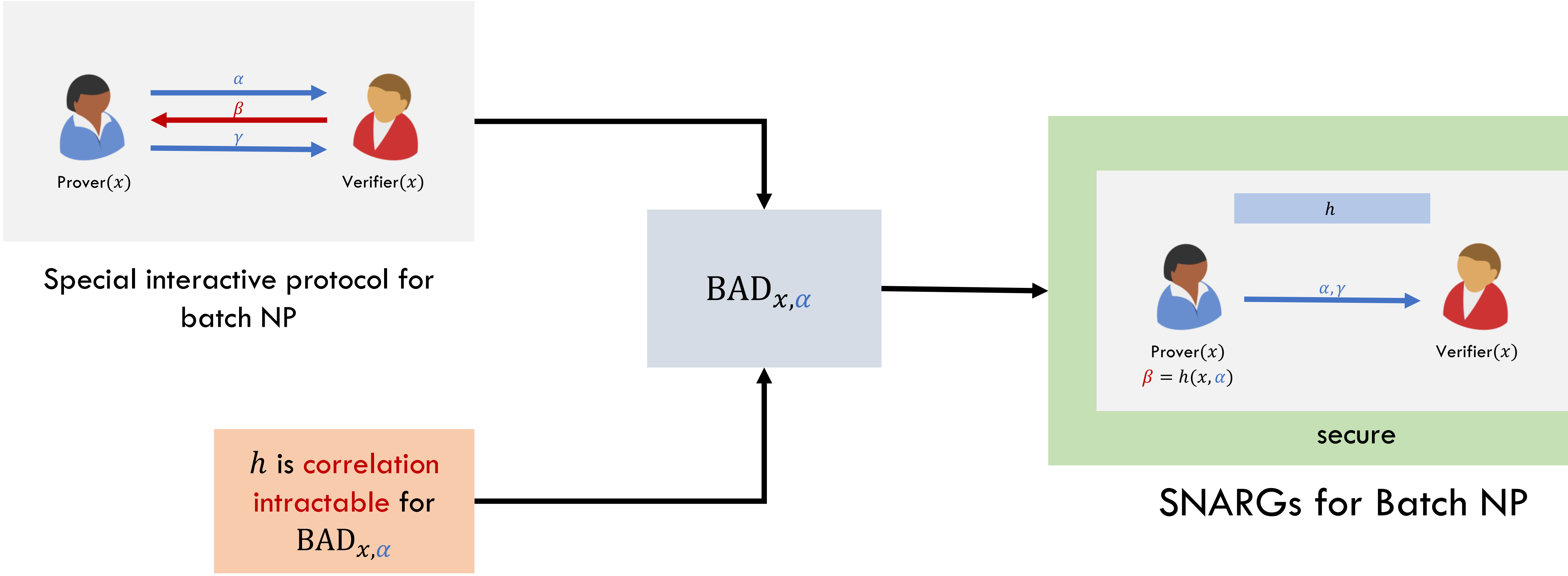


[C-Jain-Jin'21] Methodology



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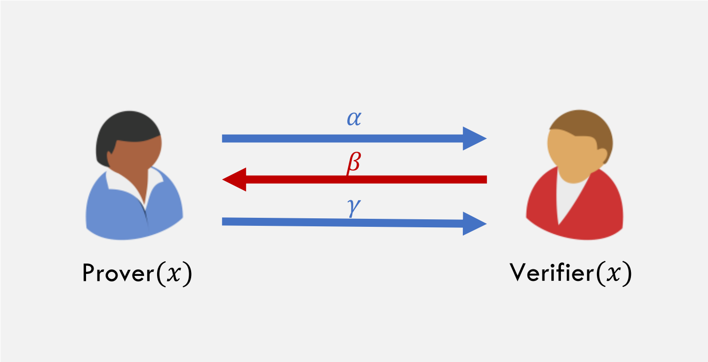

Special interactive protocol for batch NP

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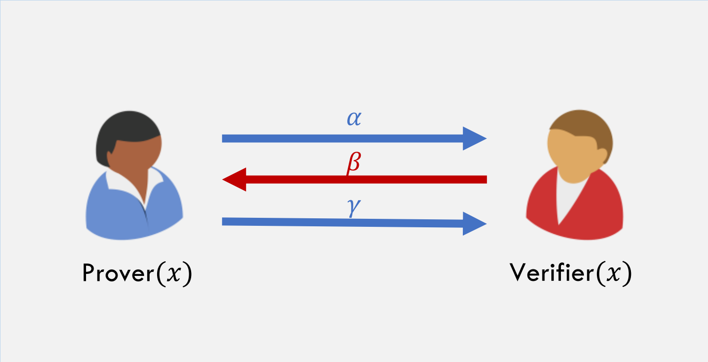

Special interactive protocol for batch NP

This work

sub-exp
DDH

see paper for details

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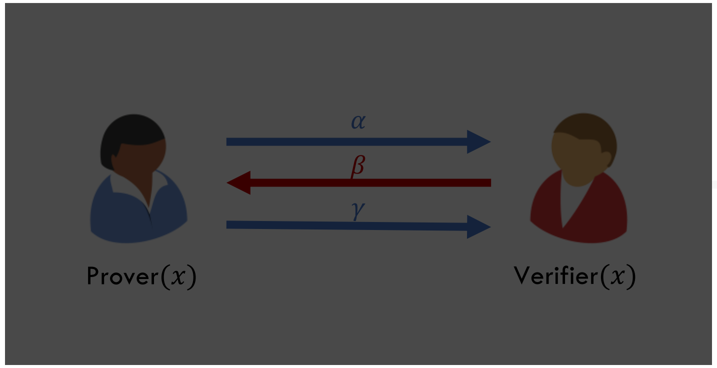

Magic Box

Special interactive protocol for batch NP

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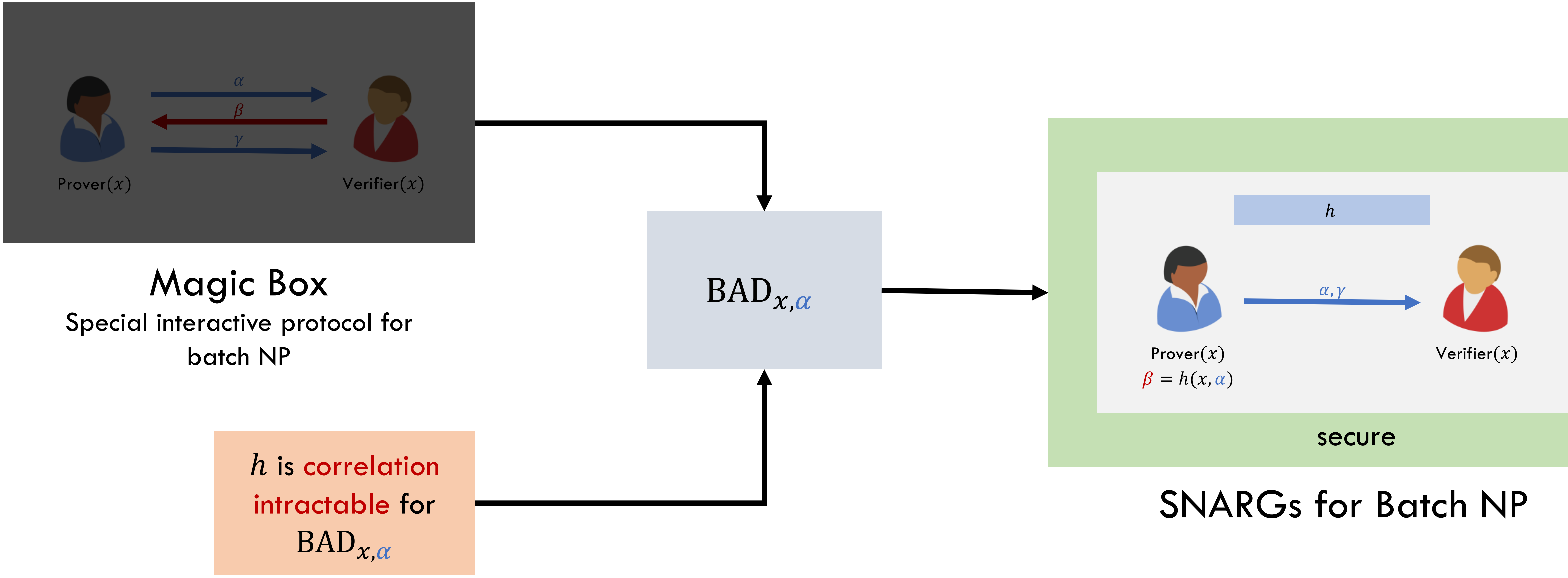

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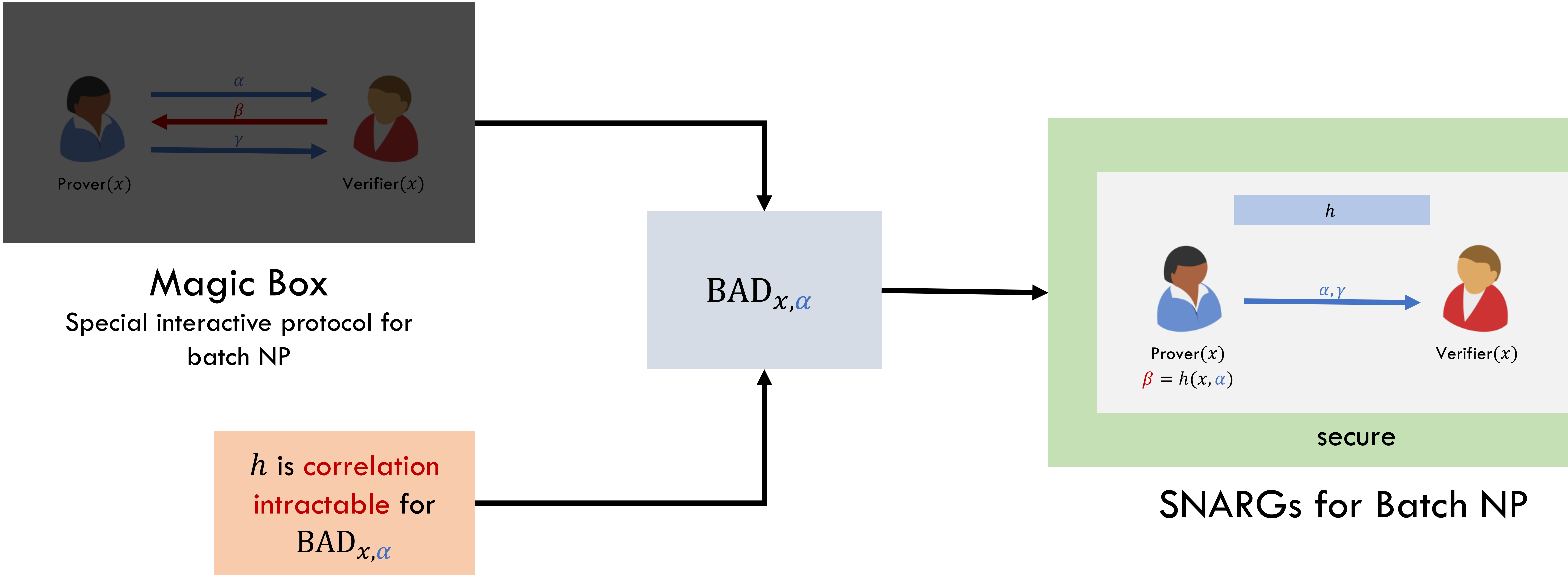

What properties does $\mathrm{BAD}_{x,\alpha}$ have?

see paper for details

Properties of $\mathrm{BAD}_{x,\alpha}$

$\mathrm{BAD}_{x,\alpha}$ is product verifiable.

$\forall\, x \notin \mathcal{L}$

$\mathrm{BAD}_{\,x,\alpha} = \{\beta \mid \exists\gamma$ s.t. Verifier accepts $(\alpha, \beta, \gamma)\}$

Properties of $\mathrm{BAD}_{x,\alpha}$

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Exponentially many bad challenges even when β sampled from polynomial size challenge space.

Properties of $\mathrm{BAD}_{x,\alpha}$

$\mathrm{BAD}_{x,\alpha} = \mathrm{BAD}^{(1)}_{x,\alpha} \times \mathrm{BAD}^{(2)}_{x,\alpha} \times \mathrm{BAD}^{(3)}_{x,\alpha} \times \mathrm{BAD}^{(4)}_{x,\alpha}$

$\mathrm{BAD}_{x,\alpha}$ is product verifiable.

Each $\mathrm{BAD}^{(i)}_{x,\alpha}$ is efficiently verifiable

$\forall\, x \notin \mathcal{L}$

$\mathrm{BAD}^{(j)}_{x,\alpha} = \{\beta \mid \exists \gamma$ s.t. Verifier accepts $(\alpha, \beta, \gamma)\}$

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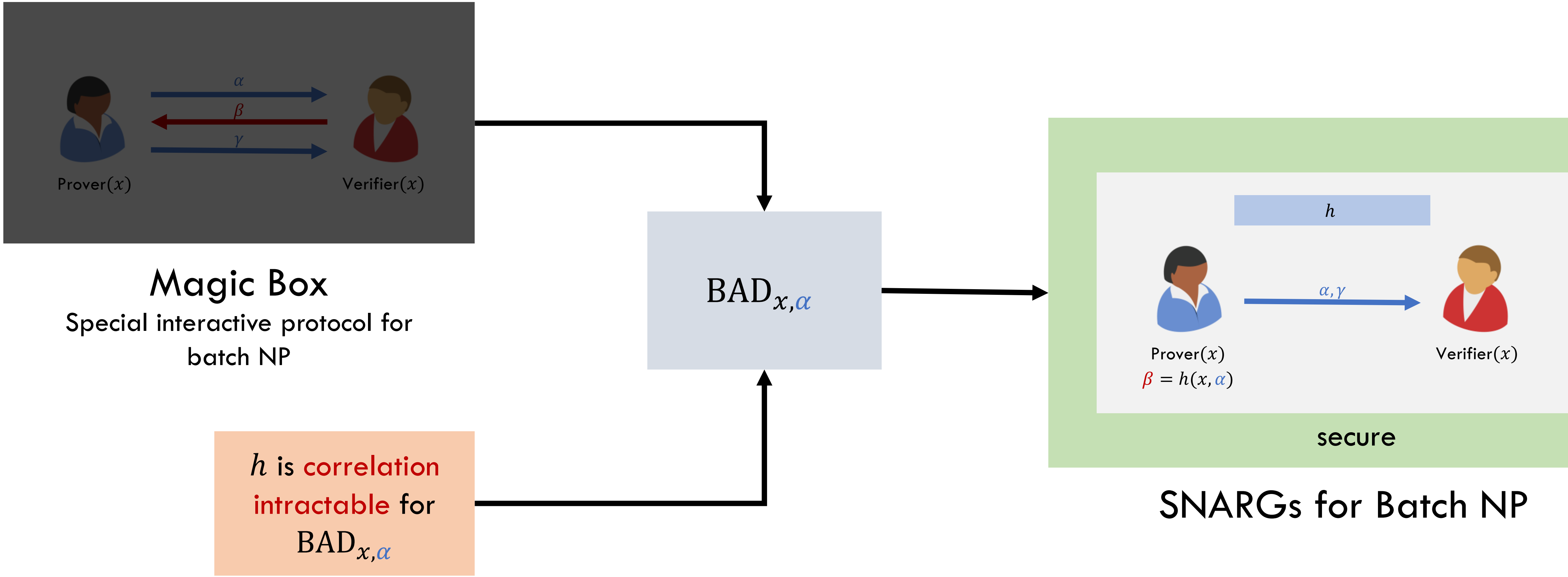

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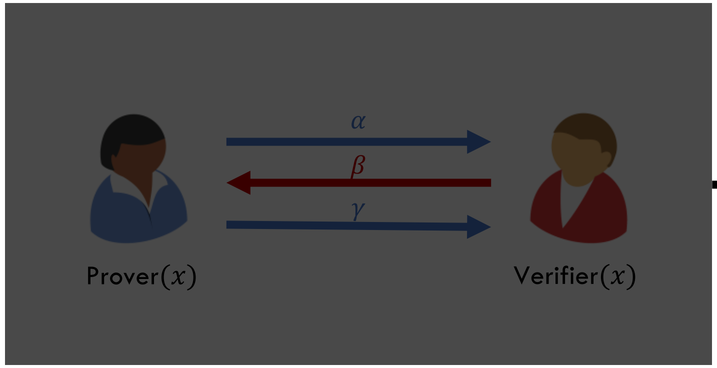

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h is correlation intractable for $\mathrm{BAD}_{x,\alpha}$

$\mathrm{BAD}_{x,\alpha}$

$\mathrm{BAD}_{x,\alpha}$ properties

1. Bad challenges are a product set
2. Challenge space is of polynomial size
3. Bad challenges are product verifiable in TC^0

TC^0 - Constant depth polynomial-size threshold circuits

see paper for details

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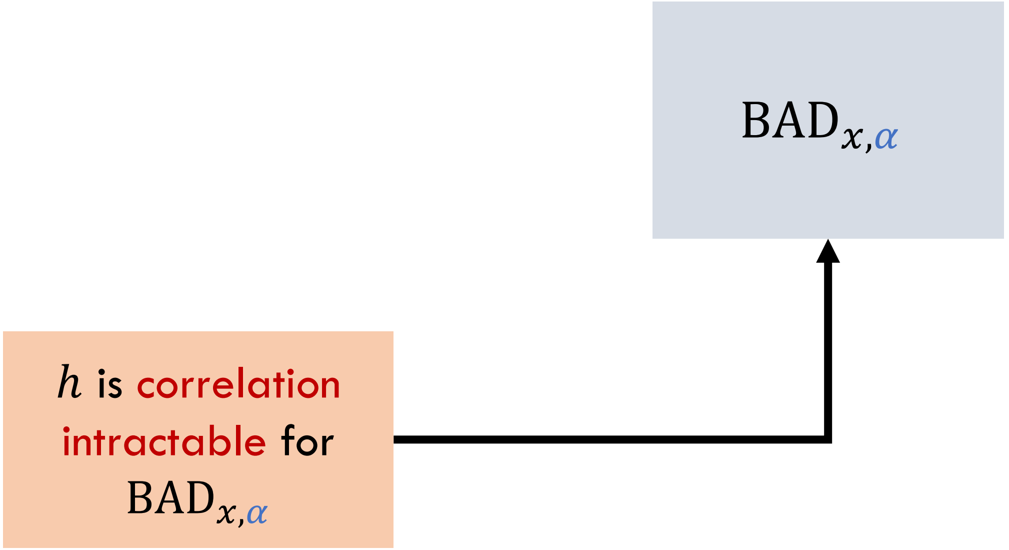

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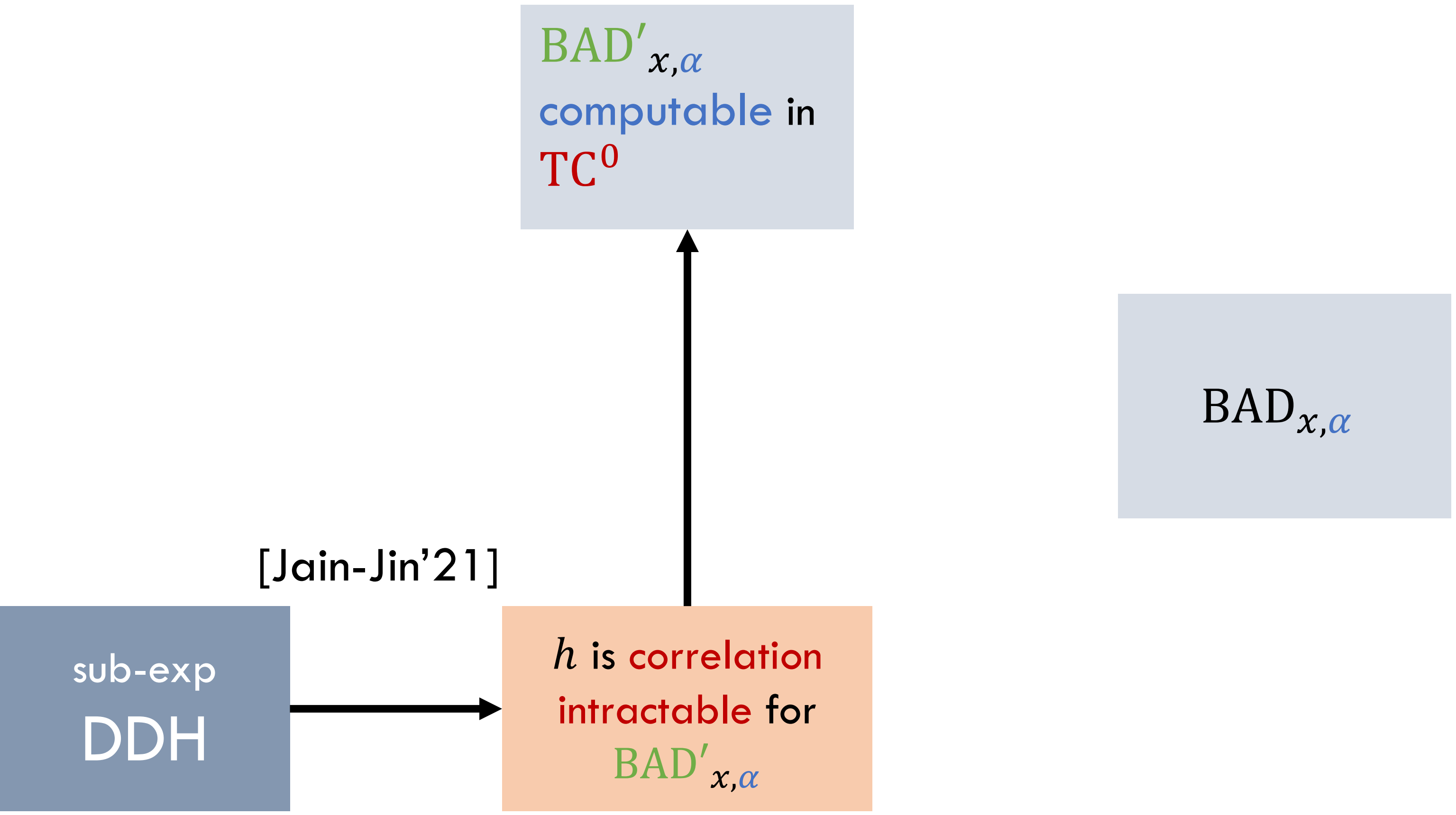
TC^0 - Constant depth polynomial-size threshold circuits

[C-Jain-Jin'21] Methodology



$\mathrm{BAD}_{x,\alpha}$ properties

1. Bad challenges are a product set
2. Challenge space is of polynomial size
3. Bad challenges are product verifiable in TC^0

TC^0 - Constant depth polynomial-size threshold circuits

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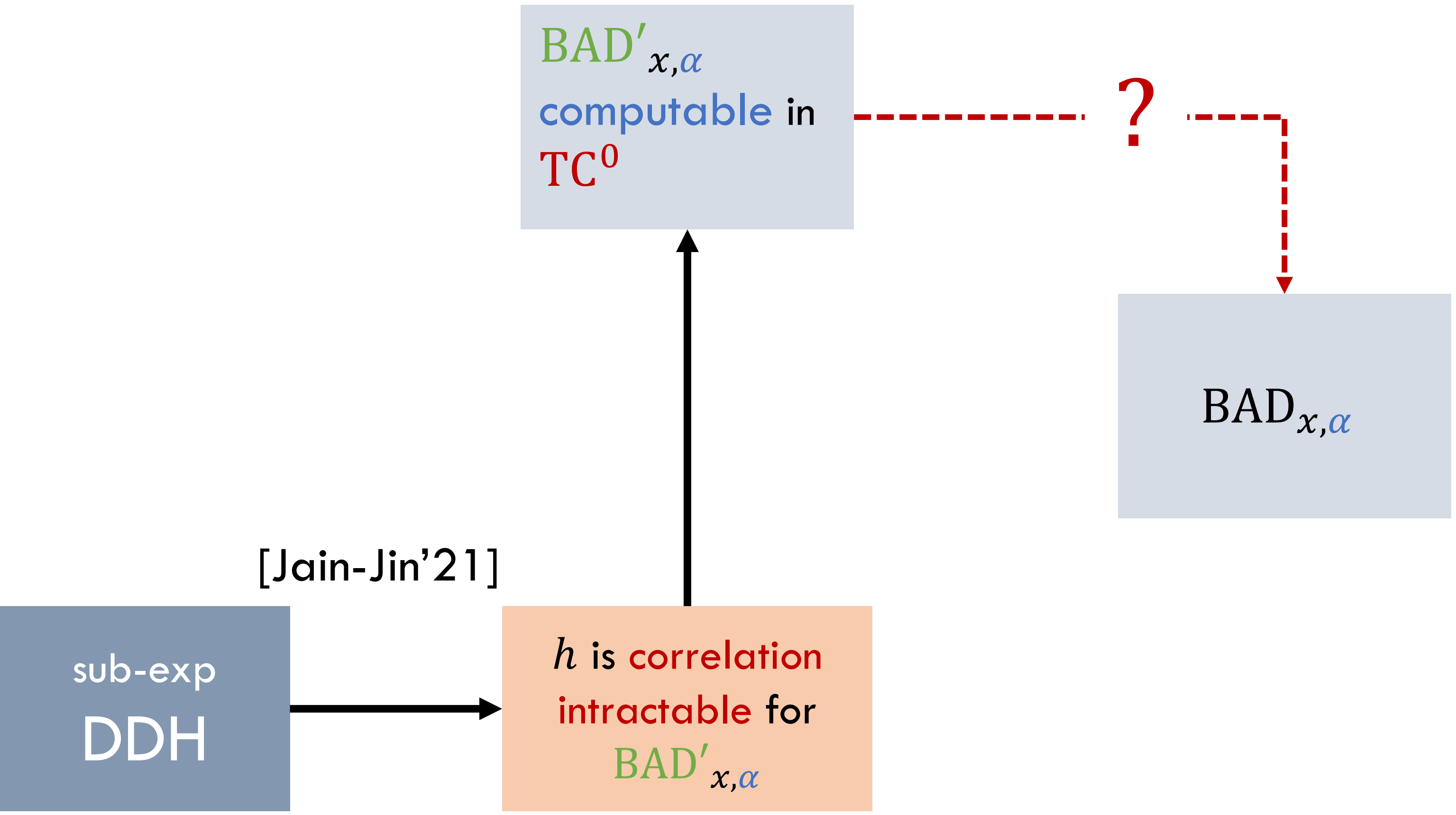

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TC^0 - Constant depth polynomial-size threshold circuits

[C-Jain-Jin'21] Methodology

$\mathrm{BAD}'_{x,\alpha}$ computable in TC^0

?

$\mathrm{BAD}_{x,\alpha}$

[Jain-Jin'21]

Difficulty [Holmgren-Lombardi-Rothblum'21]:

$\mathrm{BAD}_{x,\alpha}$ has exponentially many bad challenges.

$\mathrm{BAD}_{x,\alpha}$ properties

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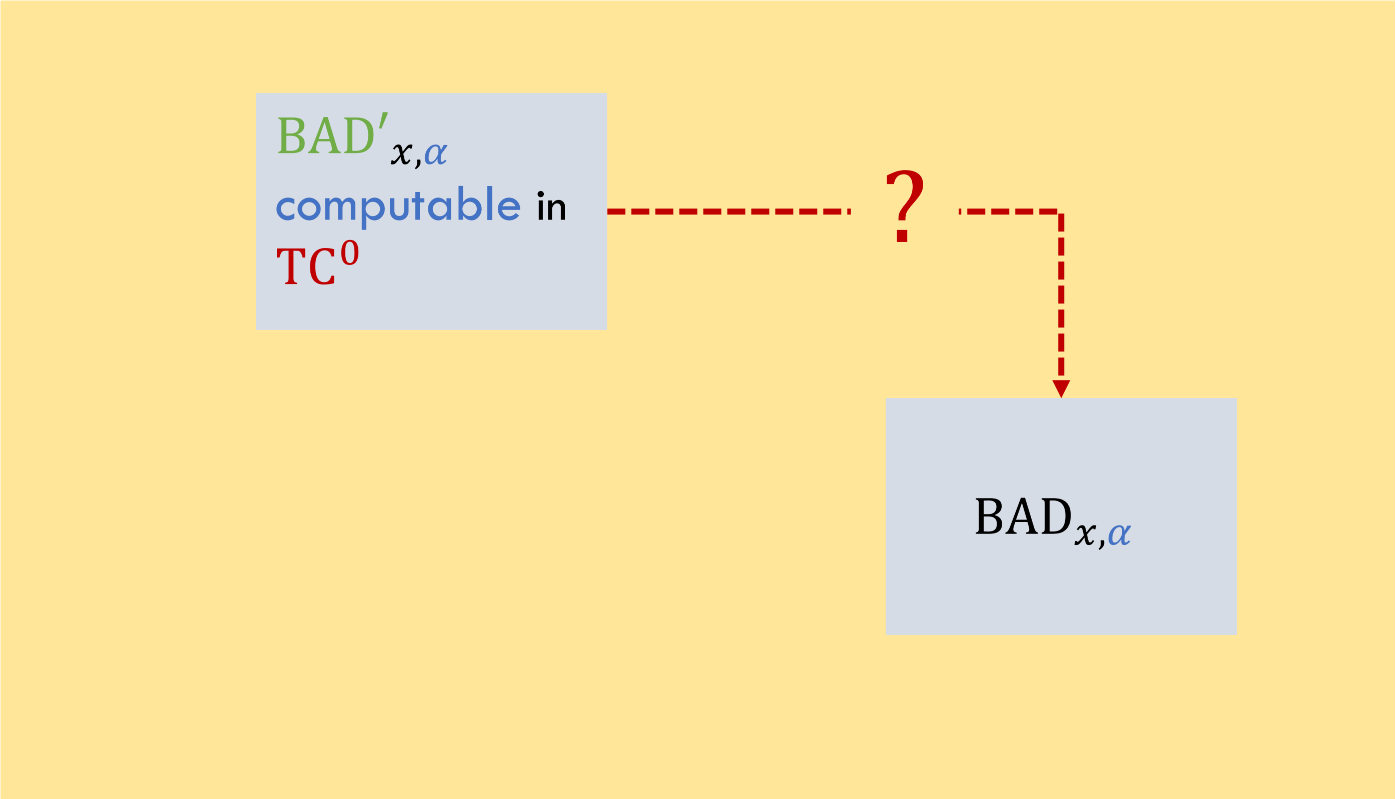

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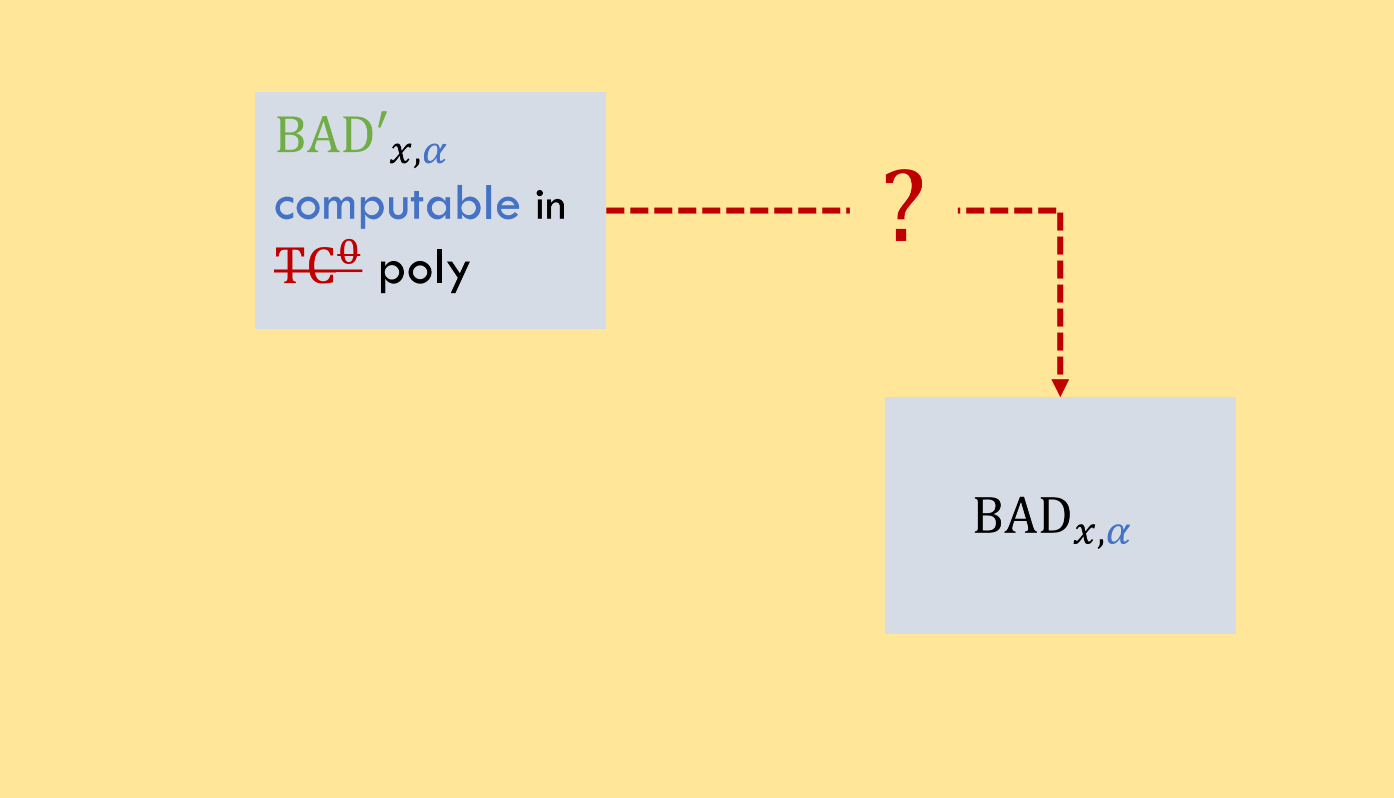


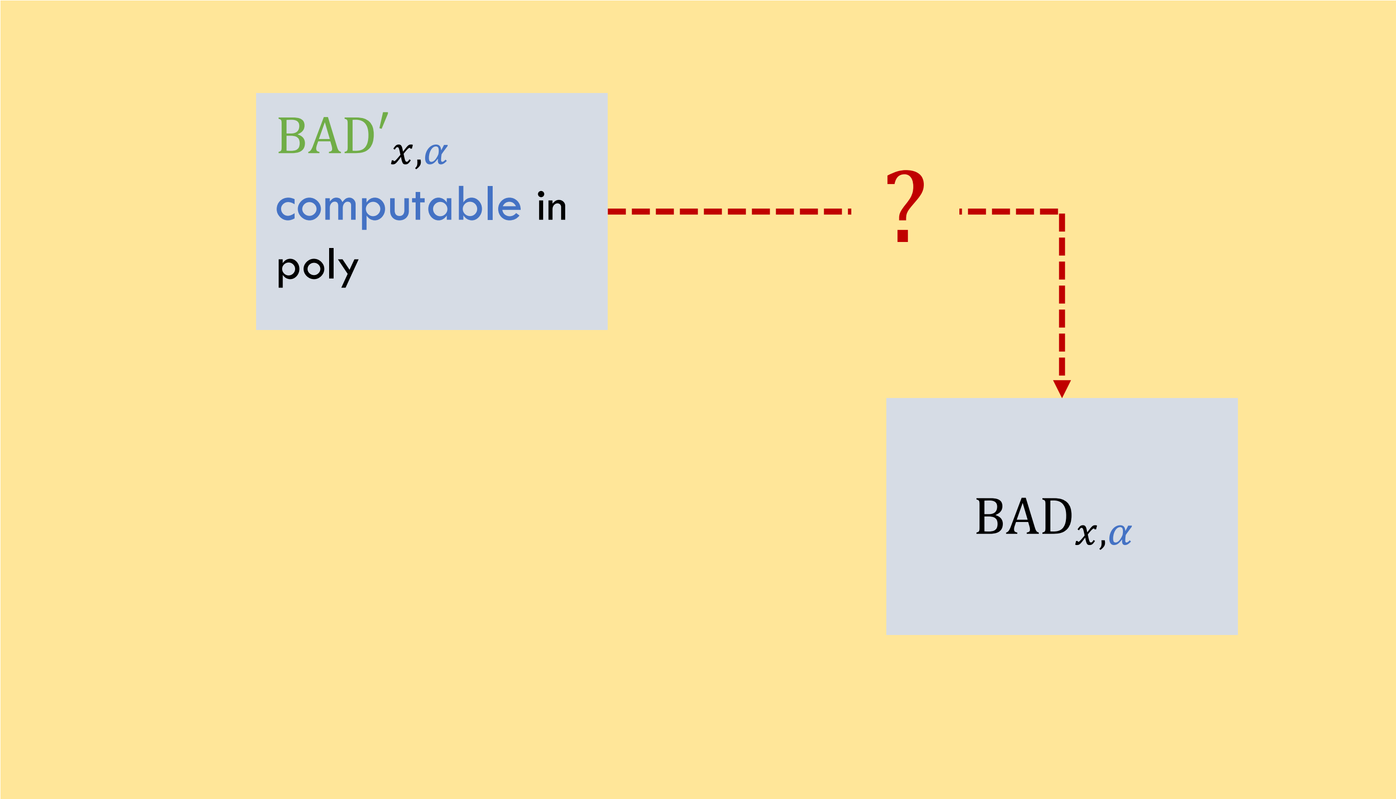
$\text{BAD}_{x,\alpha}$ properties

1. Bad challenges are a product set
2. Challenge space is of polynomial size
3. Bad challenges are product verifiable in ~~TC^0~~ poly

TC^0 - Constant depth polynomial-size threshold circuits

For this talk

[C-Jain-Jin'21] Methodology



$\mathrm{BAD}_{x,\alpha}$ properties

1. Bad challenges are a product set
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3. Bad challenges are product verifiable in poly

For this talk

Easy Case: Verifiable Unique Bad Challenge

$$\mathrm{BAD}_{x,\alpha}^{(1)}$$

Easy Case: Verifiable Unique Bad Challenge

$\mathrm{BAD}_{x,\alpha}^{(1)}$

Compute Bad Challenge

for $\beta \in \mathrm{ChallengeSpace}$

  if $\beta \in \mathrm{BAD}_{x,\alpha}^{(1)}$

    return β

ChallengeSpace polynomial size + $\mathrm{BAD}_{x,\alpha}^{(1)}$ efficiently verifiable $\Rightarrow$ $\mathrm{BAD}_{x,\alpha}^{(1)}$ efficiently computable.

Easy Case: Verifiable Unique Bad Challenge

$$\mathrm{BAD}_{x,\alpha} = \mathrm{BAD}_{x,\alpha}^{(1)} \times \mathrm{BAD}_{x,\alpha}^{(2)} \times \cdots \times \mathrm{BAD}_{x,\alpha}^{(d)}$$

Easy Case: Verifiable Unique Bad Challenge

$$\text{BAD}_{x,\alpha} = \text{BAD}_{x,\alpha}^{(1)} \times \text{BAD}_{x,\alpha}^{(2)} \times \cdots \times \text{BAD}_{x,\alpha}^{(d)}$$

Compute Bad Challenge

for $i \in [d]$

  for $\beta^{(i)} \in \text{ChallengeSpace}$

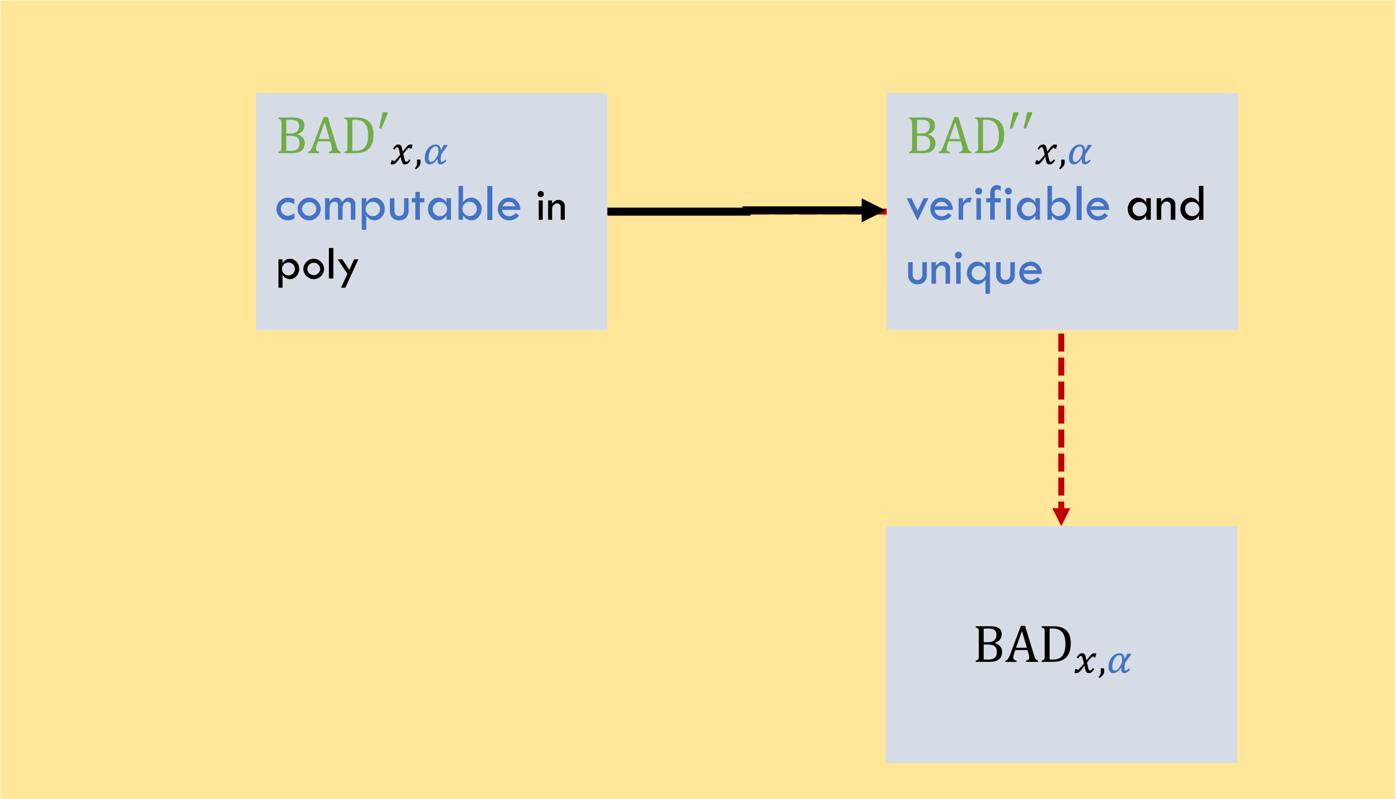
    if $\beta^{(i)} \in \text{BAD}_{x,\alpha}^{(i)}$

      store $\beta^{(i)}$

return $(\beta^{(1)}, \cdots, \beta^{(d)})$

poly repetitions + ChallengeSpace polynomial size + $\text{BAD}_{x,\alpha}^{(i)}$ efficiently verifiable $\Rightarrow$ $\text{BAD}_{x,\alpha}$ efficiently computable.

Overview So Far



$\mathrm{BAD}_{x,\alpha}$ properties

1. Bad challenges are a product set
2. Challenge space is of polynomial size
3. Bad challenges are product verifiable in poly

Reducing to Verifiable Unique Bad Challenge

No parallel repetition

$\leftarrow \ell = \log_2 |\text{ChallengeSpace}| \rightarrow$

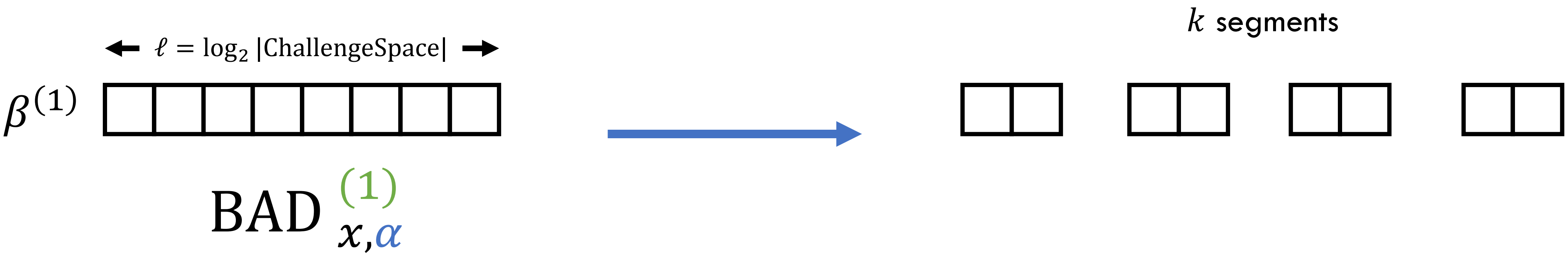
$\beta^{(1)}$ $\square\square\square\square\square\square\square\square$

$$\text{BAD}_{x,\alpha}^{(1)}$$

No restriction on number of bad challenges

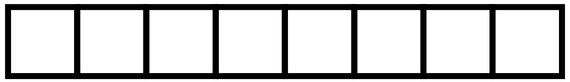
Reducing to Verifiable Unique Bad Challenge

No parallel repetition

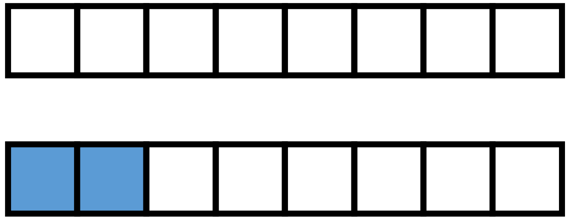


Each segment has ℓ/k bits

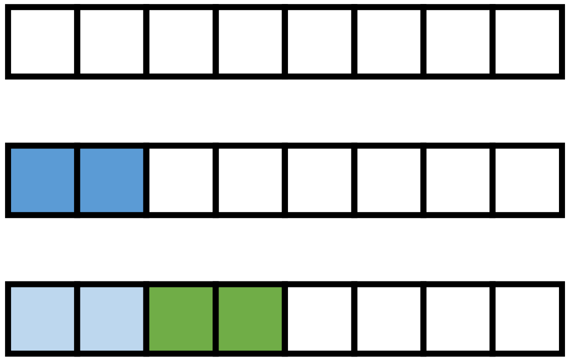
Sampling Challenges via Segments



Sampling Challenges via Segments



Sampling Challenges via Segments



Sampling Challenges via Segments

h is correlation intractable for efficiently verifiable unique bad challenge relations.

Sampling Challenges via Segments

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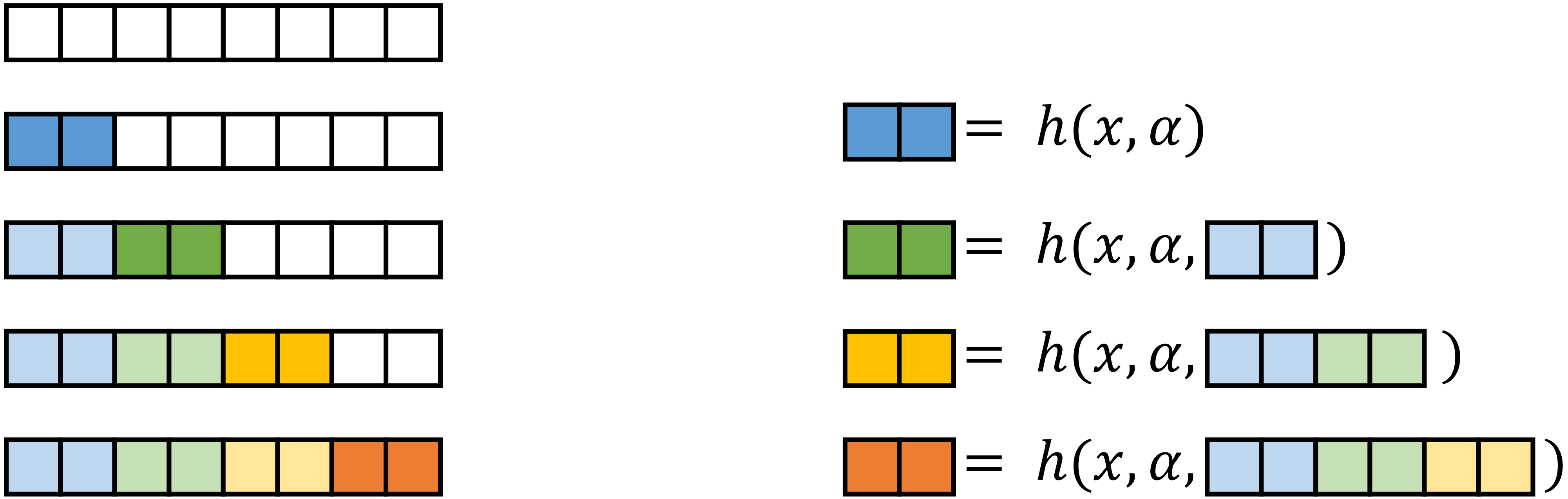
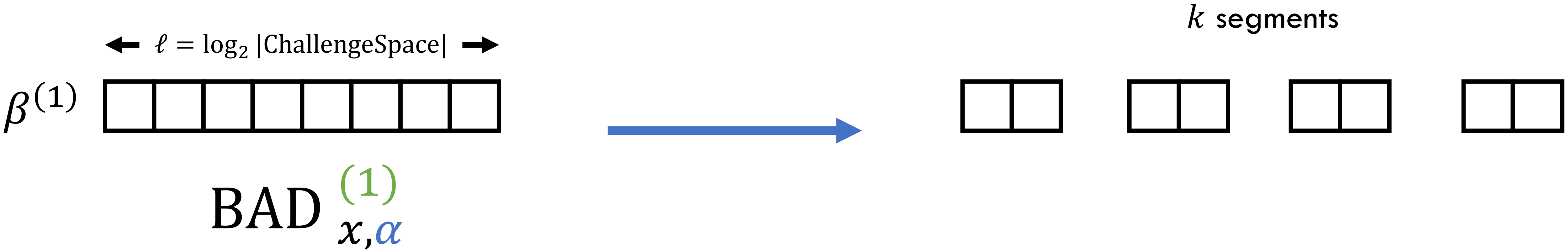
Sampling Challenges via Segments



h is correlation intractable for efficiently verifiable unique bad challenge relations.

Reducing to Verifiable Unique Bad Challenge

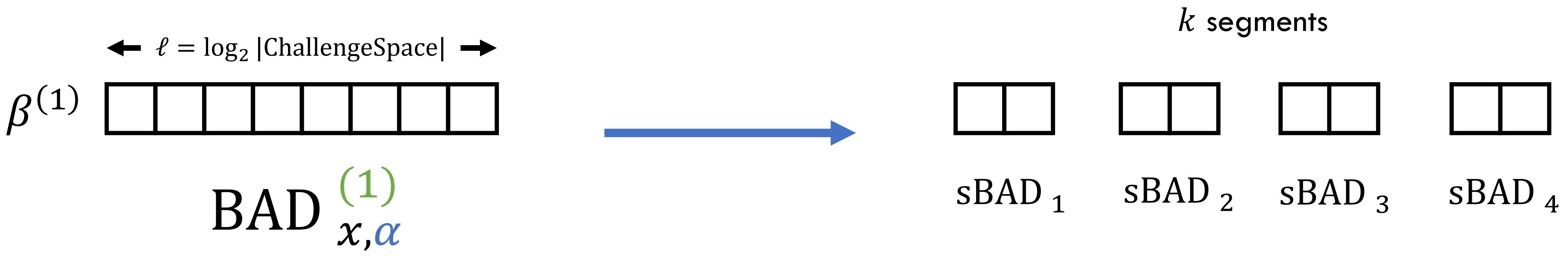
No parallel repetition



Each segment has ℓ / k bits

Reducing to Verifiable Unique Bad Challenge

No parallel repetition



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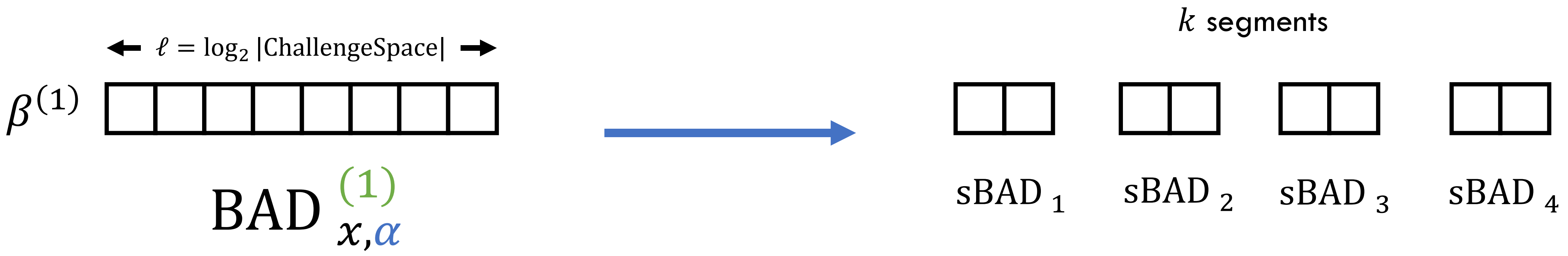
Reducing to Verifiable Unique Bad Challenge

No parallel repetition



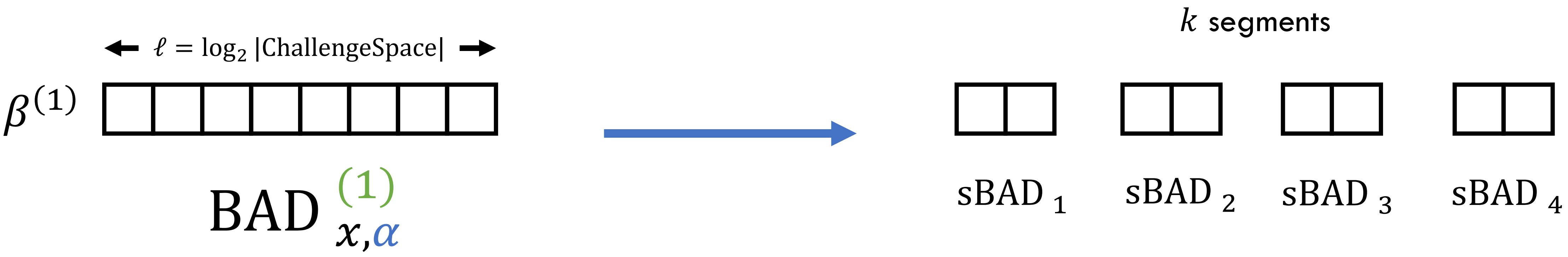
Requirements:

1. Each sBAD_j must be efficiently verifiable unique bad challenge relations.

Each segment has ℓ/k bits

Reducing to Verifiable Unique Bad Challenge

No parallel repetition



Requirements:

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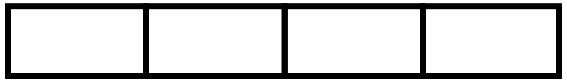
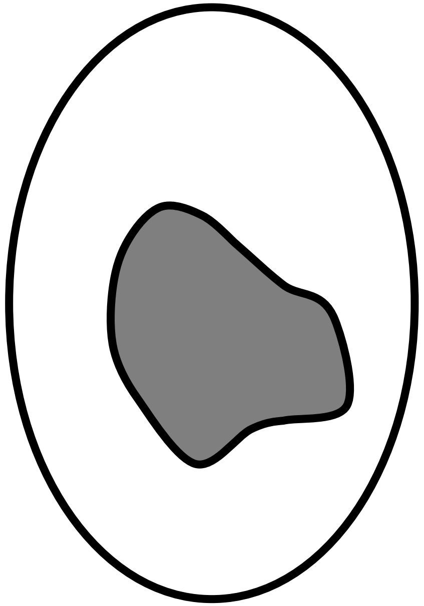
2. If a challenge is bad, then there must exist a bad segment.

Each segment has ℓ/k bits

Defining Bad Segments

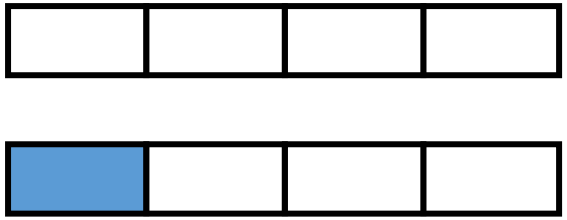


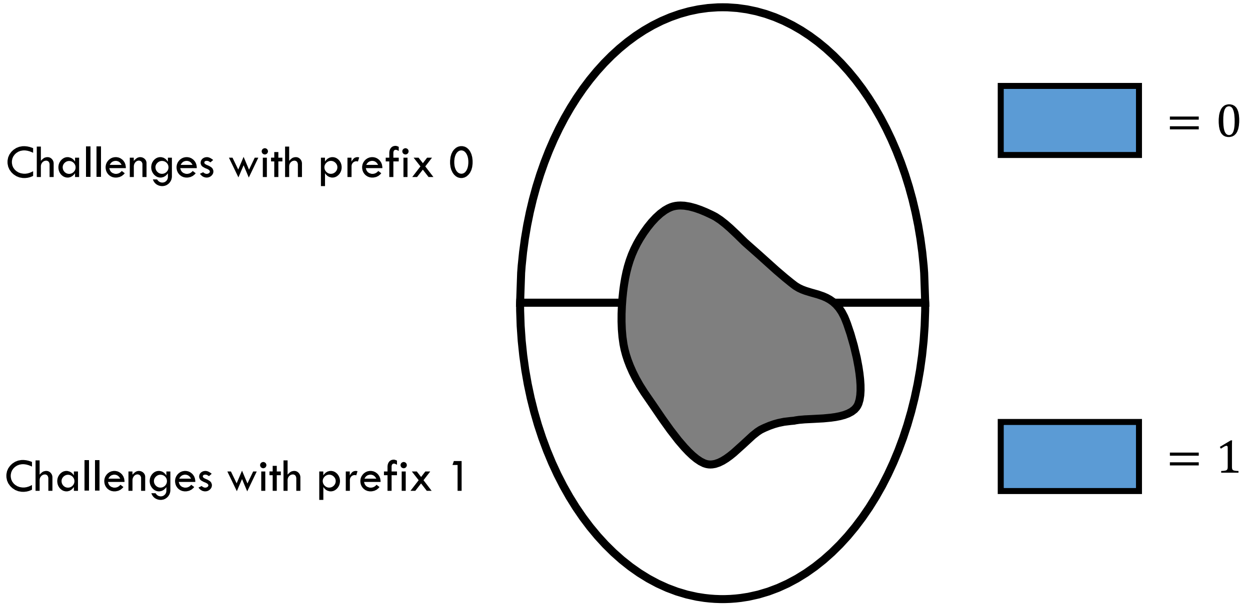


Challenge space

All bad challenges for $\mathrm{BAD}_{x,\alpha}^{(1)}$

Defining Bad Segments



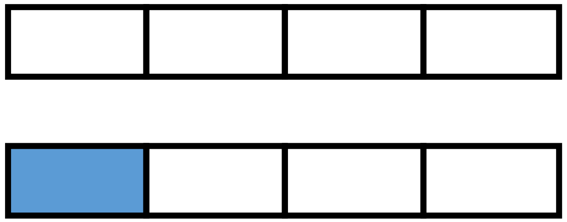


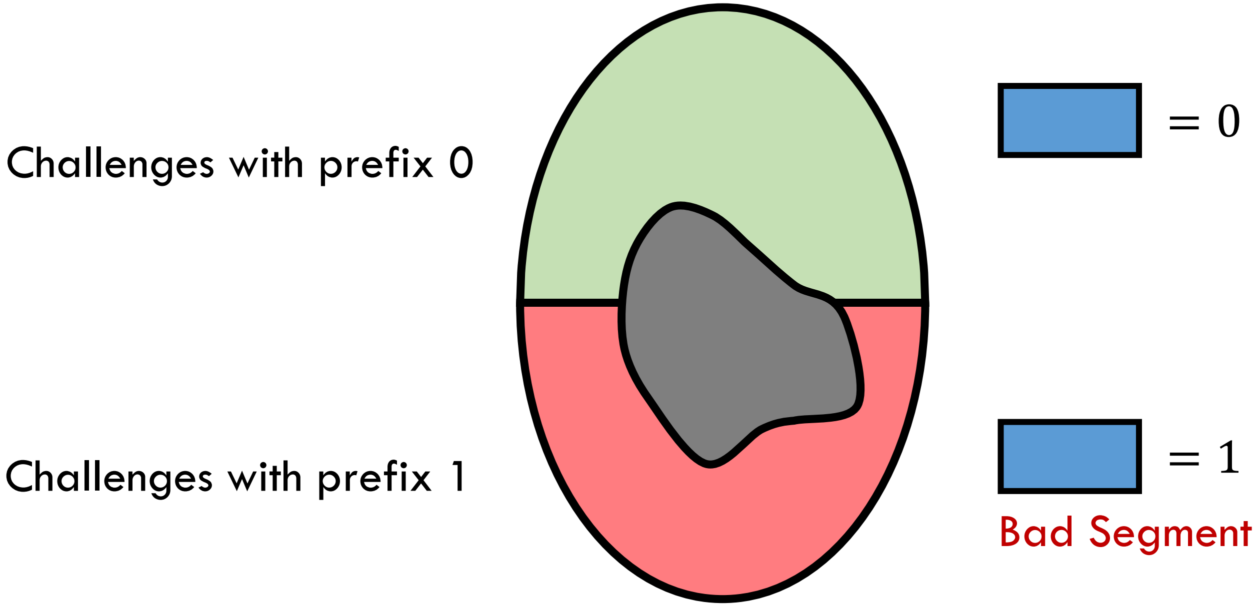
sBAD_1

is bad if

#bad challenges with prefix $>$ #bad challenges/2

Defining Bad Segments





$sBAD_1$

is bad if

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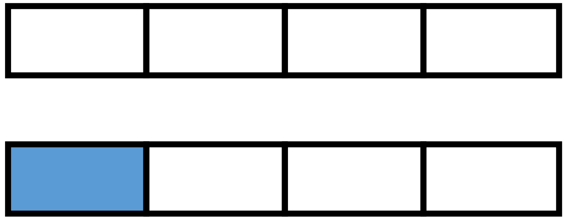
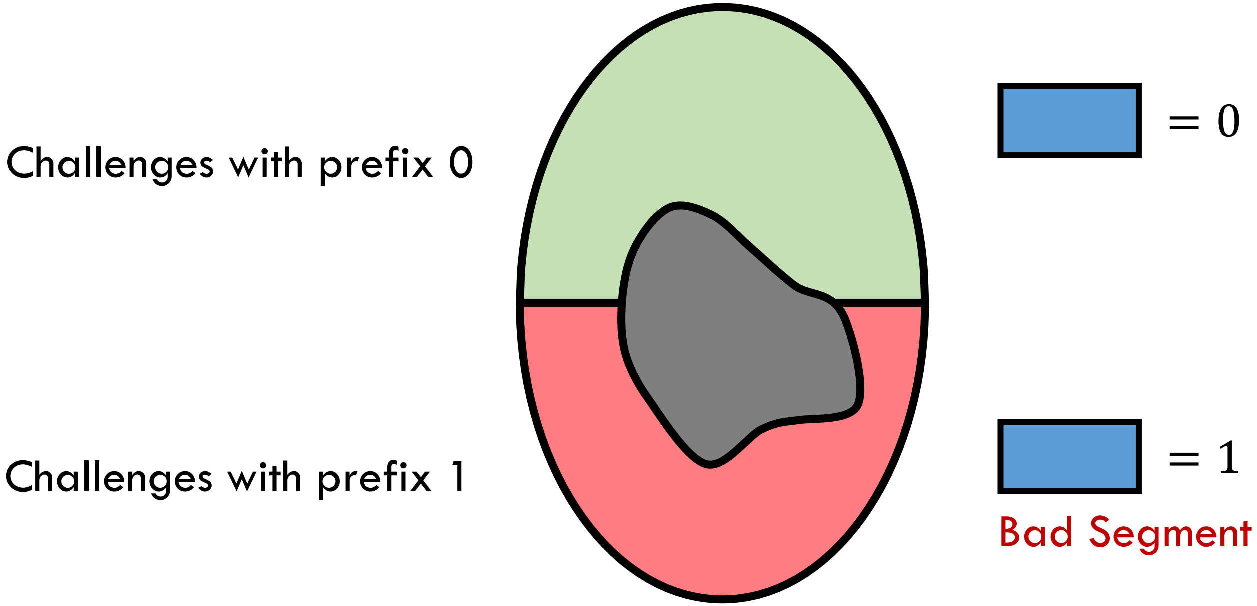
Defining Bad Segments





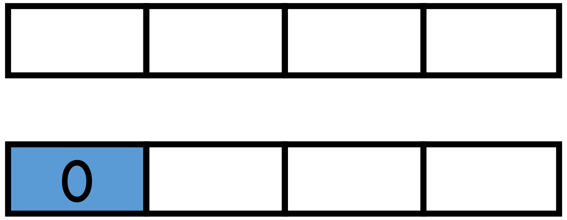
1. By pigeonhole principle, unique bad
2. ChallengeSpace polynomial size + $\mathrm{BAD}_{x,\alpha}^{(1)}$ efficiently verifiable $\Rightarrow$ sBAD_1 efficiently verifiable

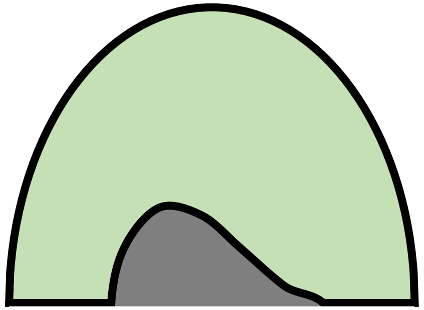
sBAD_1

is bad if

#bad challenges with prefix $>$ #bad challenges$/2$

Defining Bad Segments





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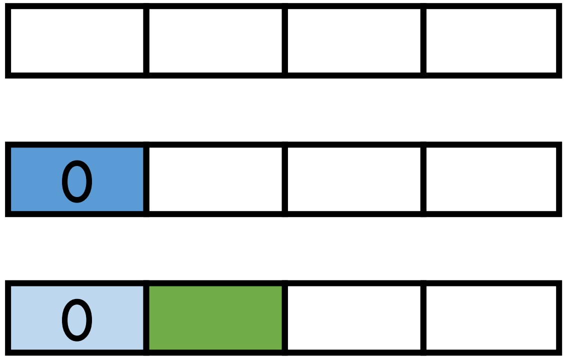
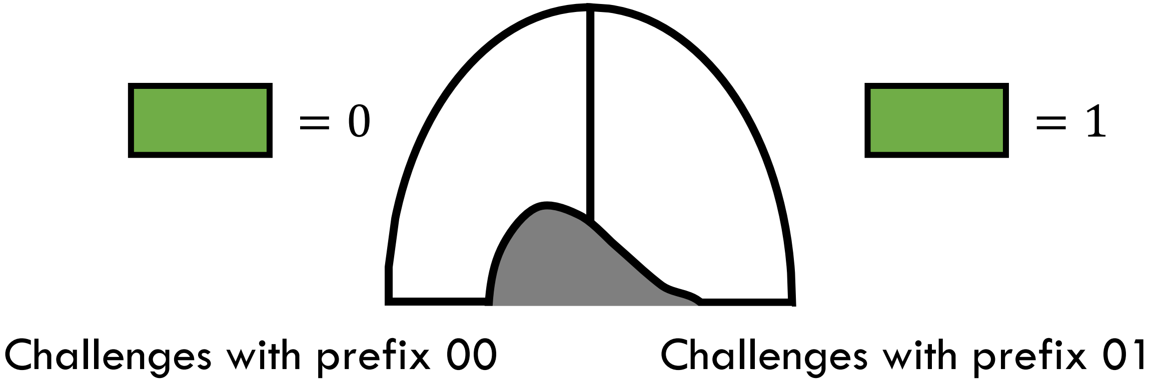
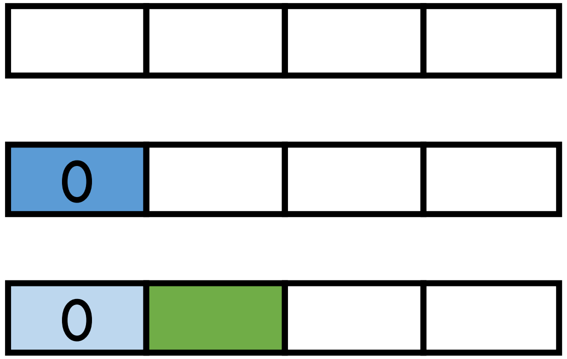
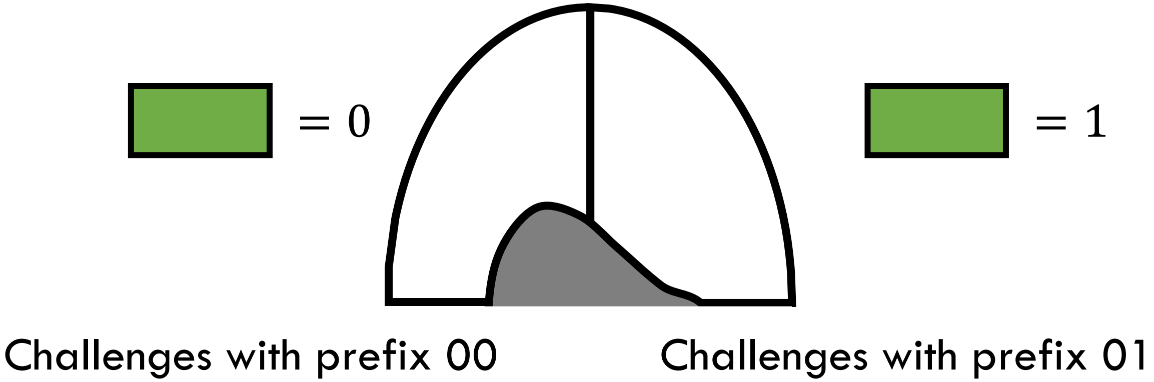




Defining Bad Segments

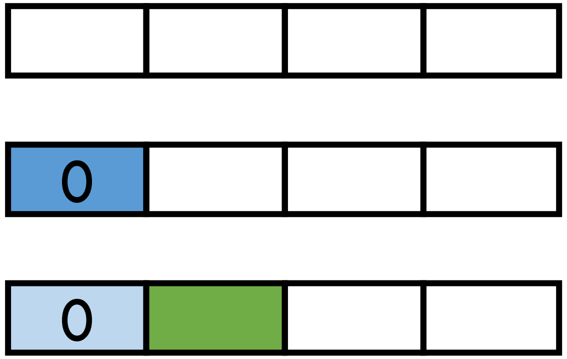


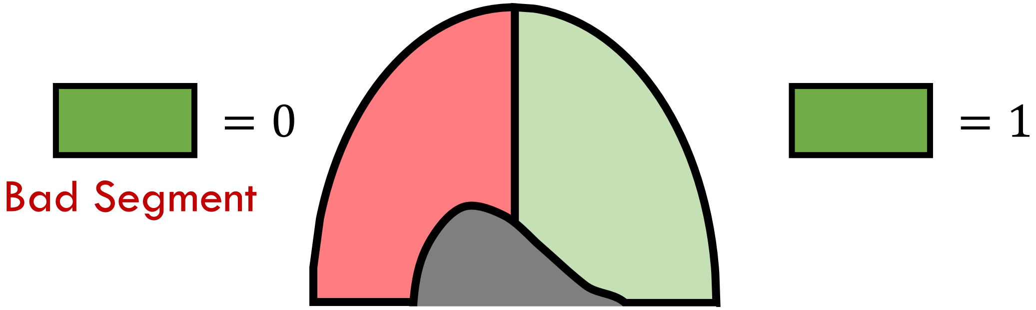


sBAD $_2$

[ ] is bad given [0] if
#bad challenges with prefix [0][ ]
> (#bad challenges with prefix [0])/2

Defining Bad Segments





sBAD_2

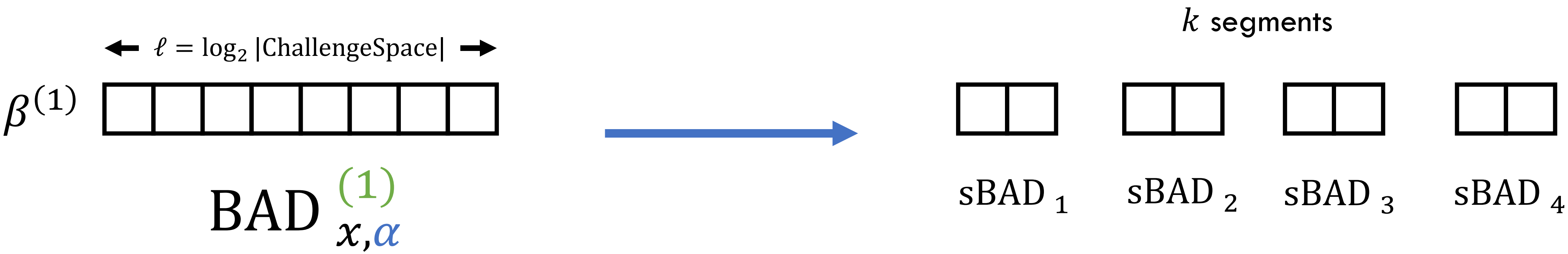
is bad given 0 if

#bad challenges with prefix 0

> (#bad challenges with prefix 0)/2

Reducing to Verifiable Unique Bad Challenge

No parallel repetition



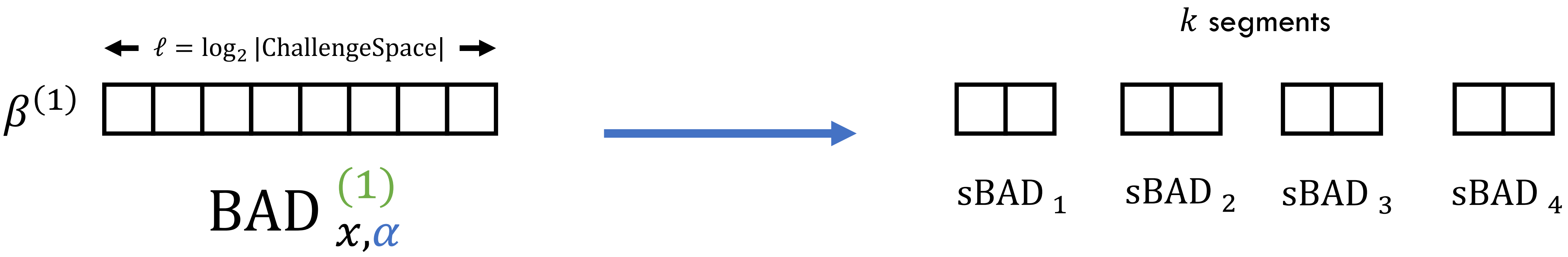
Requirements:

1. Each sBAD_j must be efficiently verifiable unique bad challenge relations.
2. If a challenge is bad, then there must exist a bad segment.

Each segment has ℓ/k bits

Reducing to Verifiable Unique Bad Challenge

No parallel repetition



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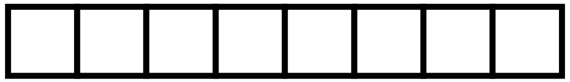
Each segment has ℓ / k bits

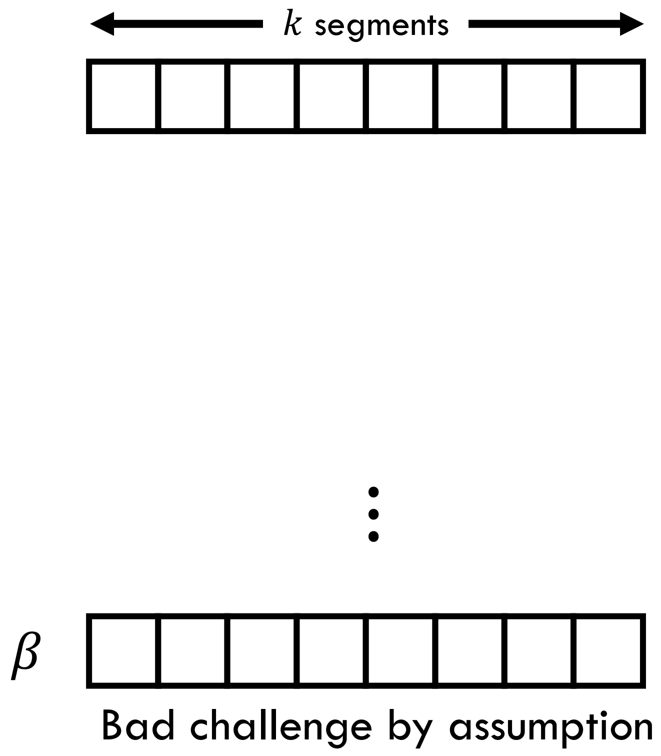
Existence of a bad segment

β



Bad challenge by assumption

Existence of a bad segment



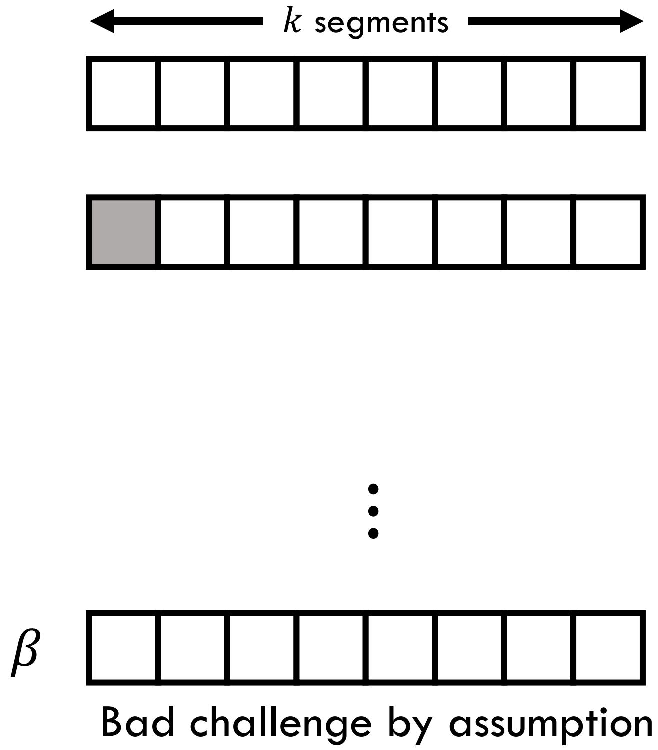
#bad challenges remaining

T

T = #bad challenges $\mathrm{BAD}_{x,\alpha}^{(1)}$

k such that $2^k > T$

Existence of a bad segment



#bad challenges remaining

T

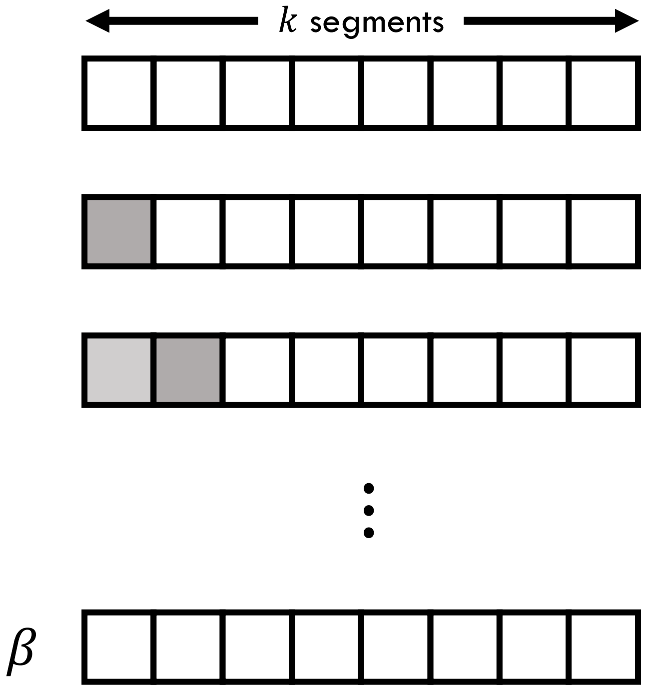
$< T/2$

If each segment is good

$T = $ #bad challenges $\mathrm{BAD}_{x,\alpha}^{(1)}$

k such that $2^k > T$

Existence of a bad segment



Bad challenge by assumption

#bad challenges remaining

T

$< T/2$

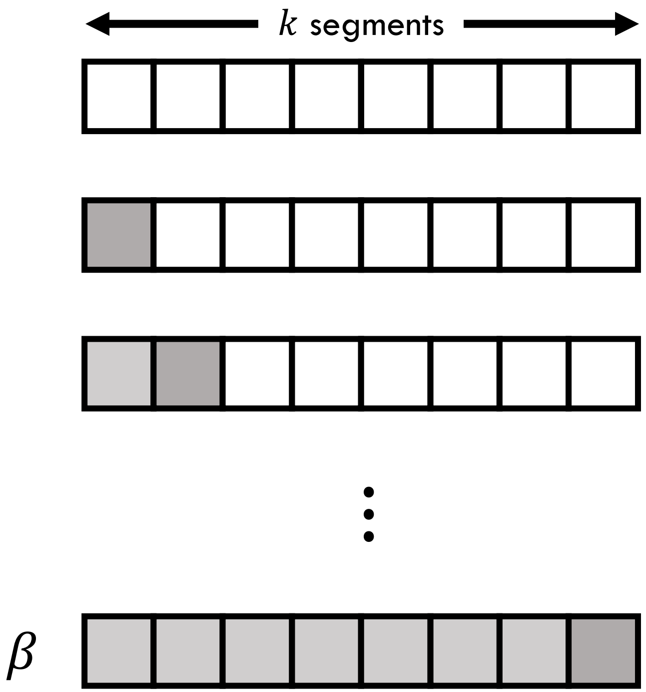
$< T/4$

If each segment is good

T = #bad challenges $\mathrm{BAD}_{x,\alpha}^{(1)}$

k such that $2^k > T$

Existence of a bad segment



Bad challenge by assumption

#bad challenges remaining

T

$< T/2$

$< T/4$

⋮

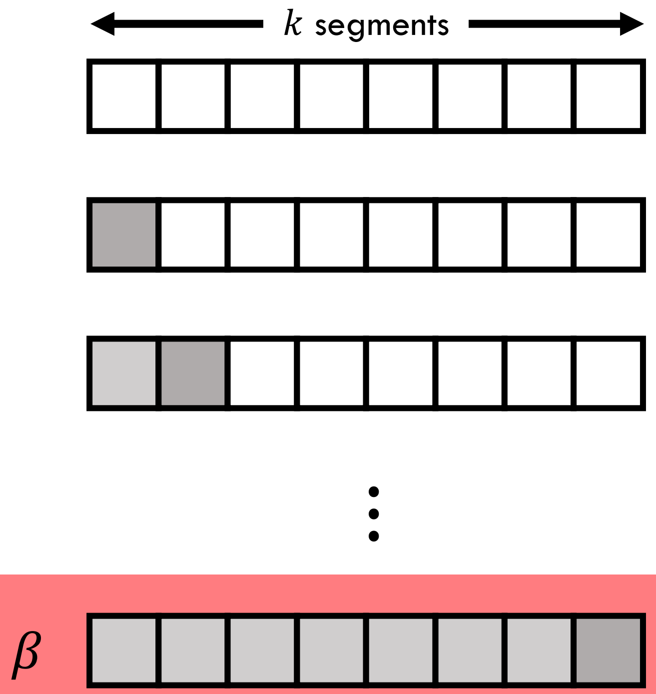
< 1

If each segment is good

$T =$ #bad challenges $\mathrm{BAD}_{x,\alpha}^{(1)}$

k such that $2^k > T$

Existence of a bad segment



Bad challenge by assumption

#bad challenges remaining

T

$< T/2$

$< T/4$

⋮

< 1

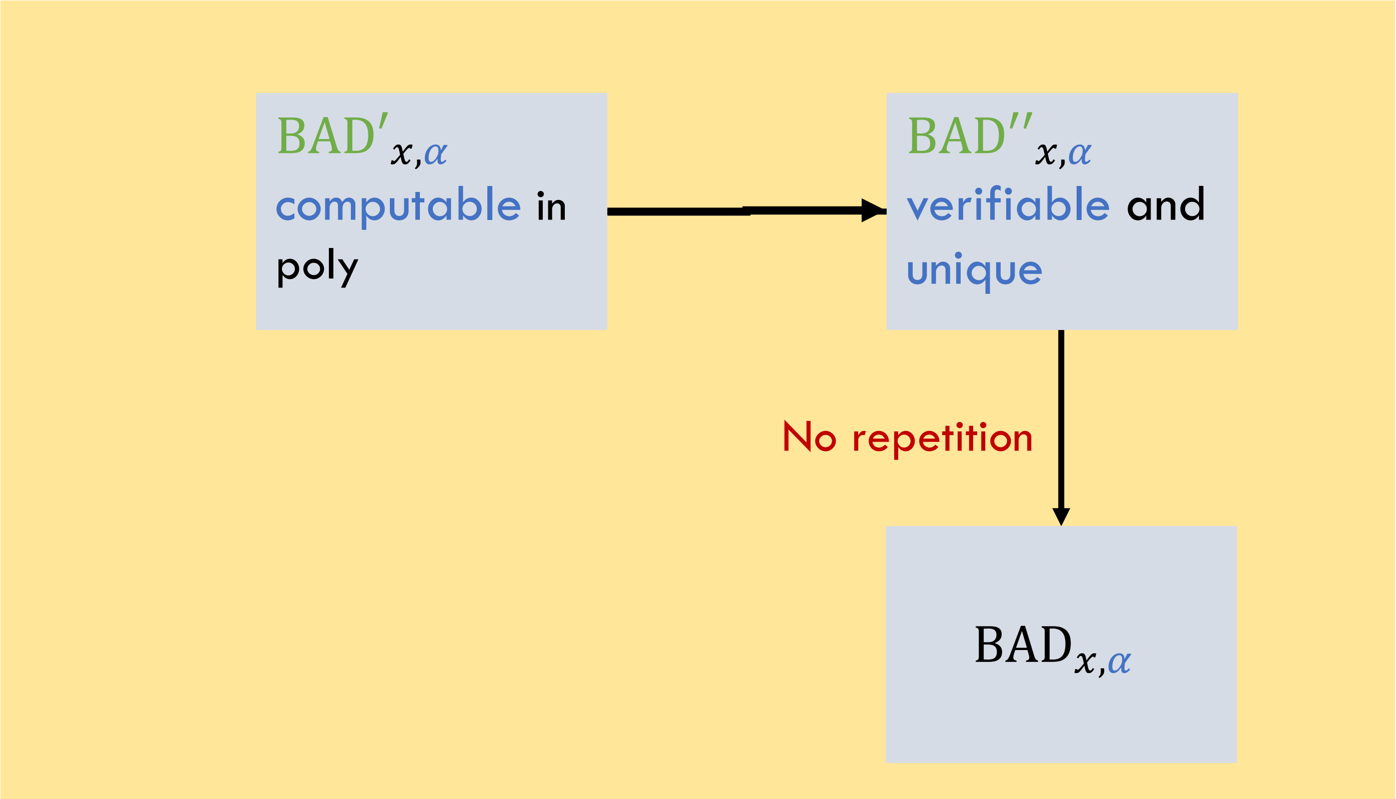
$T = \#\text{bad challenges } \mathrm{BAD}_{x,\alpha}^{(1)}$

k such that $2^k > T$

contradiction

If each segment is good

Overview So Far



$\mathrm{BAD}_{x,\alpha}$ properties

1. Bad challenges are a product set
2. Challenge space is of polynomial size
3. Bad challenges are product verifiable in poly

Concluding Remarks

See paper for:

1. Extension to parallel repetition.
2. Choice of parameters for size of segments, number of repetitions.
3. New somewhere extractable hash scheme necessary for “Magic box”.

Recap: Our Results

Theorem 1

Assuming sub-exponential hardness of DDH, there exists SNARGs for batch NP where

$$|\Pi| = \text{poly}(\log k, |C|)$$

Theorem 2

Assuming sub-exponential hardness of DDH, there exists SNARGs for P where

$|\text{CRS}|, |\Pi|, |$  $| = \text{polylog}(T)$

Thank you. Questions?

Arka Rai Choudhuri

arkarai.choudhuri@ntt-research.com

ia.cr/2022/1486