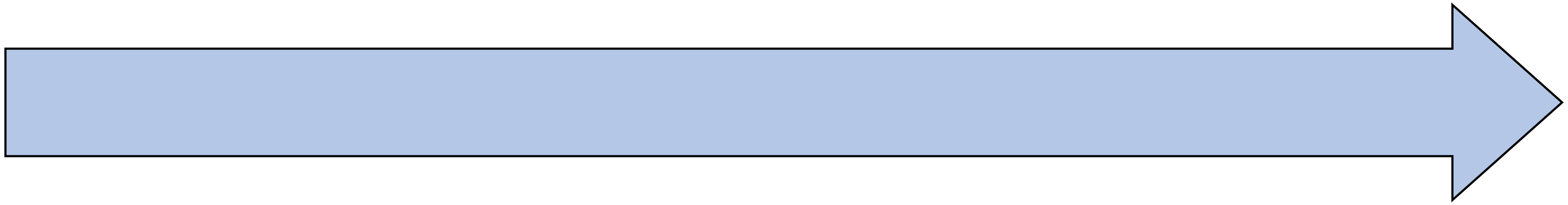
Finding a Nash Equilibrium is No Easier than Breaking Fiat-Shamir

Arka Rai Choudhuri   Pavel Hubáček   Chethan Kamath
Krzysztof Pietrzak   Alon Rosen   Guy Rothblum

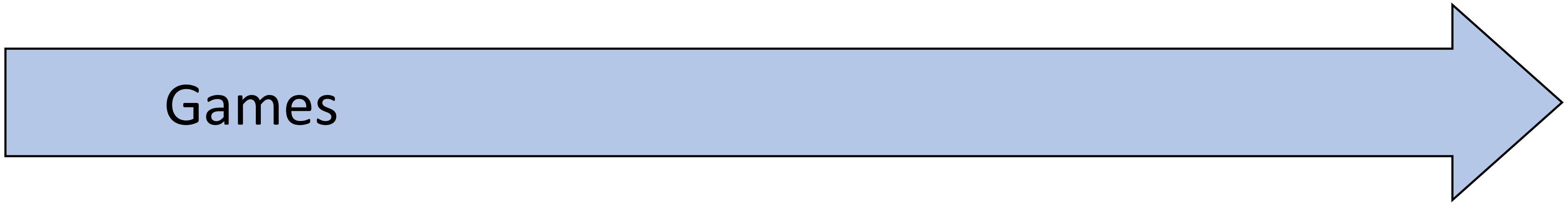
JHU Theory Seminar

What is the Cryptographic Hardness in PPAD?

The Story Line



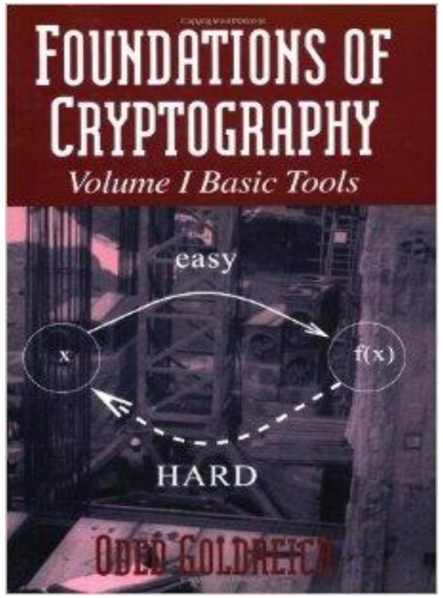
The Story Line



The Story Line

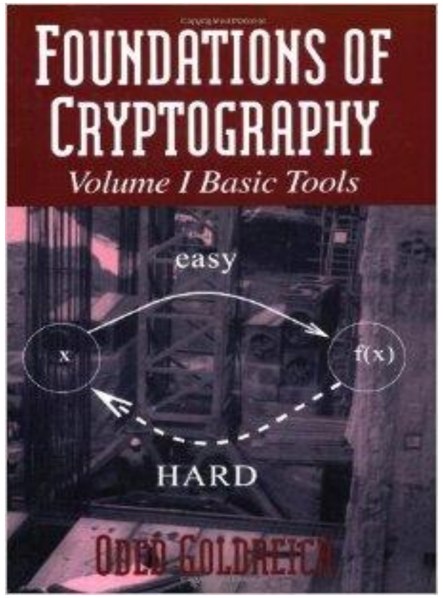
Games Complexity

The Story Line



Games Complexity Crypto

The Story Line



Games Complexity Crypto

The Story Line

Games Complexity Crypto

Game Theory and Nash Equilibrium

Game Theory and Nash Equilibrium

Game Theory and Nash Equilibrium

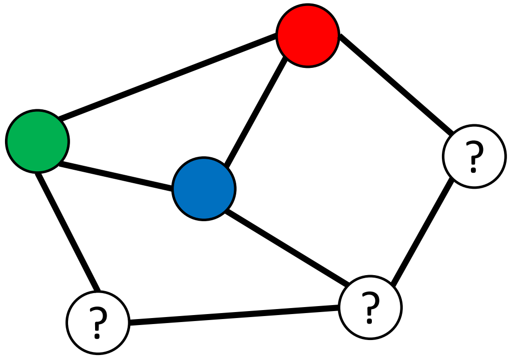
	Left	Right
Left	1 \ -1	-1\ 1
Right	-1 \ 1	1 \ -1

Game Theory and Nash Equilibrium

	Left	Right
Left	1 \ -1	-1\ 1
Right	-1 \ 1	1 \ -1

[Nash'51]: A (mixed) equilibrium always exists

How hard is finding a Nash Equilibrium?



Reduction?

	Left	Right
Left	1 \ -1	-1\ 1
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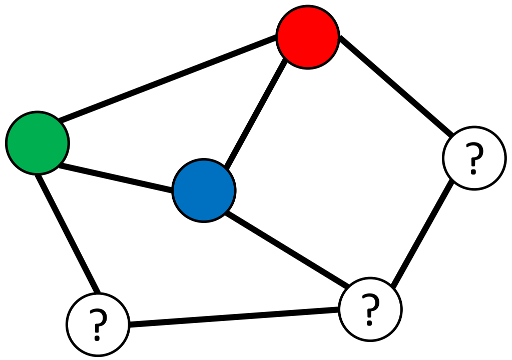
How hard is finding a Nash Equilibrium?



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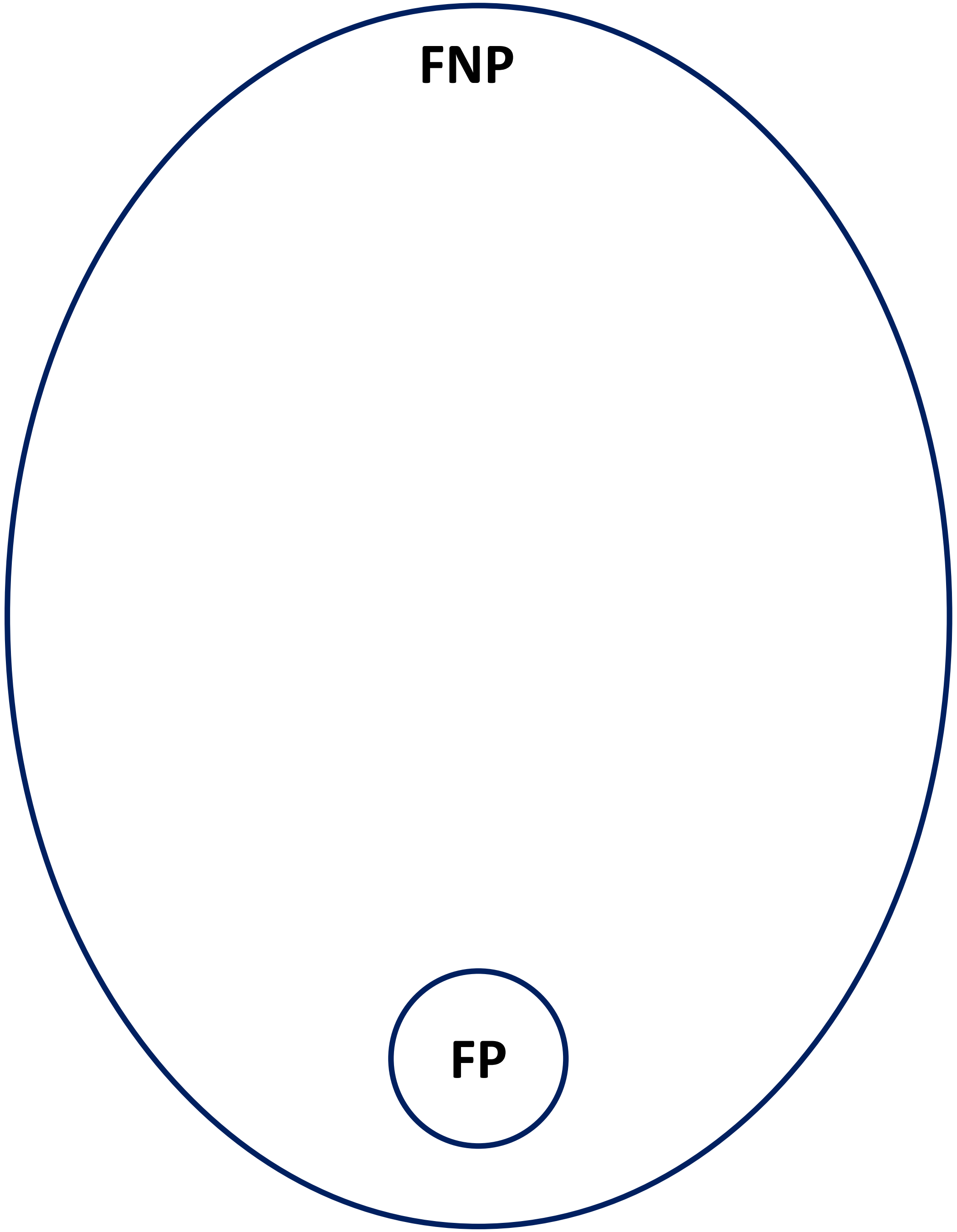
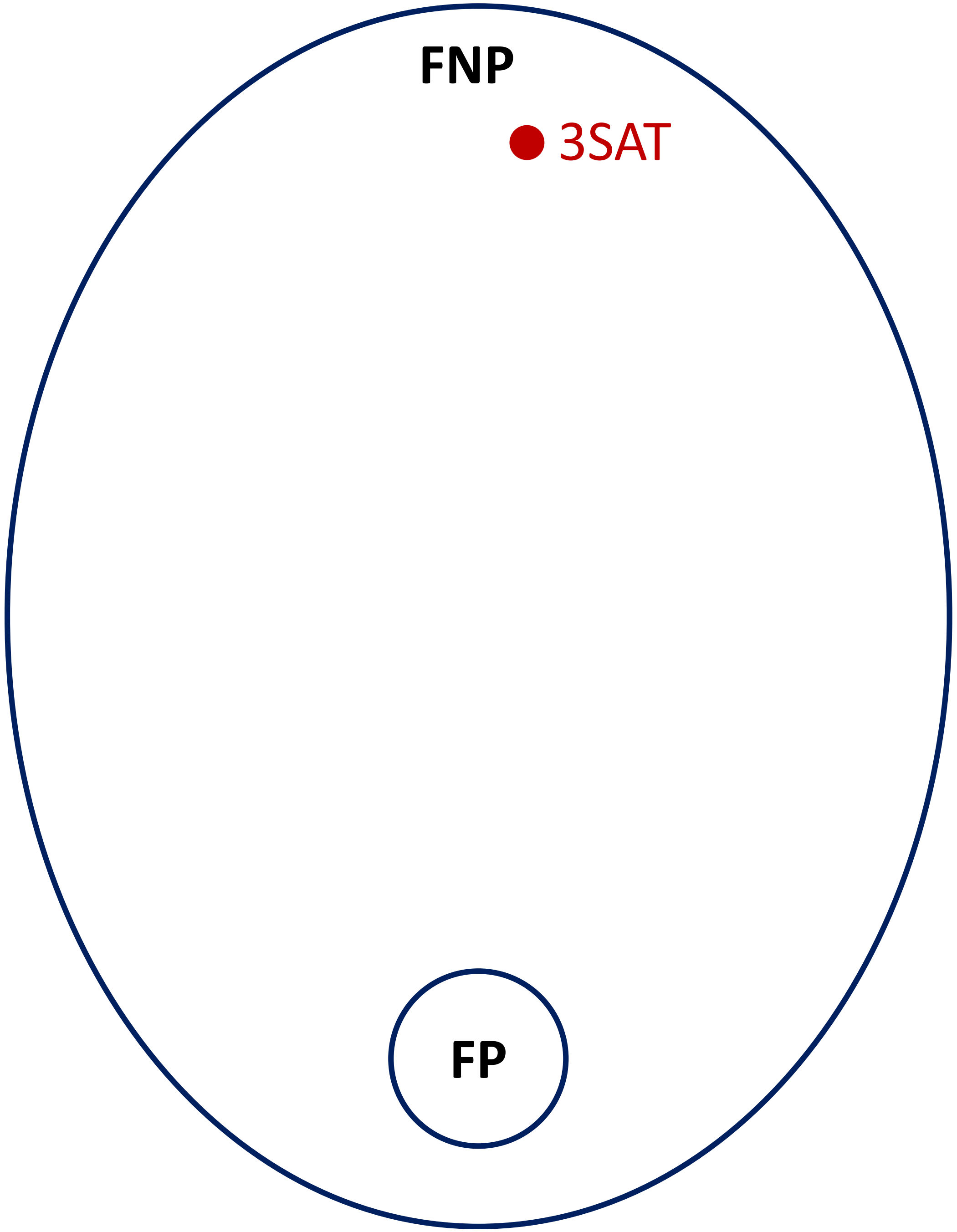
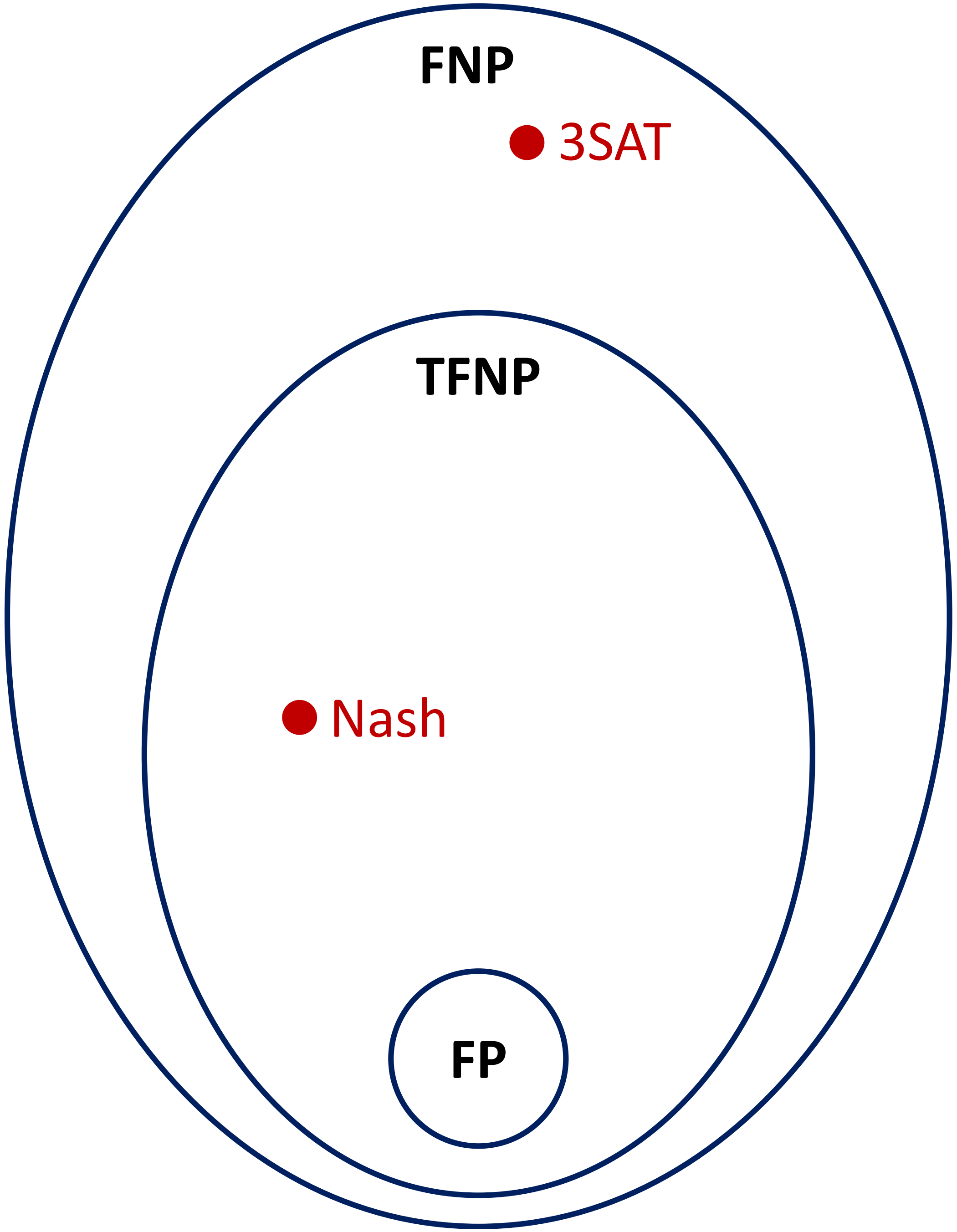
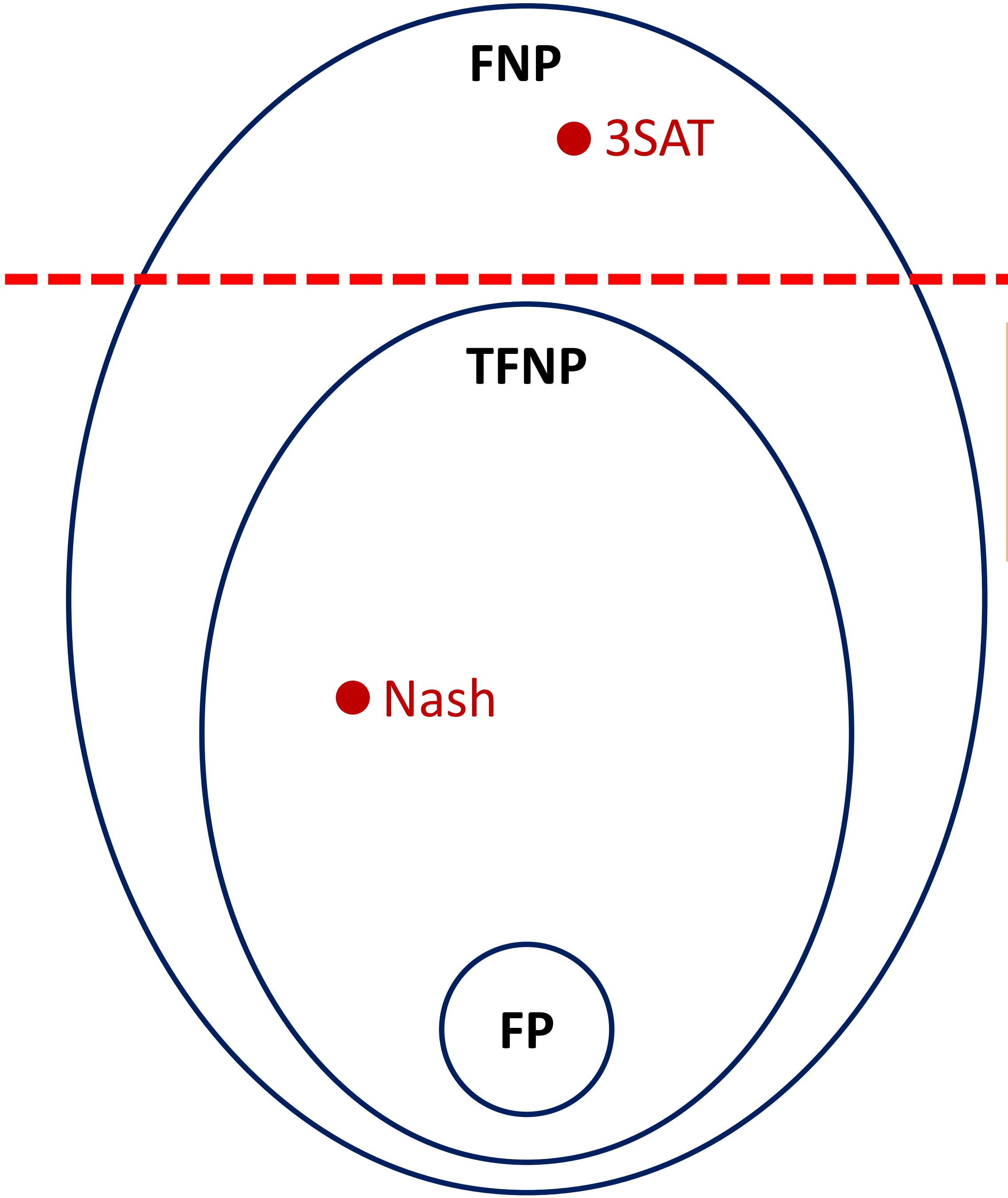

FNP
FP

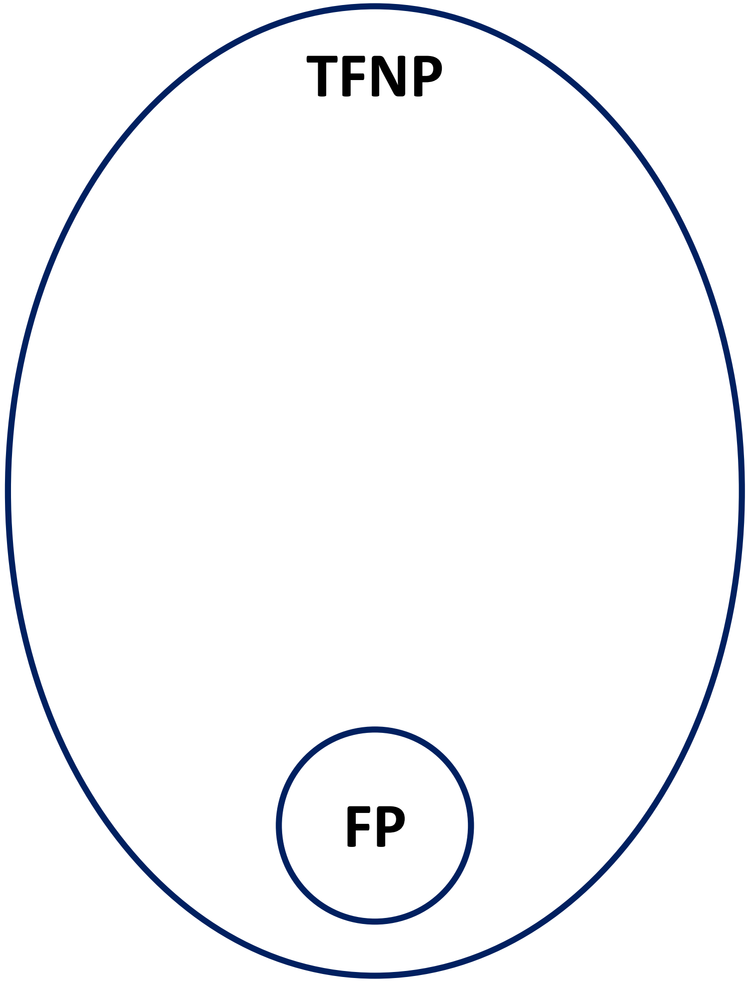

FNP
3SAT
FP


FNP
3SAT
TFNP
Nash
FP



Not NP-hard unless NP=coNP
[Megiddo-Papadimitriou’89]

The Class PPAD [Papadimitriou'94]



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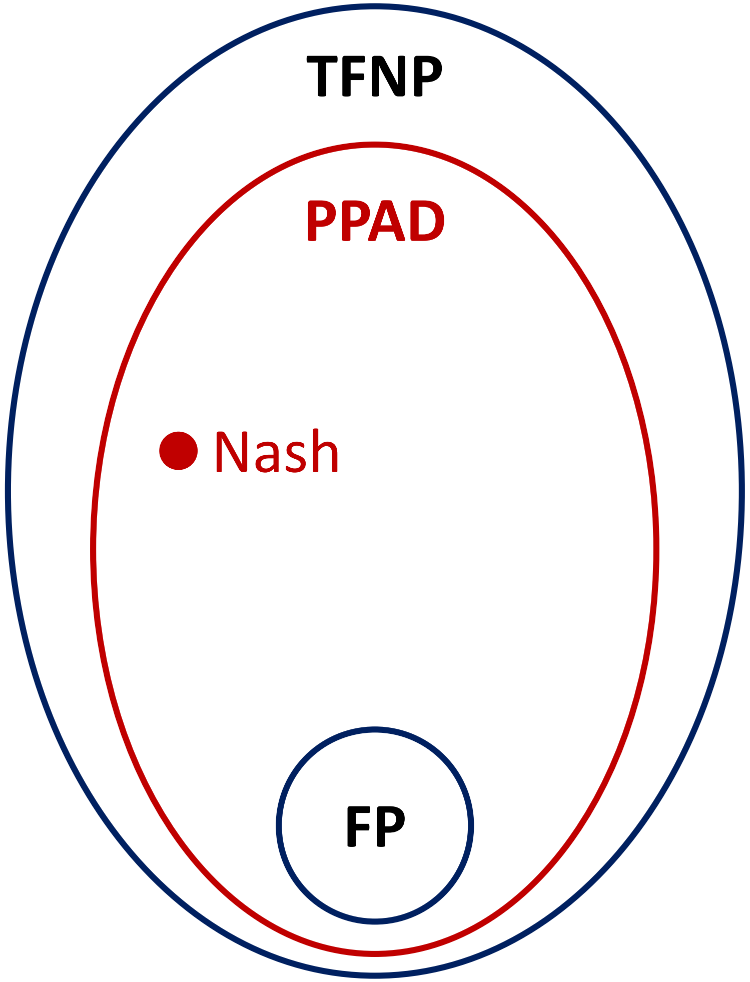

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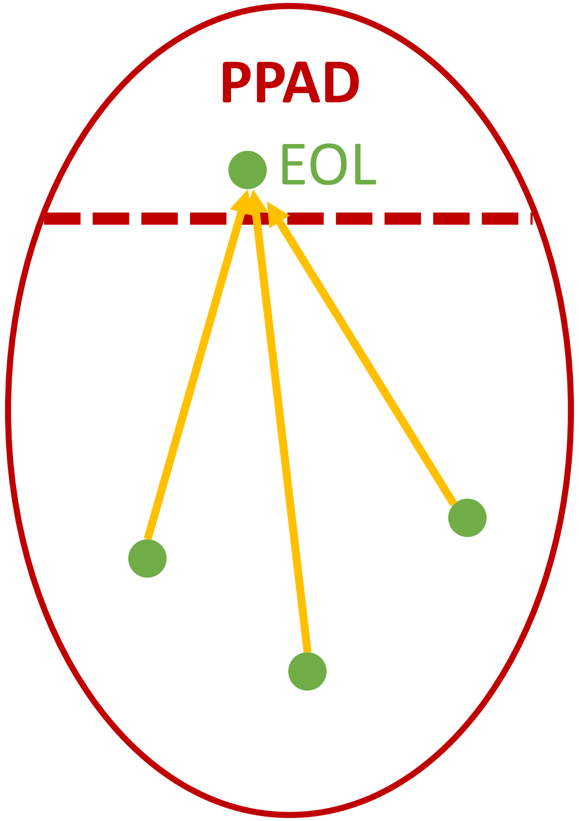
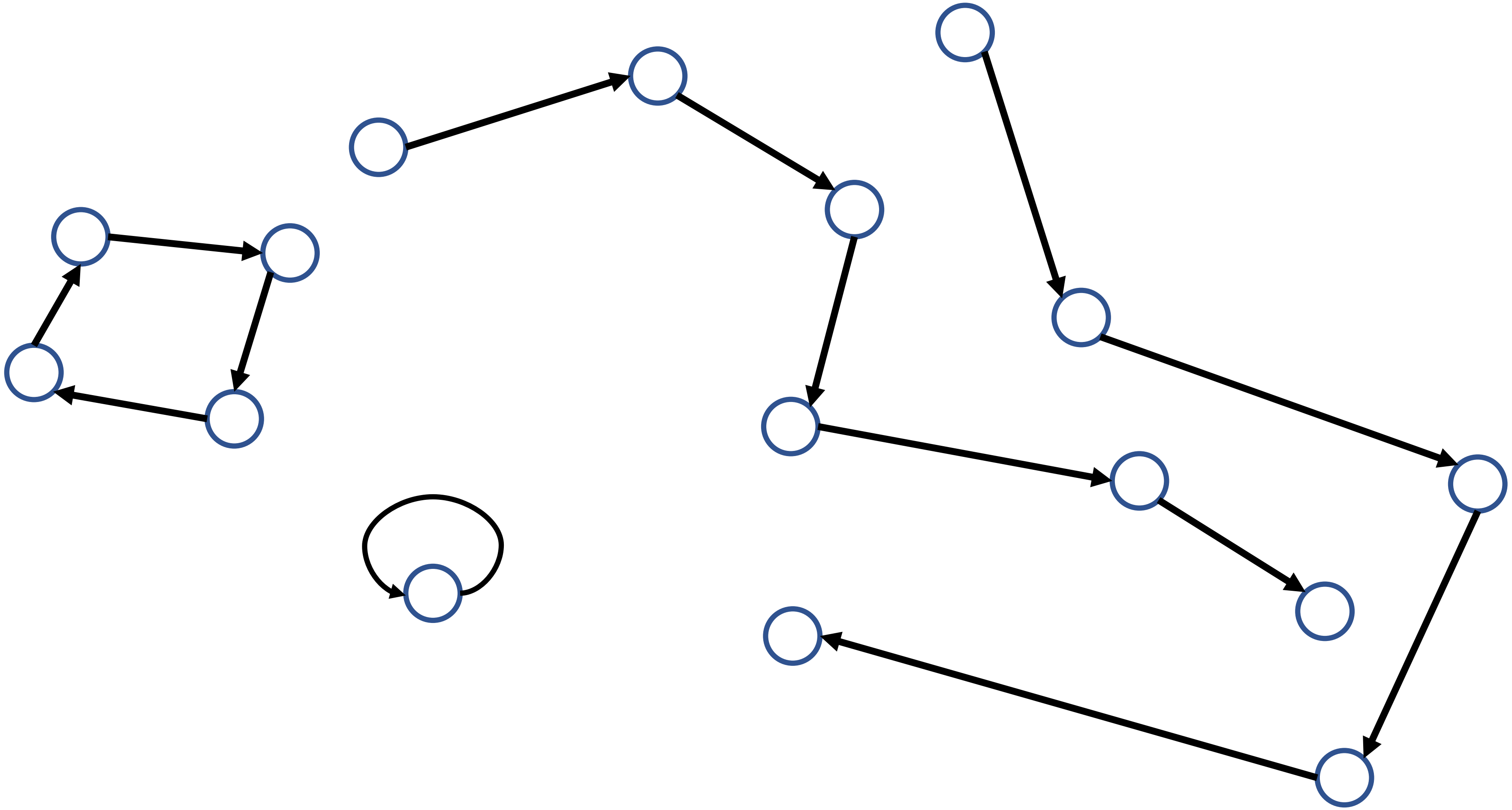


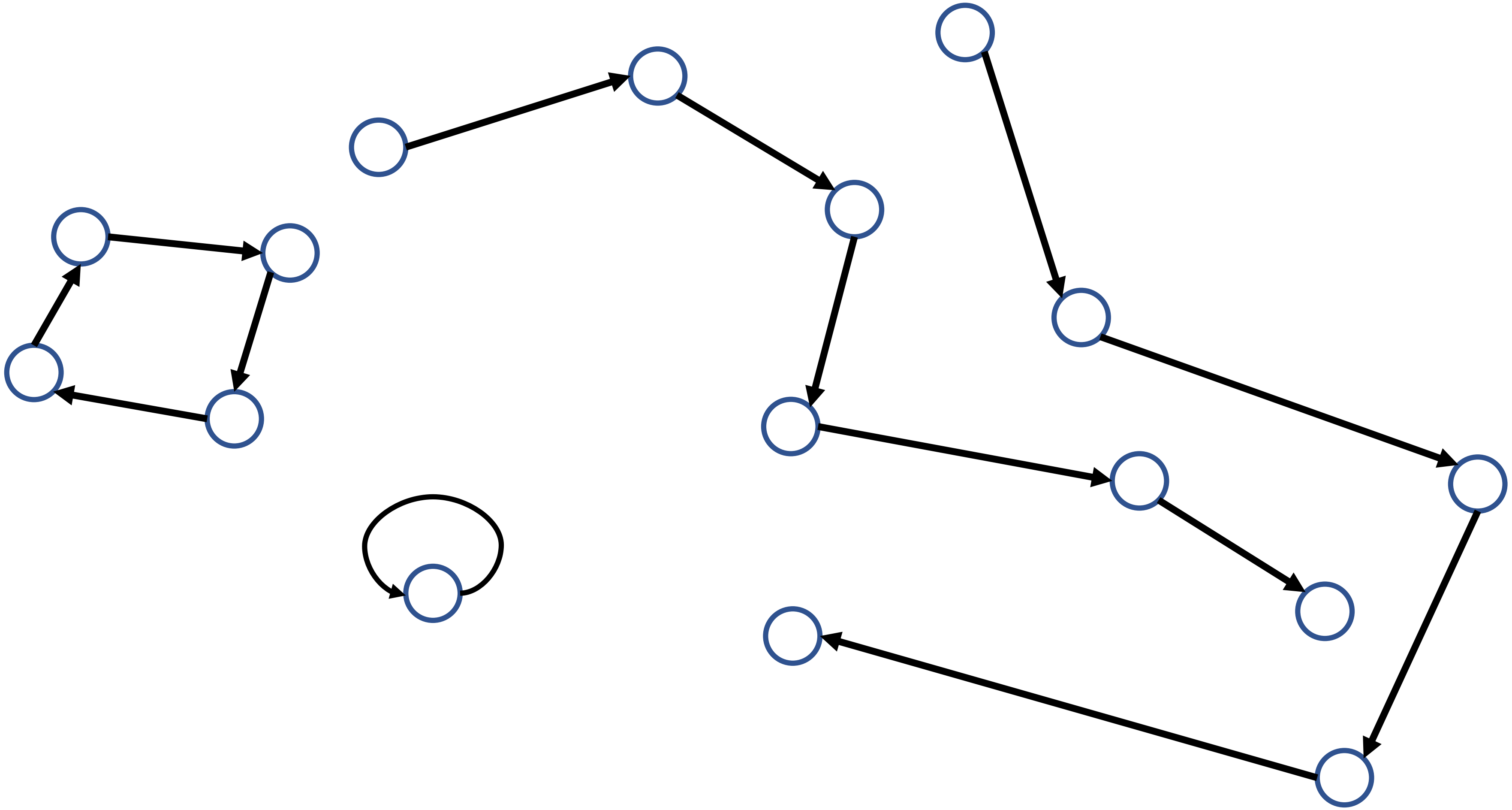
Defined through its complete problem:
END OF THE LINE (EOL)

End of the Line (EOL)



End of the Line (EOL)

Directed graph



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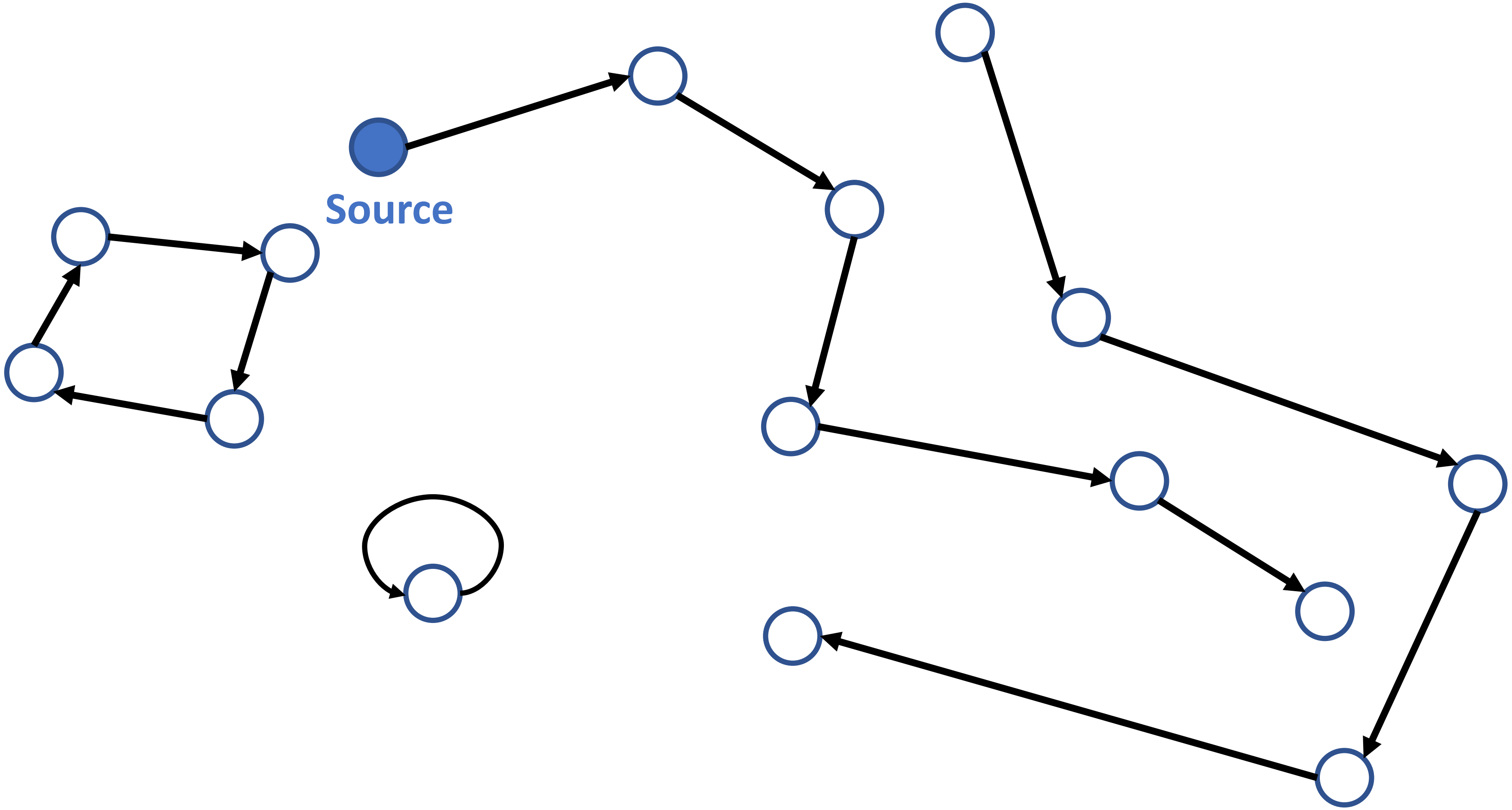
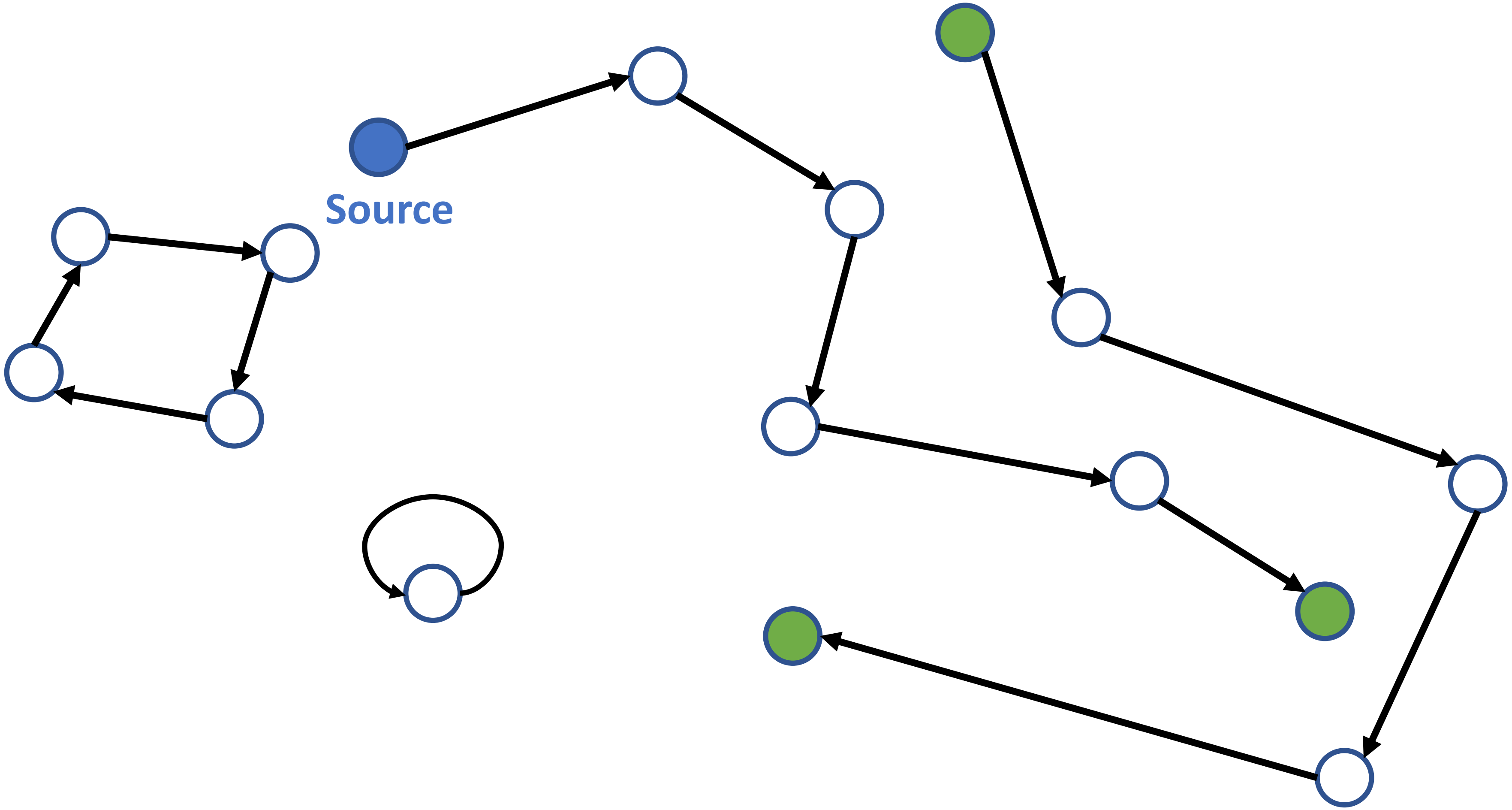


Directed graph

Every node has in and out degree ≤ 1

End of the Line (EOL)



Directed graph

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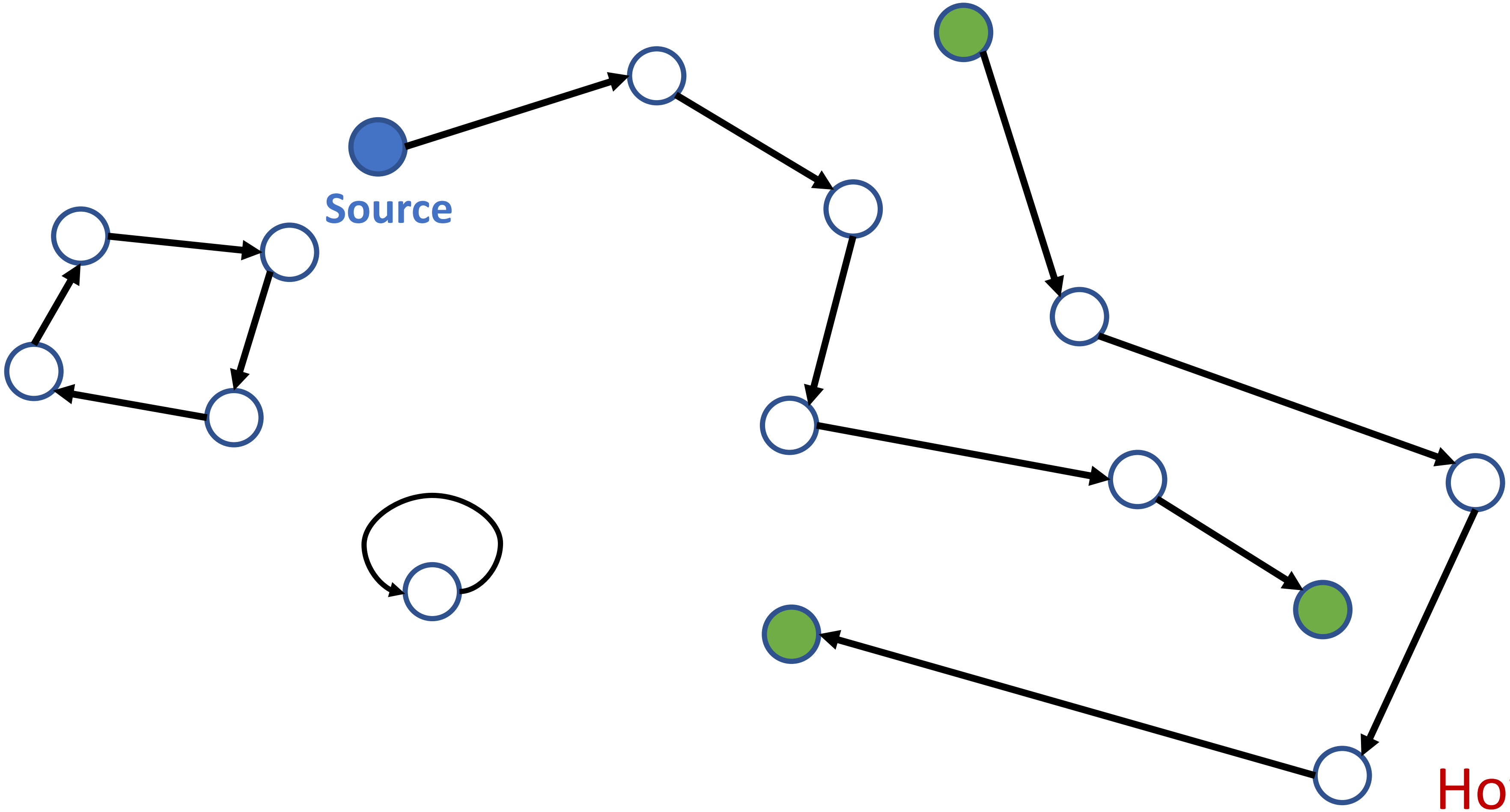
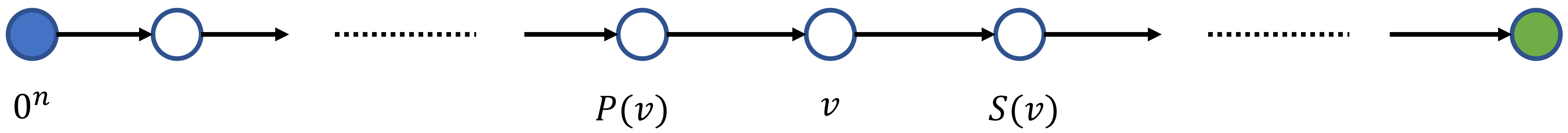


Directed graph

Every node has in and out degree ≤ 1

How easy is it to solve this?

End of the Line (EOL)



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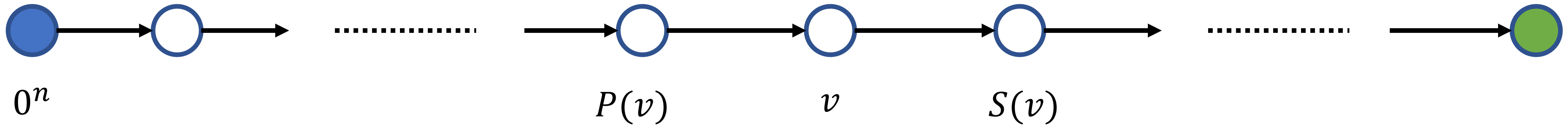

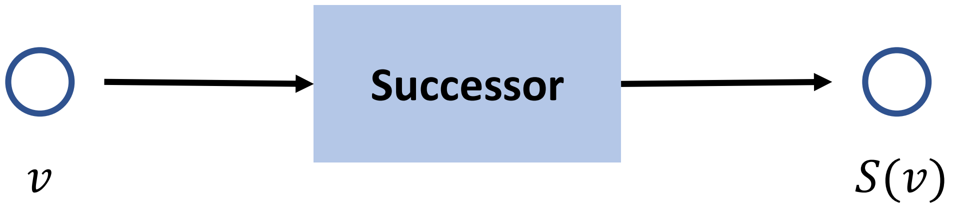

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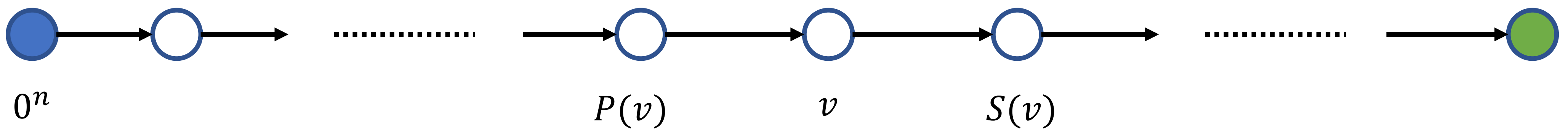
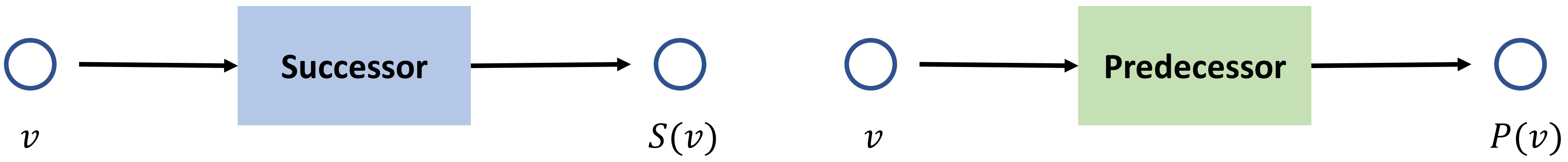
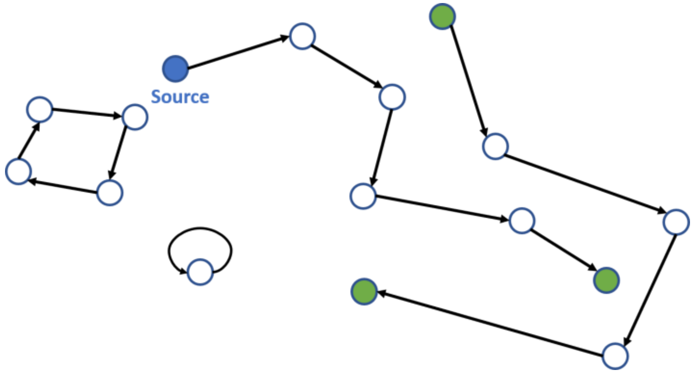




PPAD and NASH [Papadimitriou’94]



Reduction

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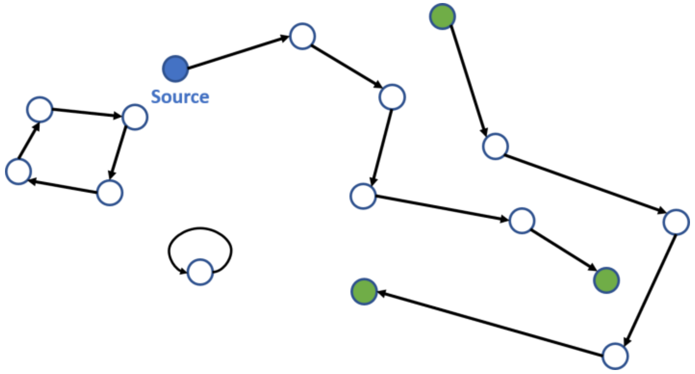

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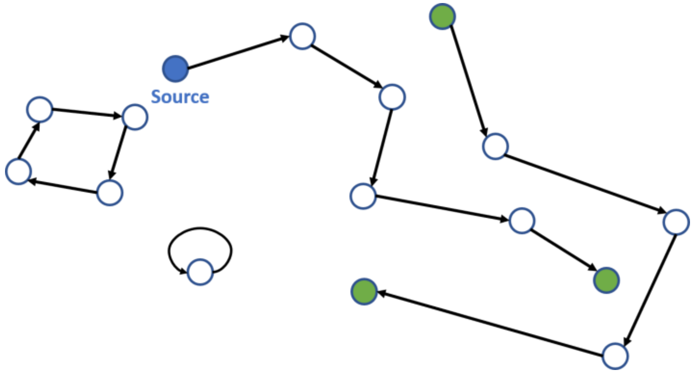

Reduction

Reduction

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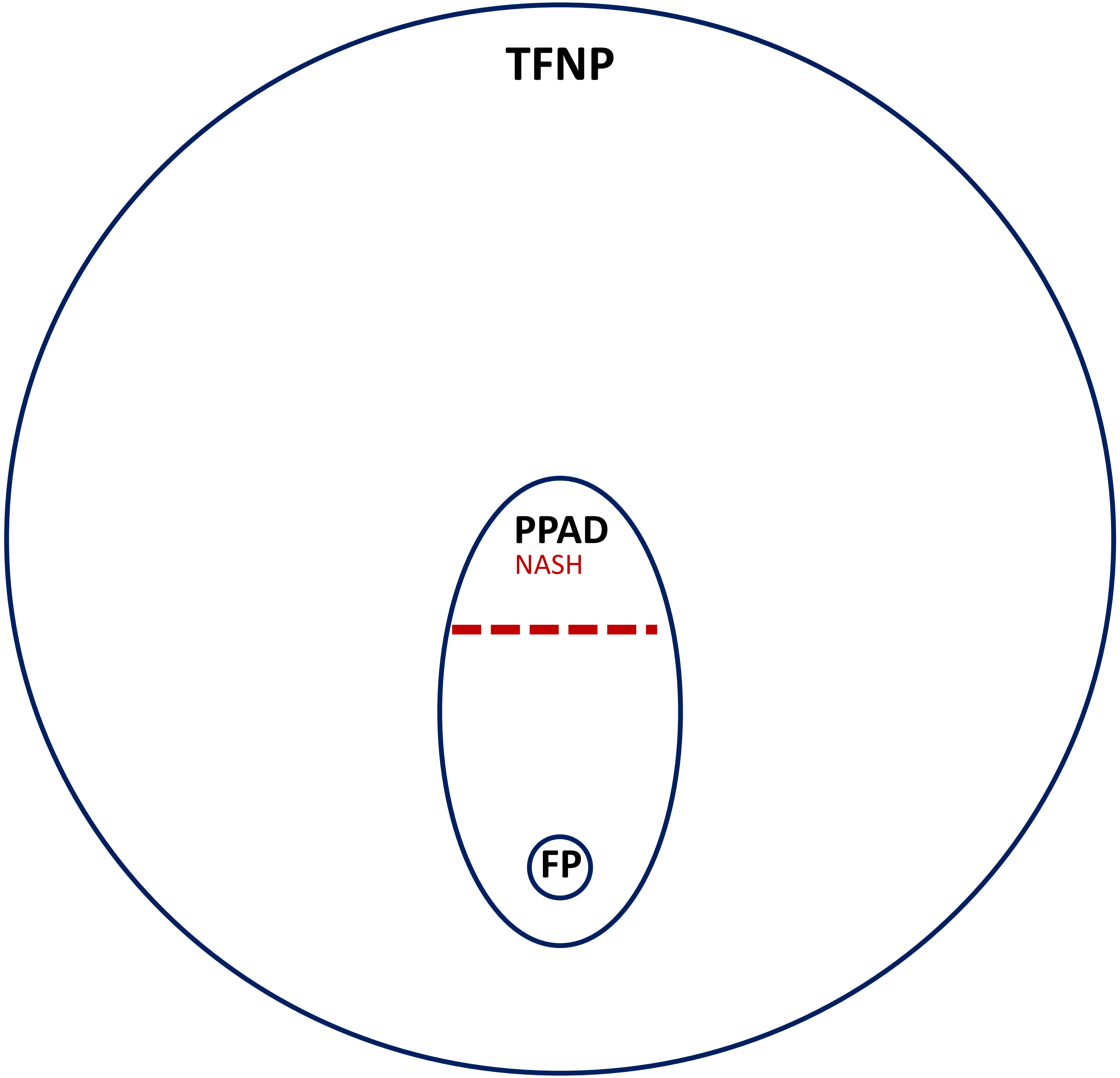
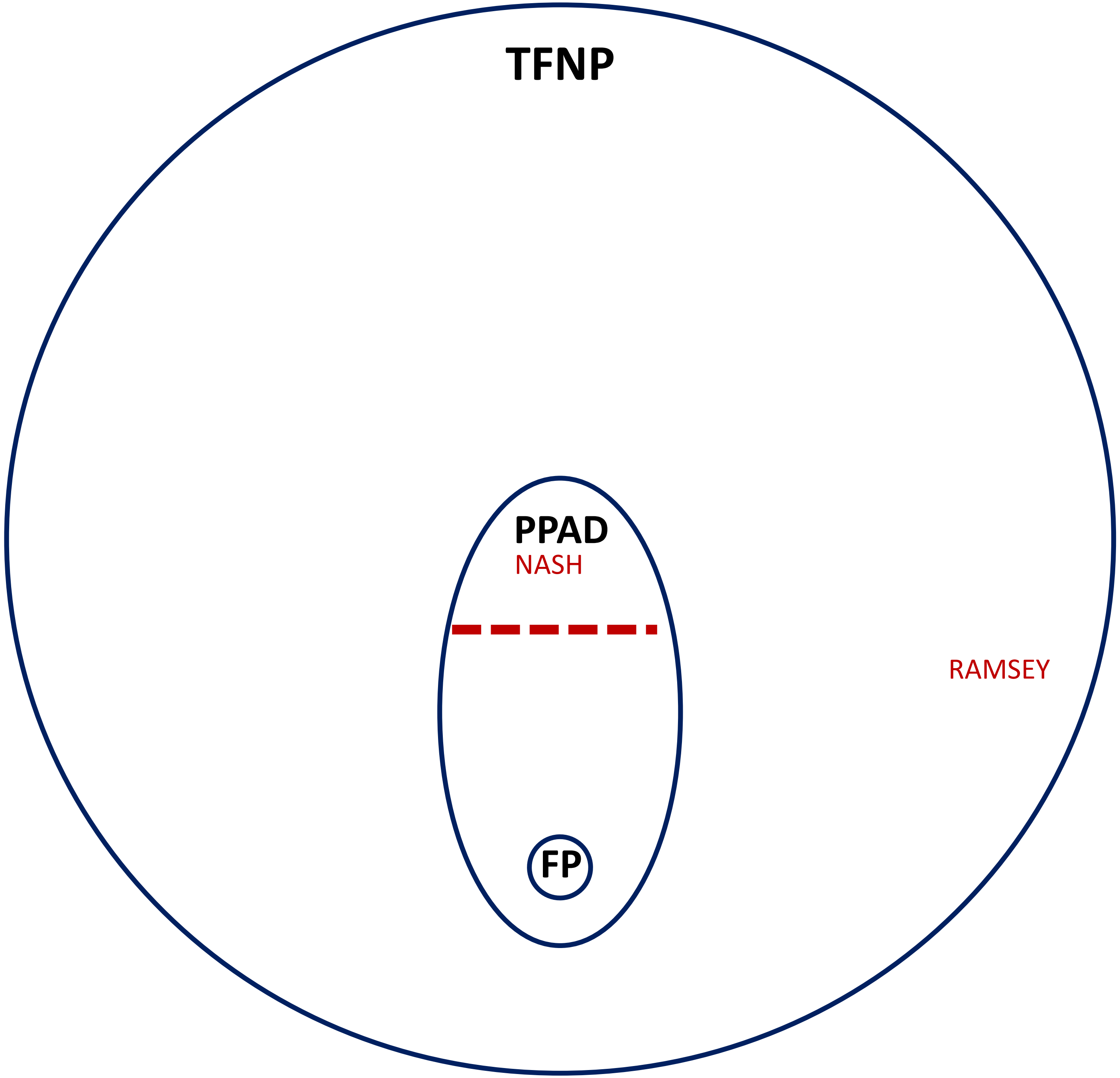
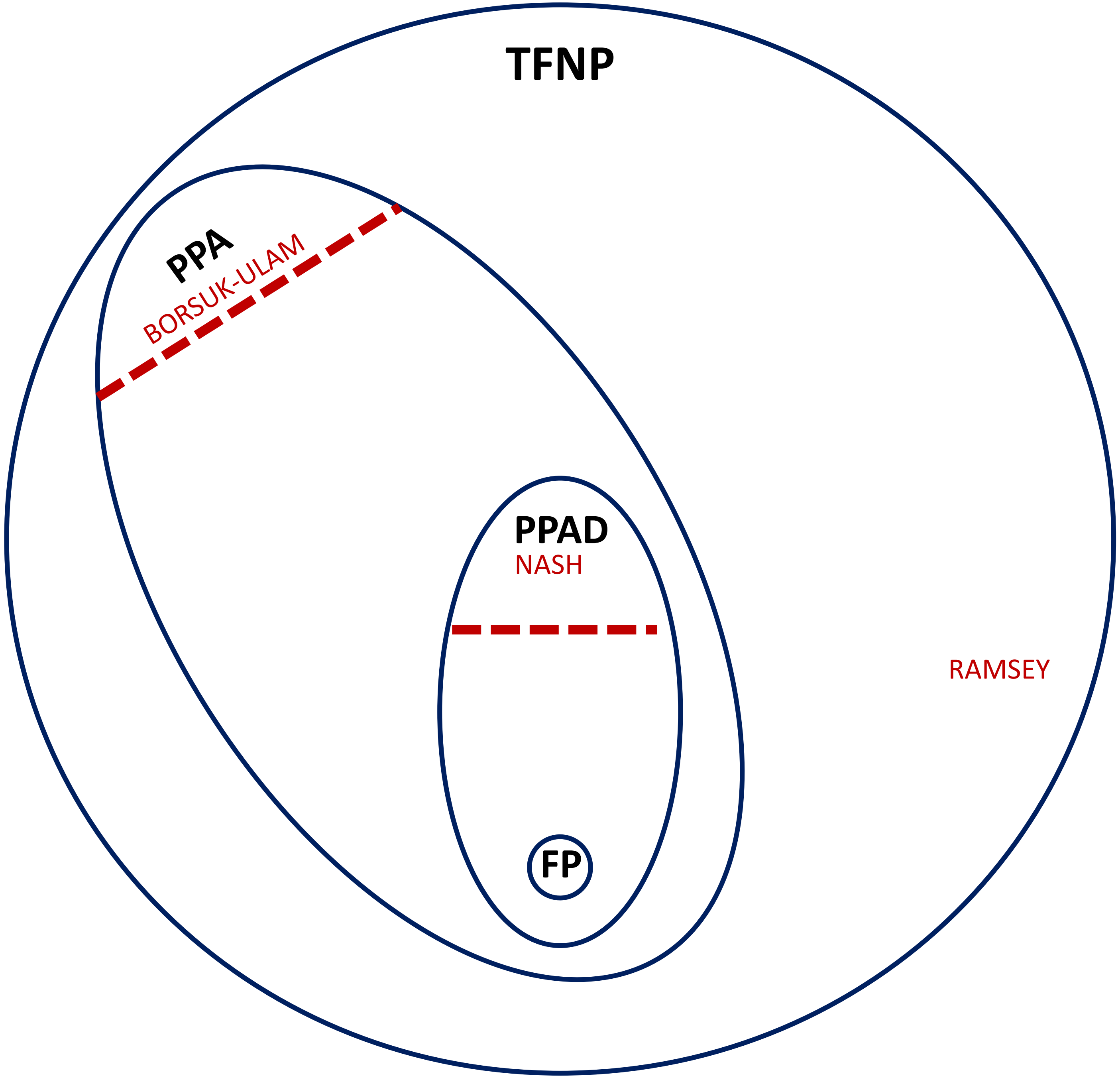
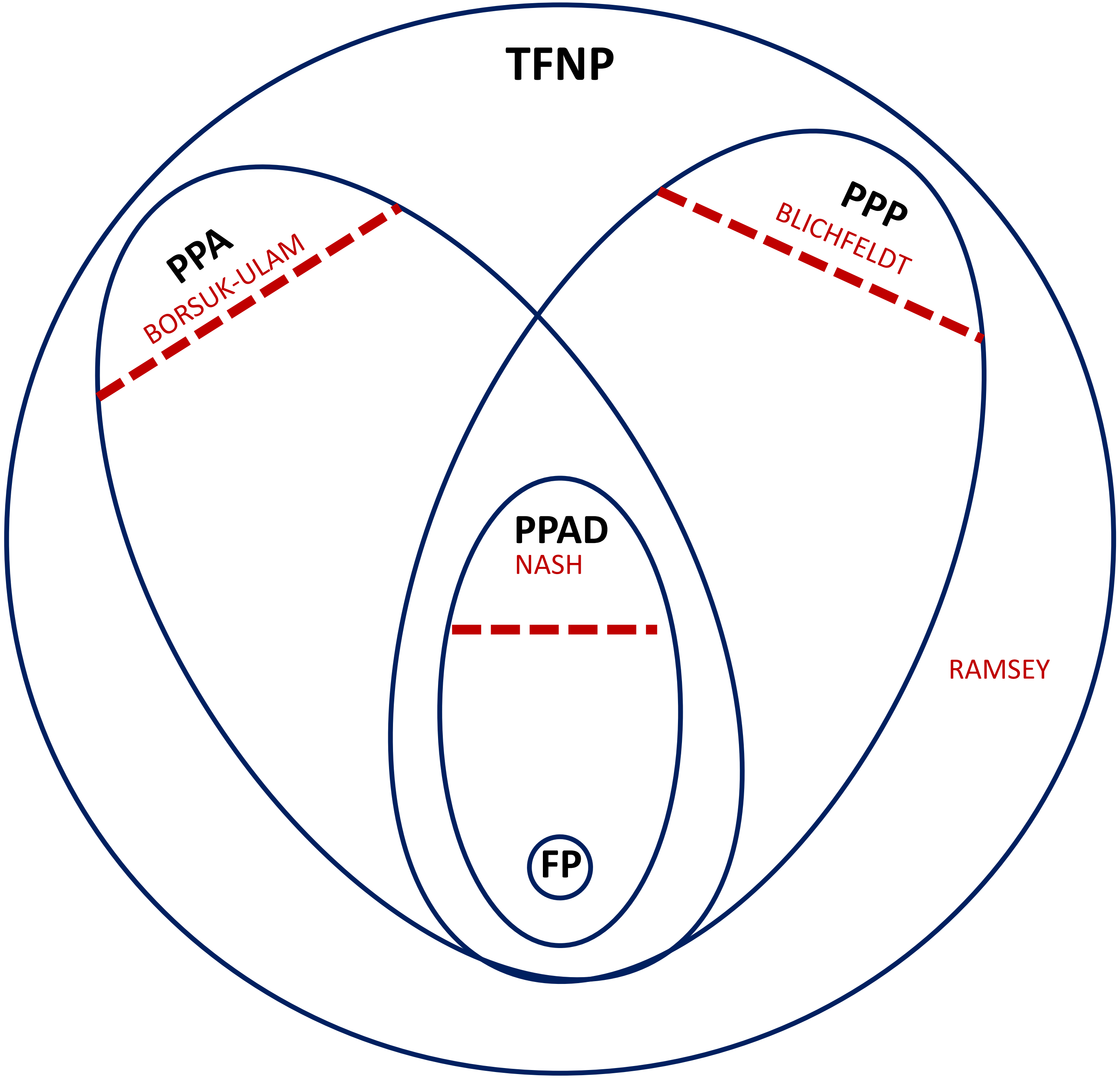
[Daskalakis-Goldberg-Papadimitriou 05],
[Chen-Deng 05]


TFNP
PPAD
NASH
FP


TFNP
PPAD
NASH
FP
RAMSEY



TFNP
PPA
BORSUK-ULAM
PPAD
NASH
FP
RAMSEY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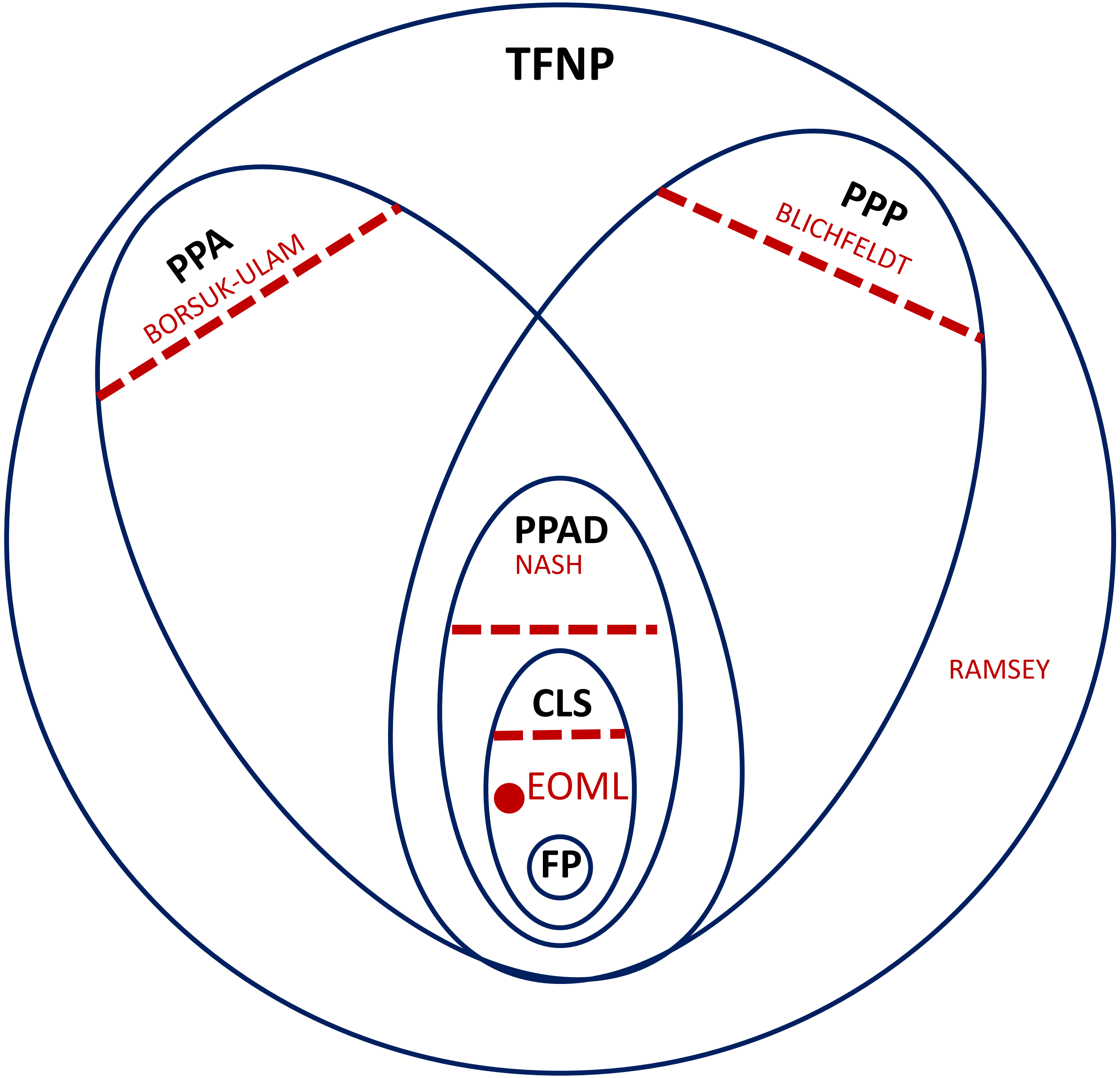
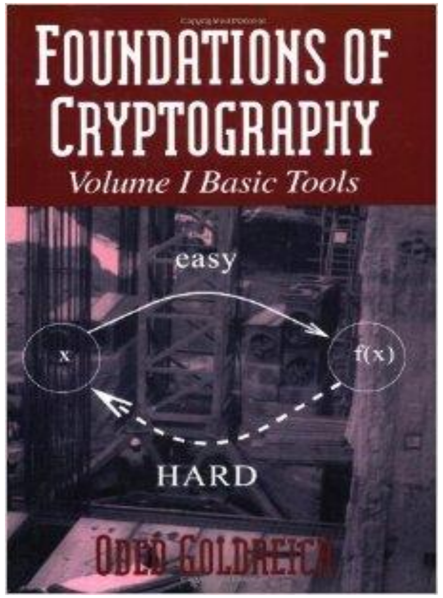
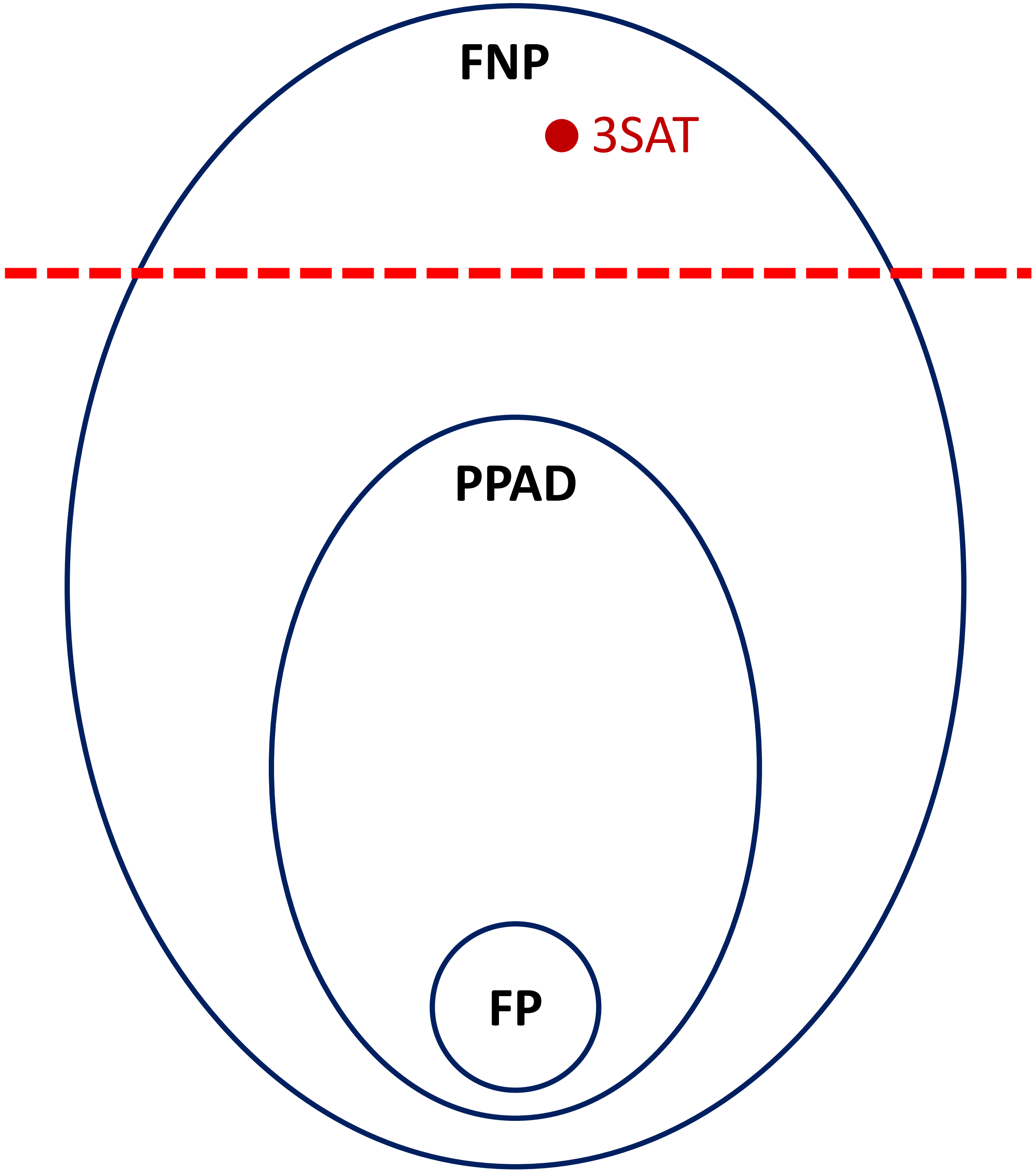
PPA
BORSUK-ULAM
PPP
BLICHFELDT
PPAD
NASH
FP
RAMSEY

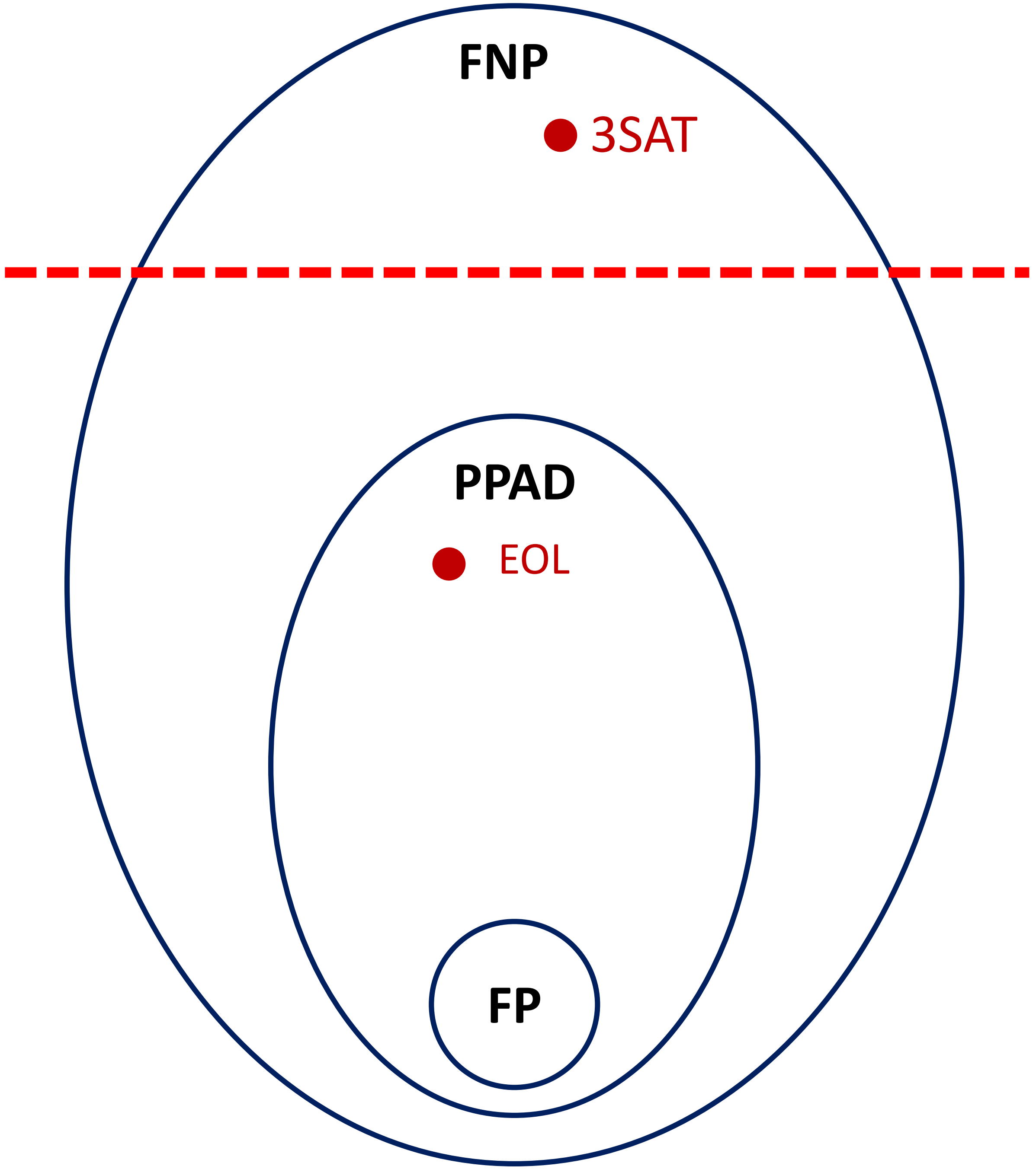

TFNP
PPA
BORSUK-ULAM
PPP
BLICHFELDT
PPAD
NASH
CLS
EOML
FP
RAMSEY

The Story Line



Games → Complexity → **Crypto**


FNP
3SAT
PPAD
FP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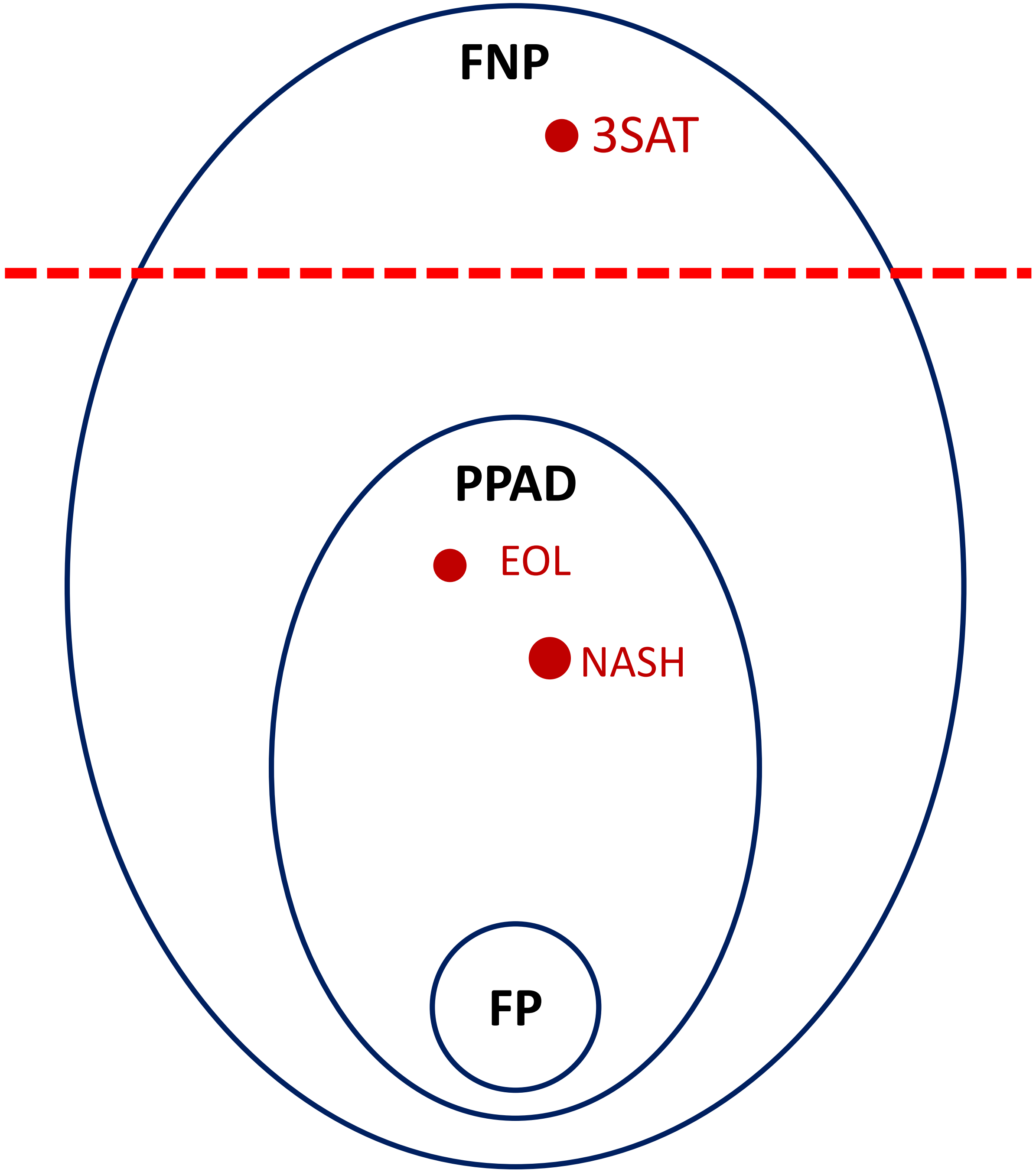
PPAD
EOL
FP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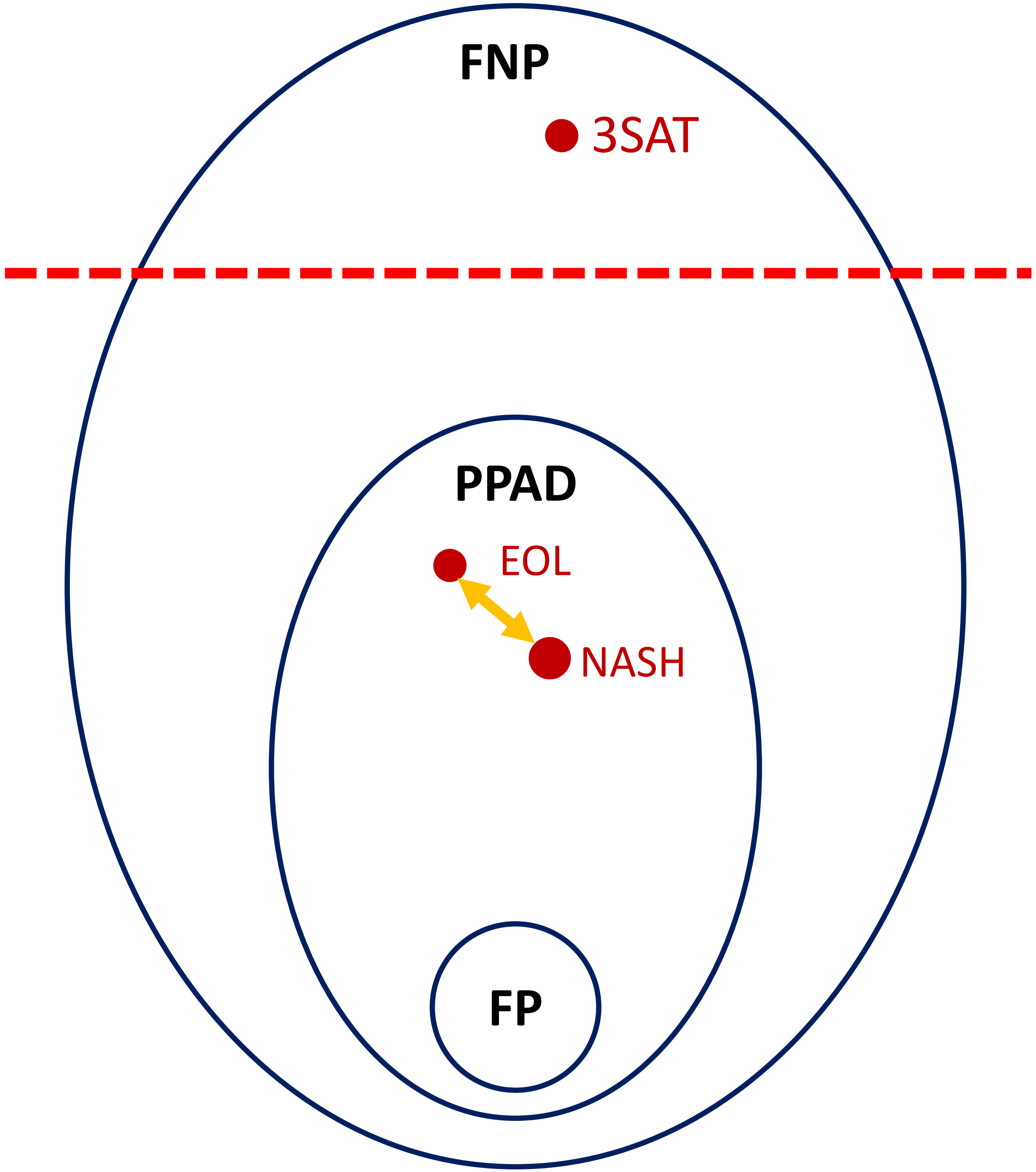
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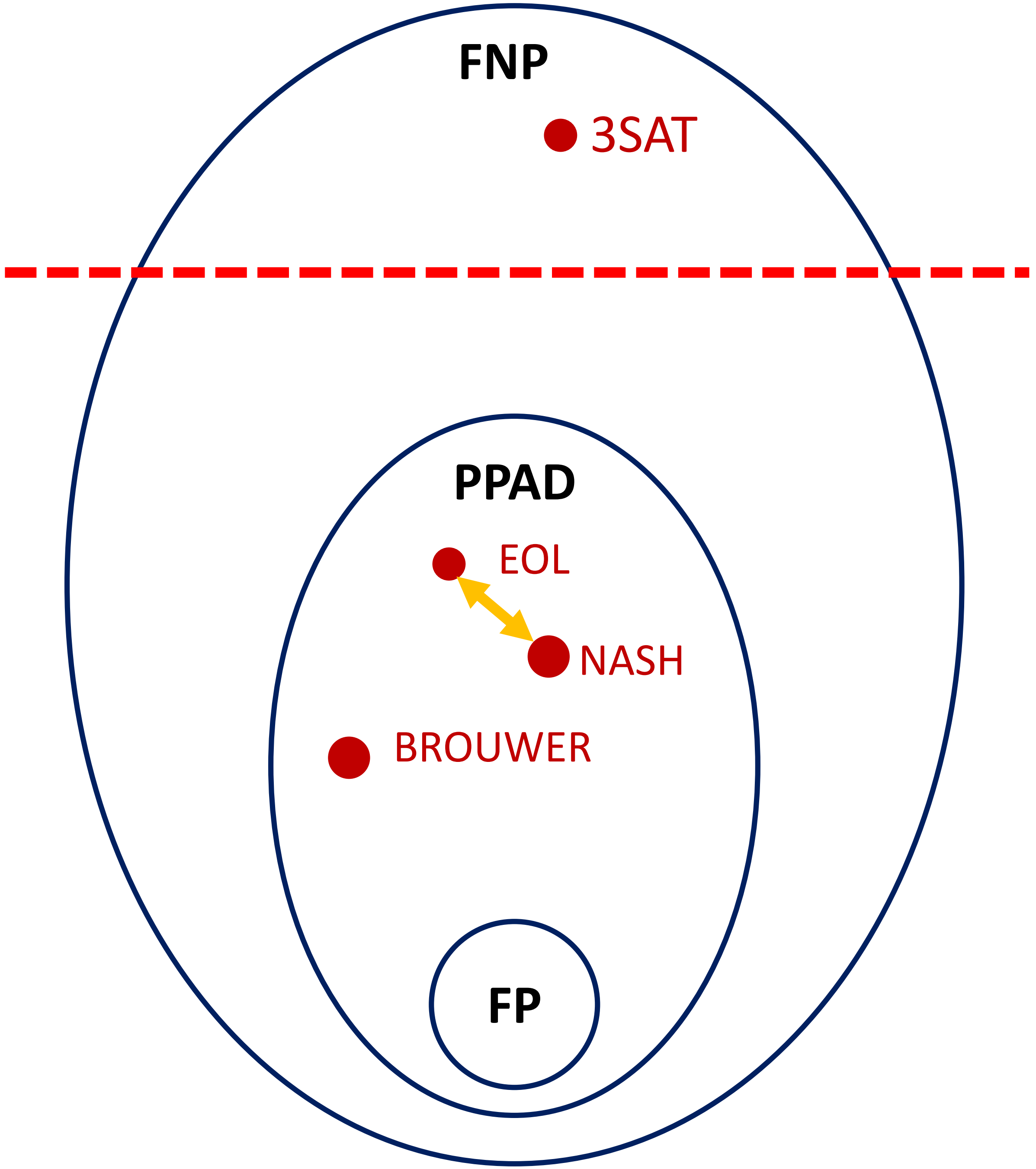
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BROUWER
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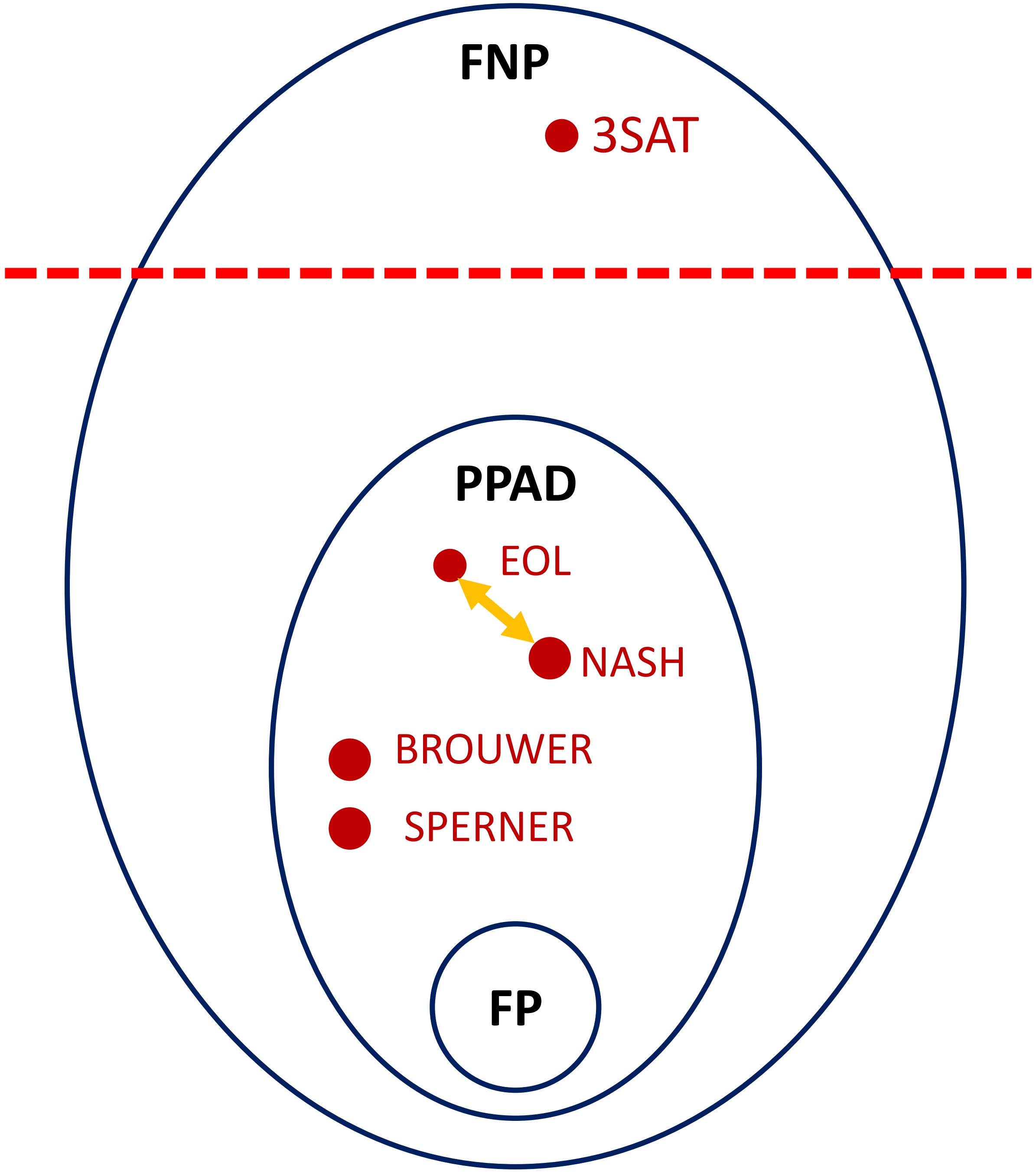
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PPAD
EOL
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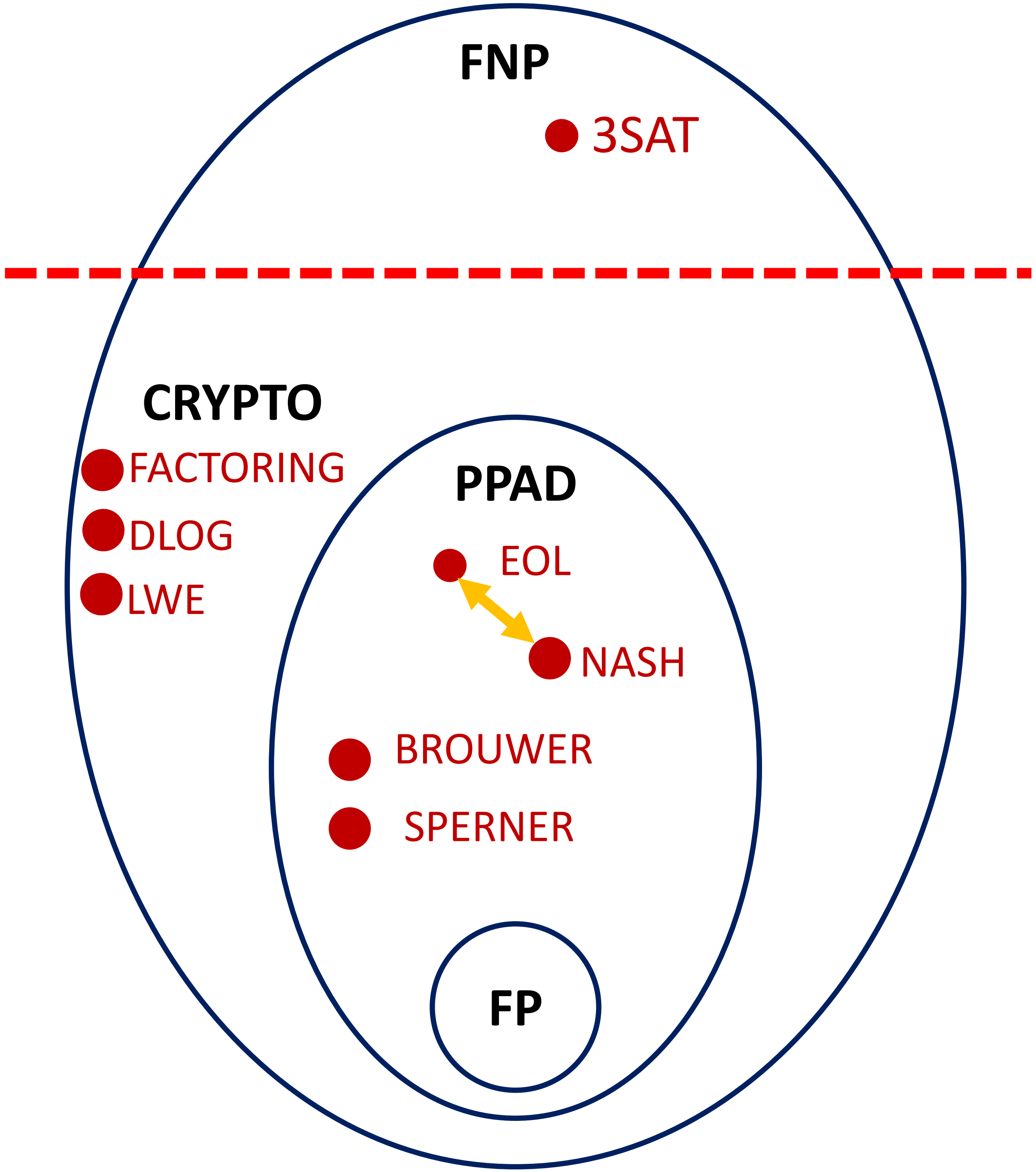
BROUWER
SPERNER
FP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DLOG
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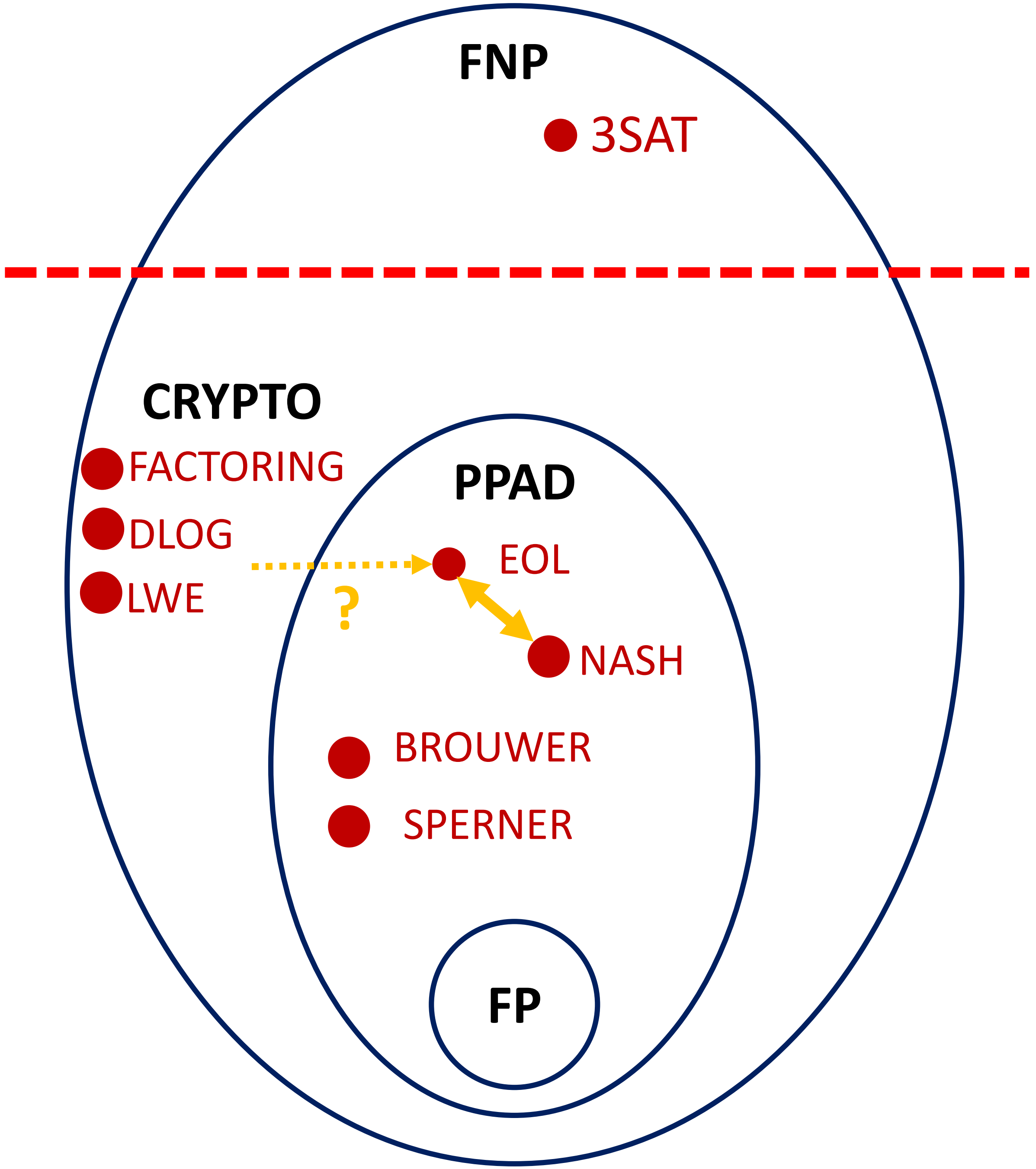
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FACTORING
DLOG
LWE
PPAD
EOL
?
NASH
BROUWER
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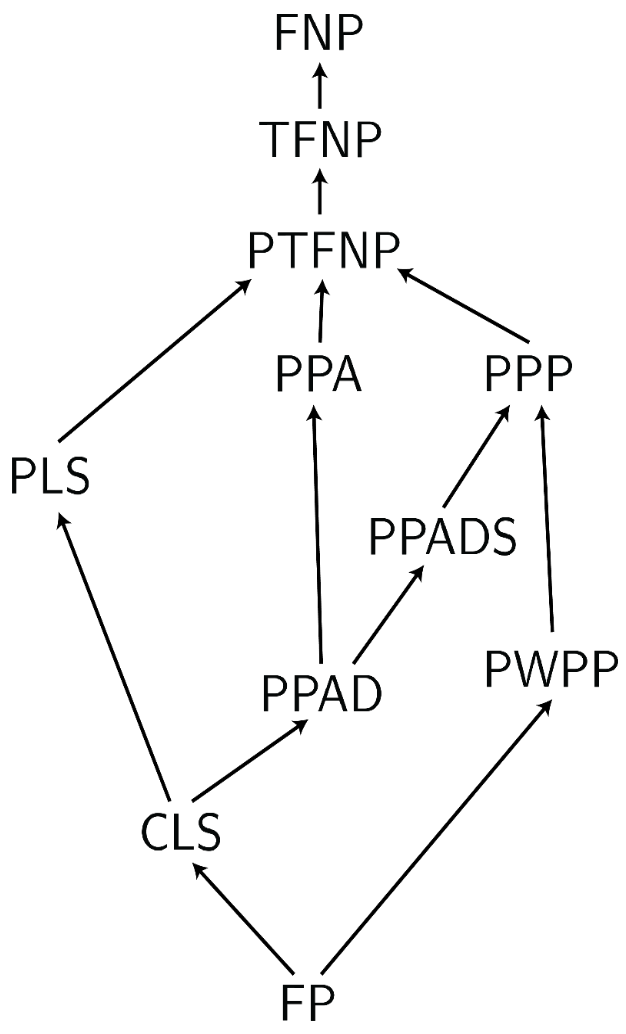
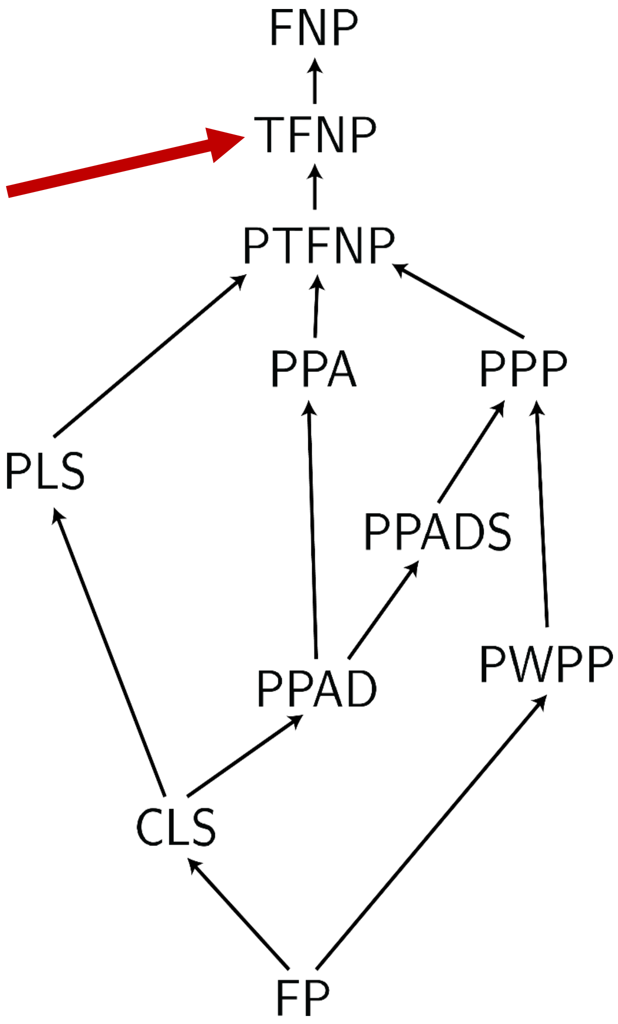
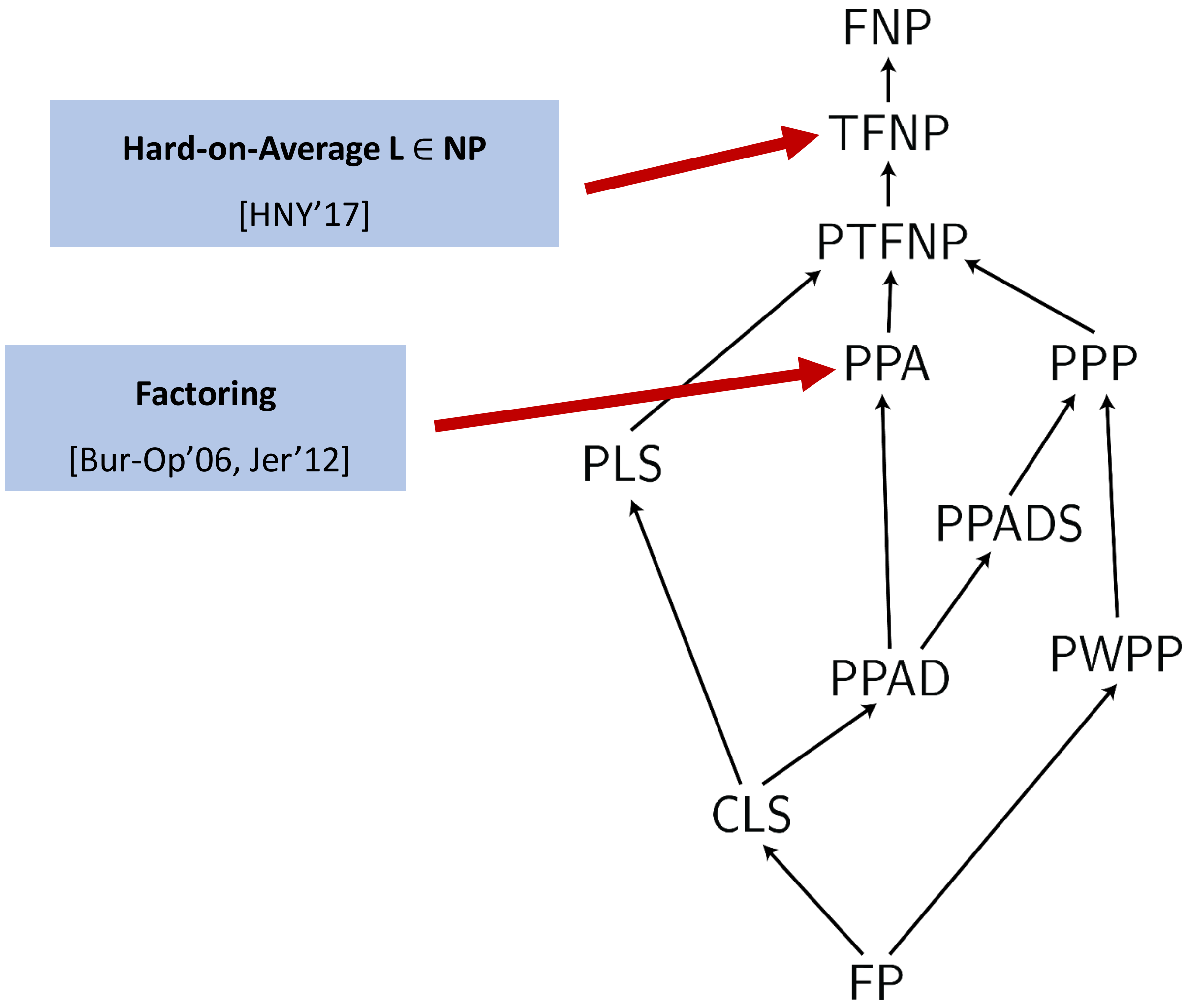
FP

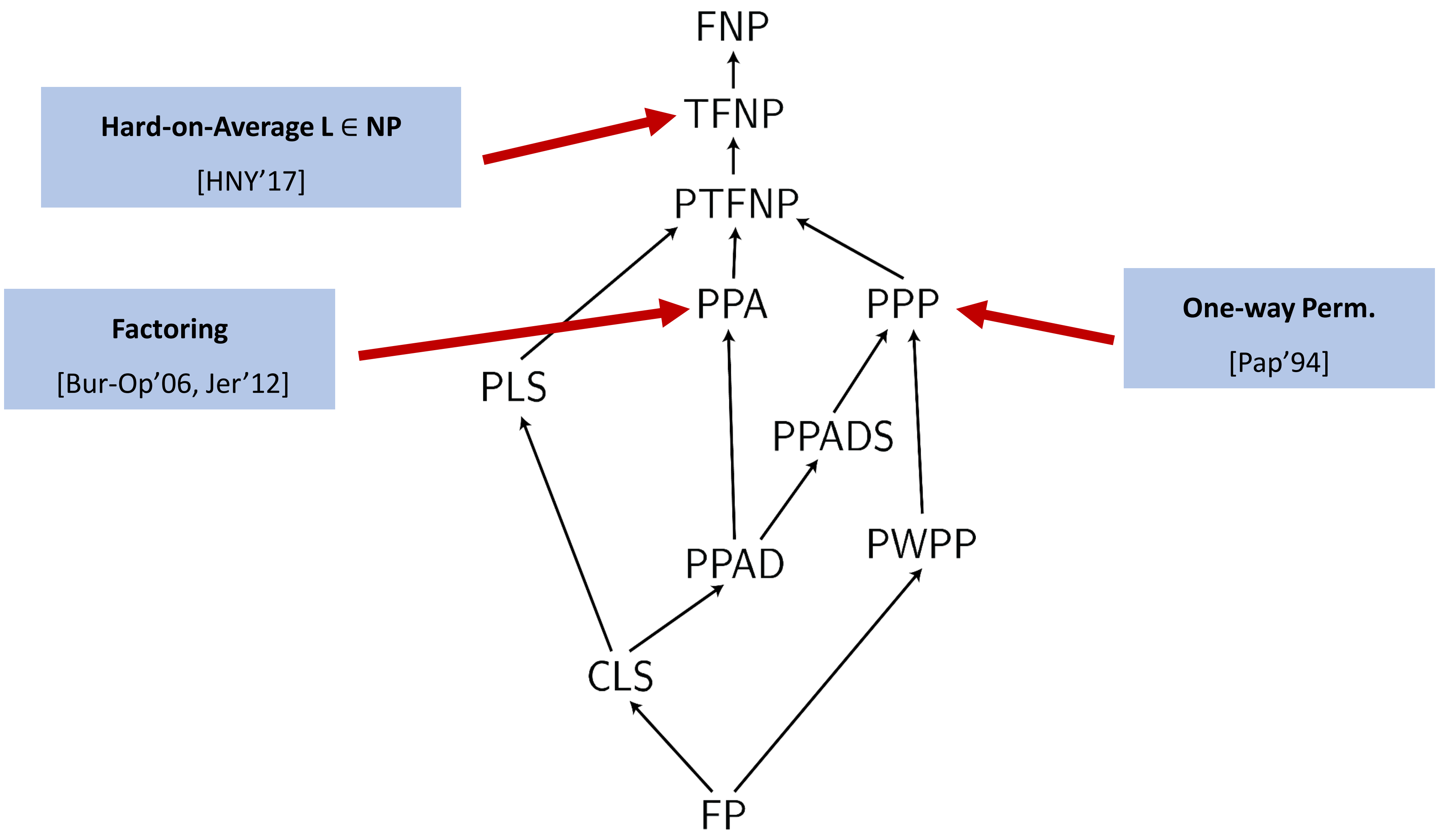

FNP
TFNP
PTFNP
PPA
PPP
PLS
PPADS
PPAD
PWPP
CLS
FP

Hard-on-Average L ∈ NP

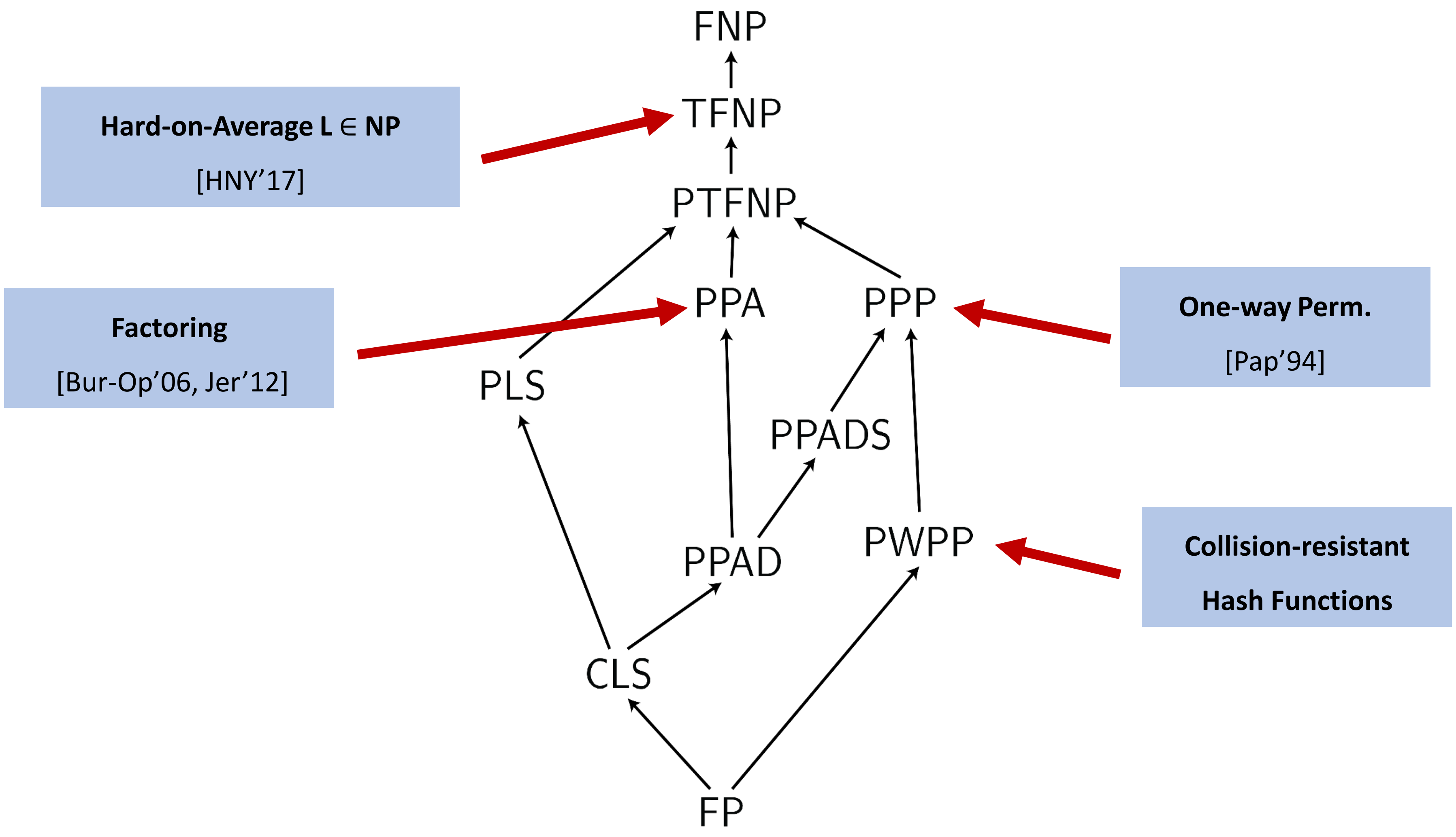
[HNY'17]




Hard-on-Average L ∈ NP
[HNY'17]
Factoring
[Bur-Op'06, Jer'12]
FNP
TFNP
PTFNP
PPA
PPP
PLS
PPADS
PPAD
PWPP
CLS
FP



FNP
TFNP
PTFNP
PPA
PPP
PLS
PPADS
PPAD
PWPP
CLS
FP
Hard-on-Average L ∈ NP
[HNY'17]
Factoring
[Bur-Op'06, Jer'12]
One-way Perm.
[Pap'94]



FNP
TFNP
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Hard-on-Average L ∈ NP
[HNY'17]
Factoring
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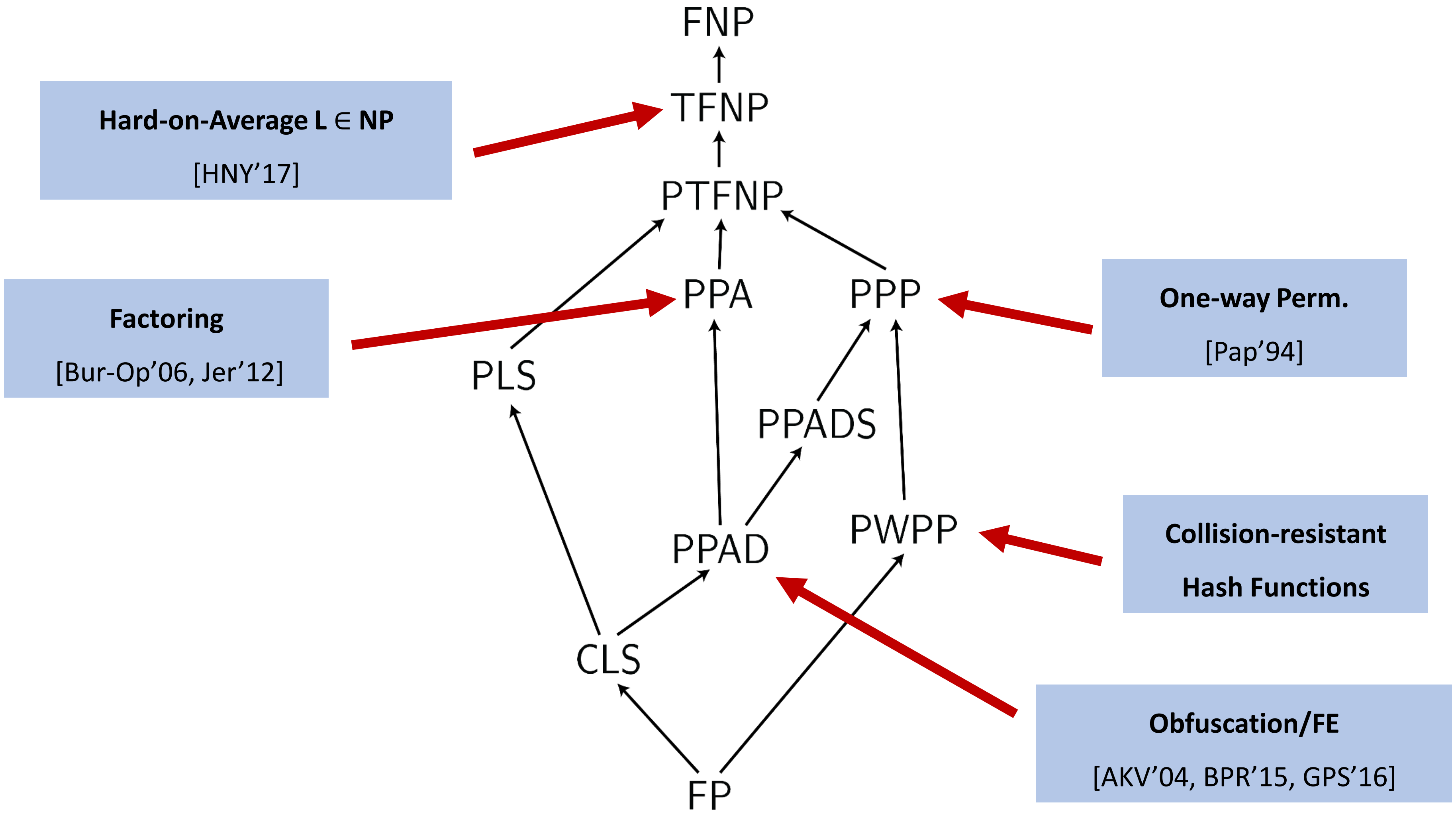
One-way Perm.
[Pap'94]
Collision-resistant
Hash Functions


FNP
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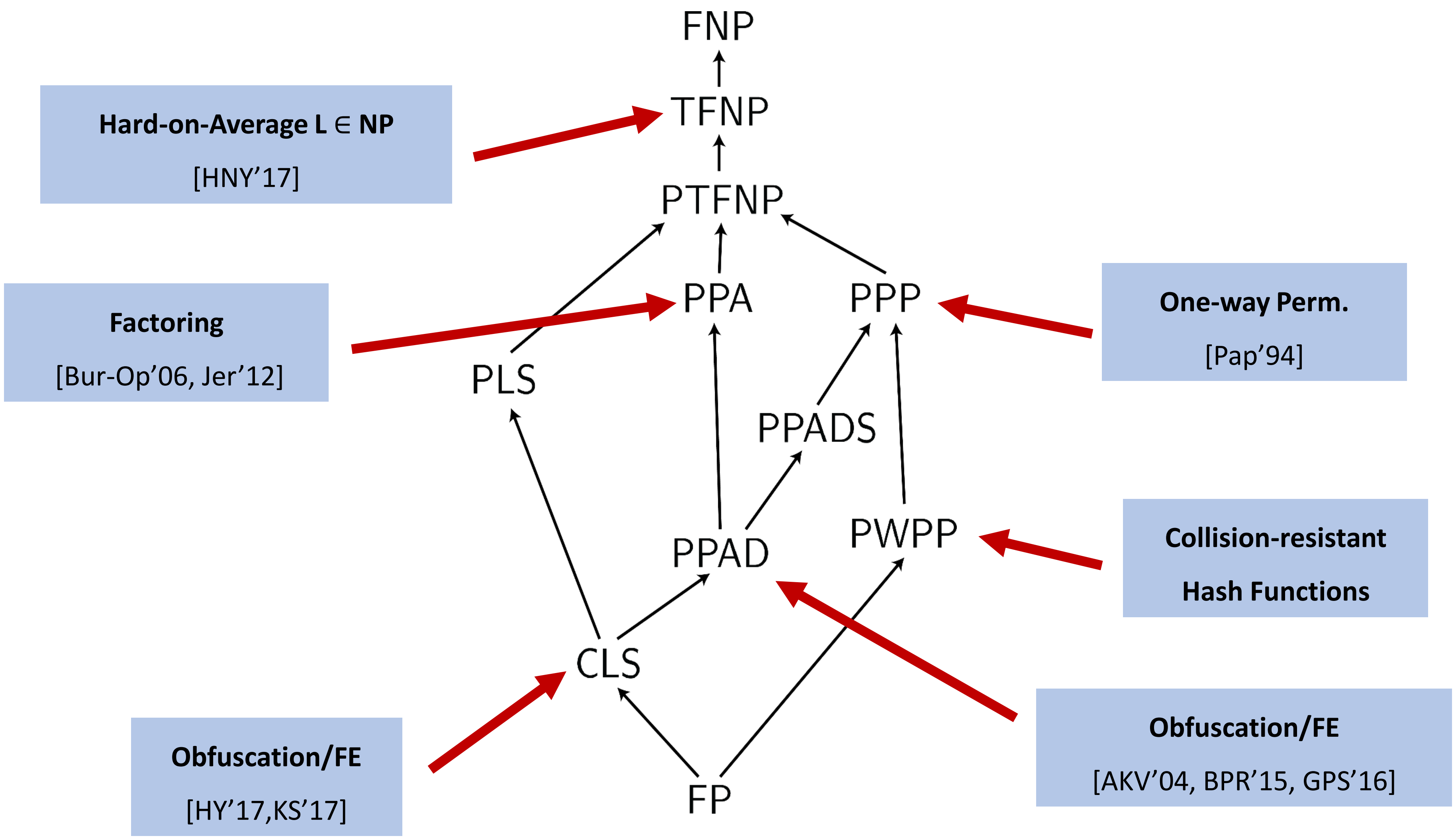
[AKV'04, BPR'15, GPS'16]



FNP
TFNP
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Hard-on-Average L ∈ NP
[HNY'17]
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[HY'17,KS'17]
Obfuscation/FE
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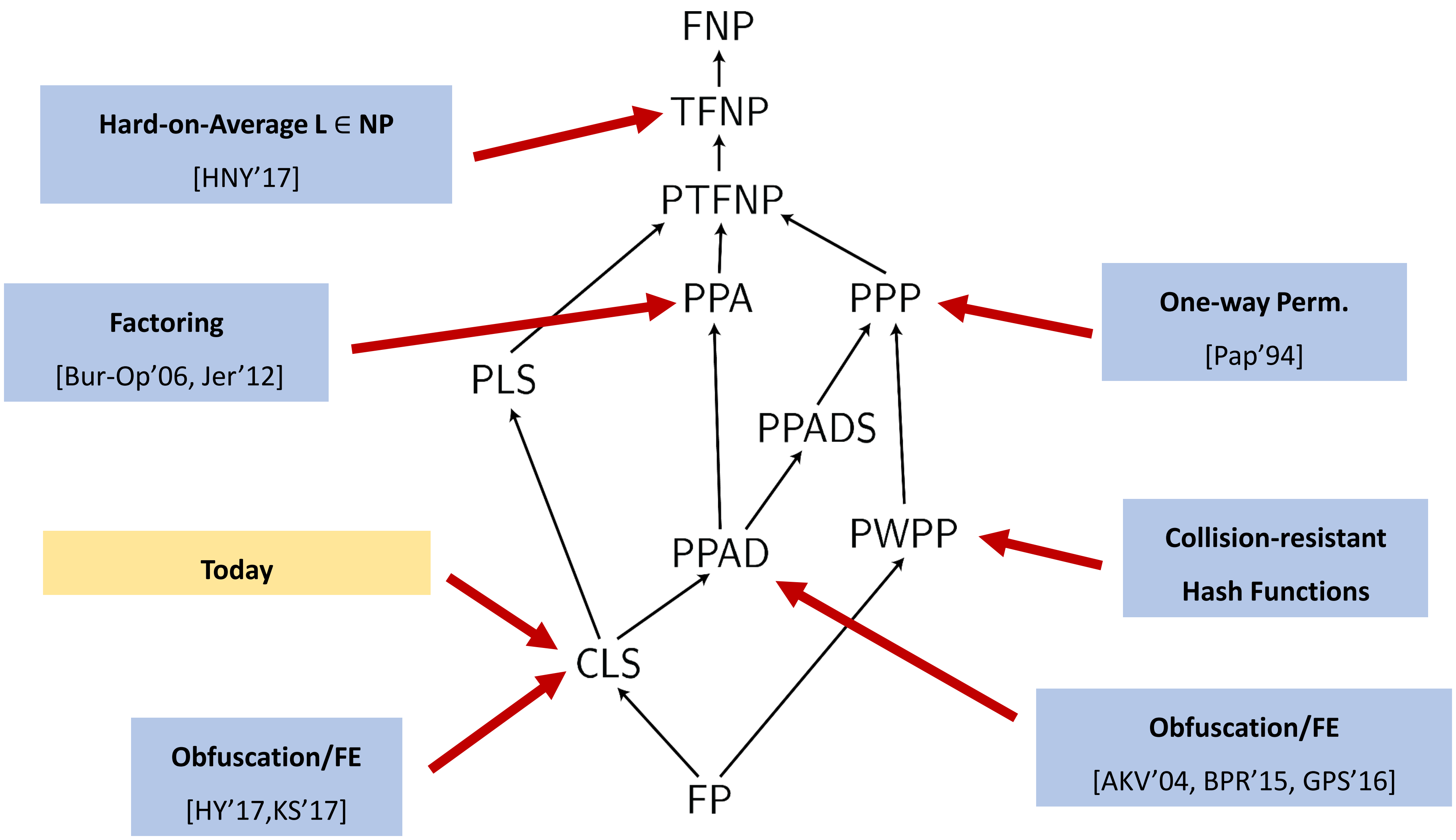
FNP
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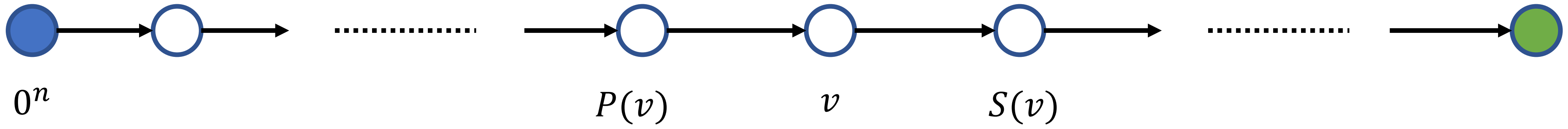
EOL Hardness from Obfuscation

[Bitansky-Paneth Rosen'15]



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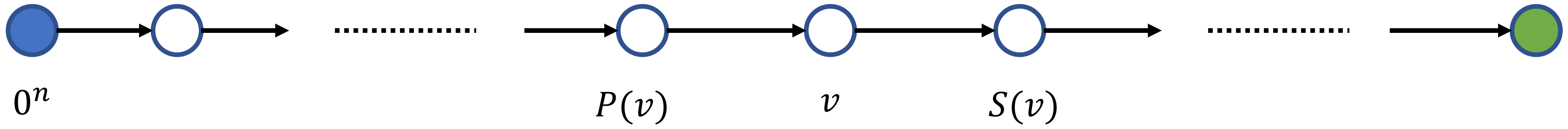
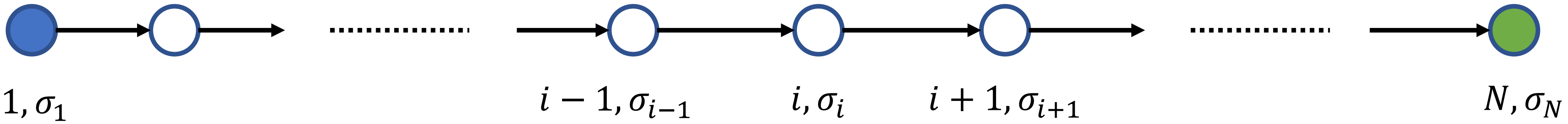


PPAD/CLS hardness can be based on
indistinguishability obfuscation (iO)

Sink of Verifiable Line (SVL)

[Abbott-Kane-Valiant’04,Bitansky-Paneth Rosen’15]



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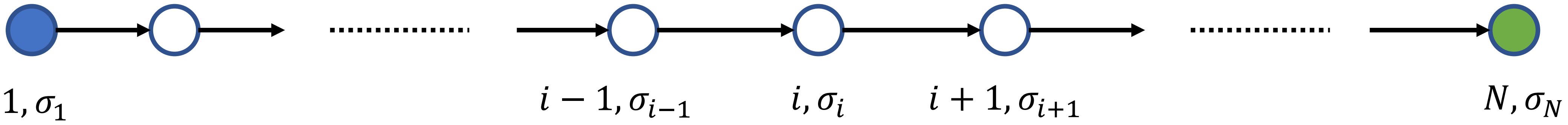

If the path is verifiable, then **Predecessor** is for free.

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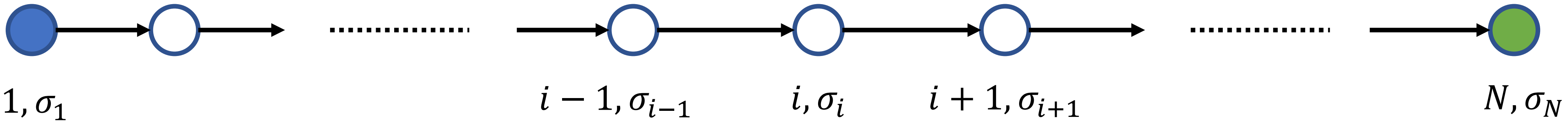

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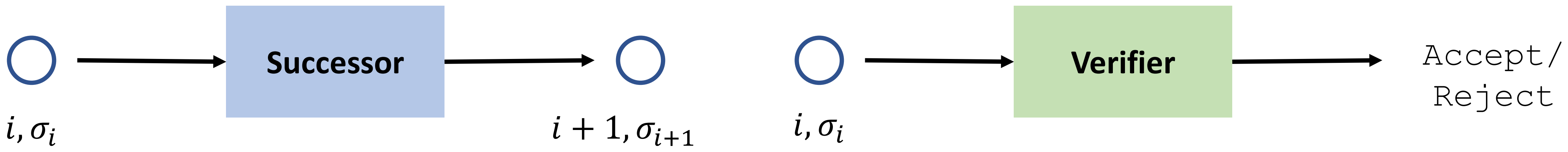

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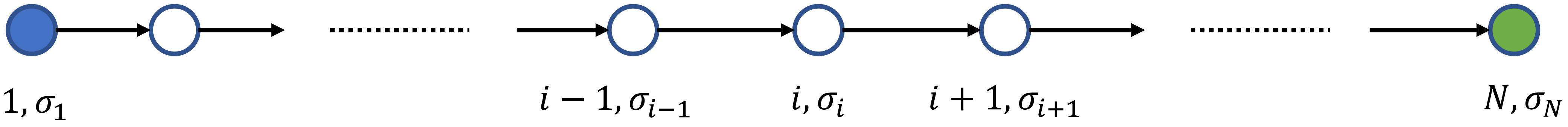

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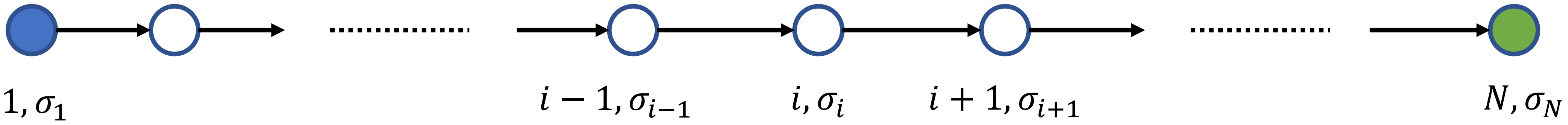
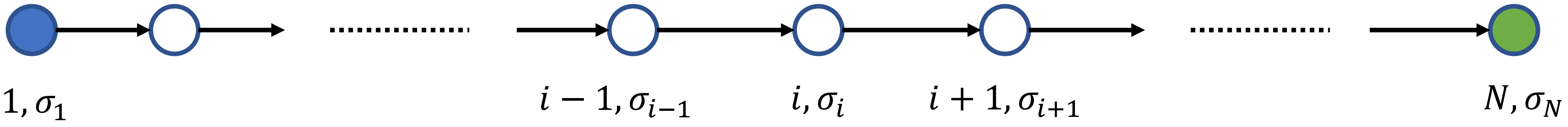


If the path is verifiable, then **Predecessor** is for free.

Based on reversible computation [Bennett’84]

Sink of Verifiable Line (SVL)

[Abbott-Kane-Valiant’04,Bitansky-Paneth Rosen’15]



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[AKV’04,BPR’15]

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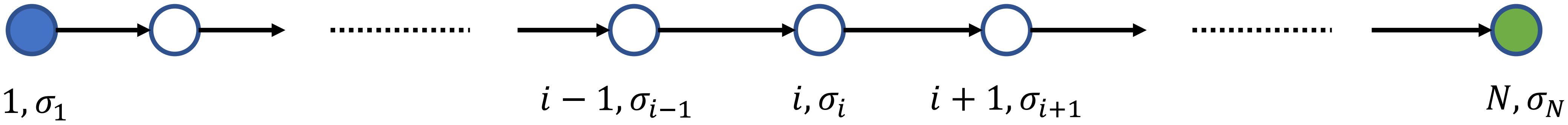

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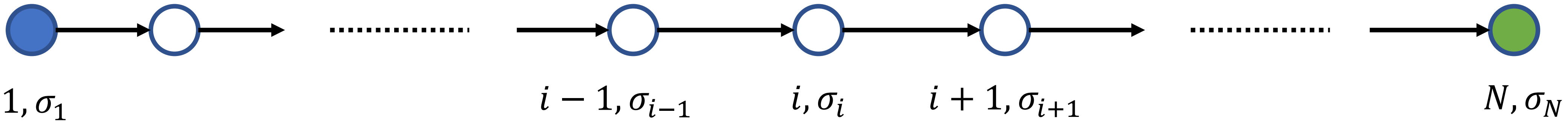
Based on reversible computation [Bennett’84]

iO hard → SVL hard → EOL hard

[BPR’15,GPS’16] [AKV’04,BPR’15]

Sink of Verifiable Line (SVL)

[Abbott-Kane-Valiant’04,Bitansky-Paneth Rosen’15]



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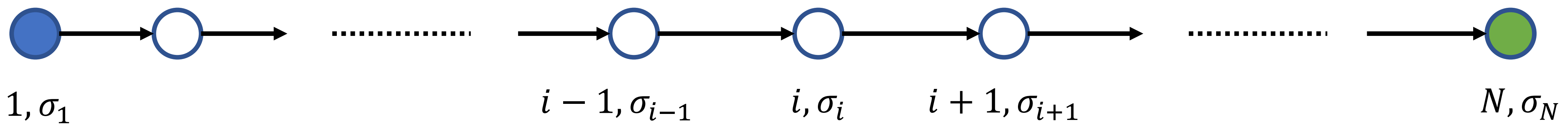

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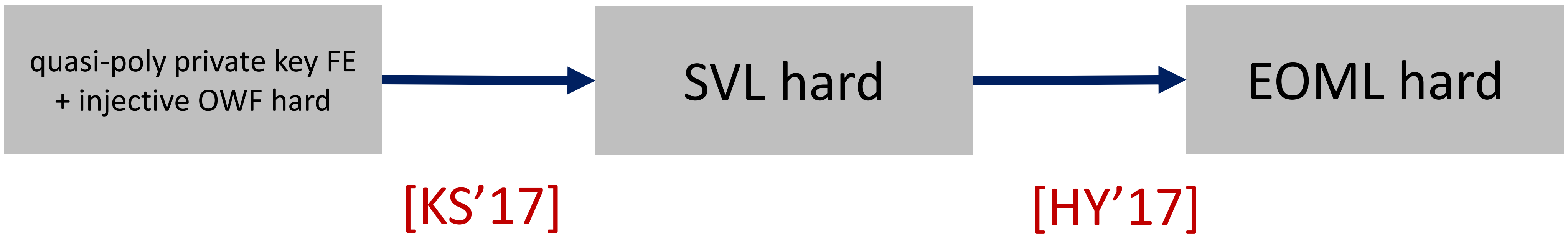
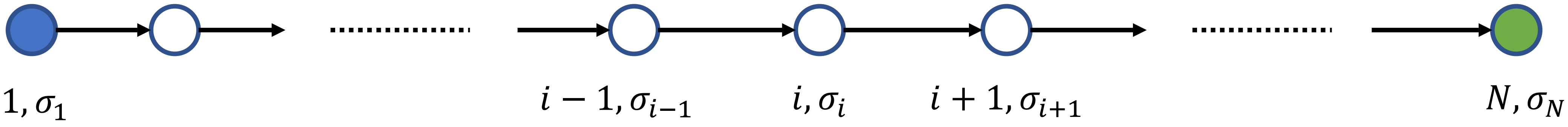


Sink of Verifiable Line (SVL)

[Abbott-Kane-Valiant'04,Bitansky-Paneth Rosen'15]



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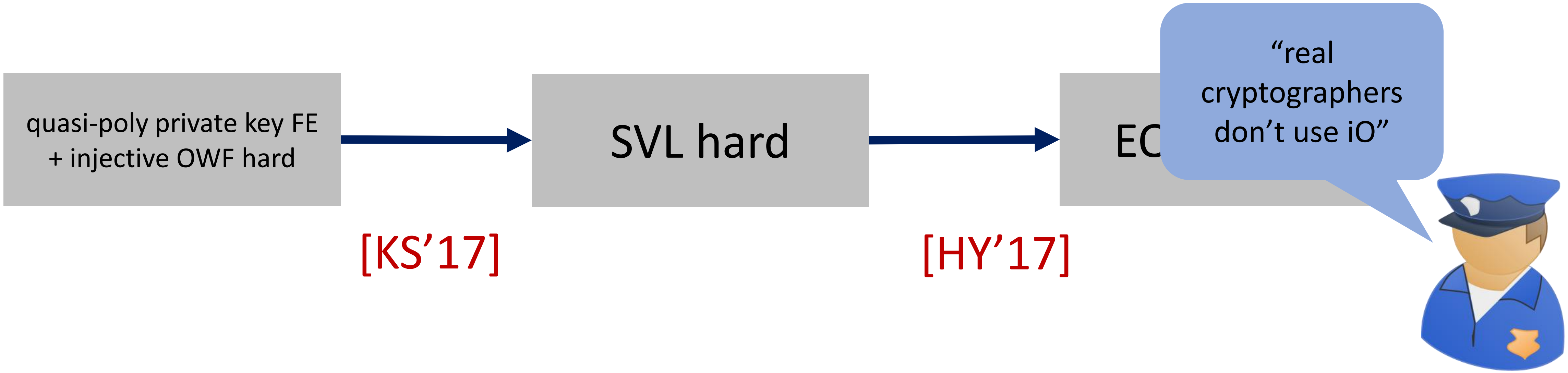


Our Result

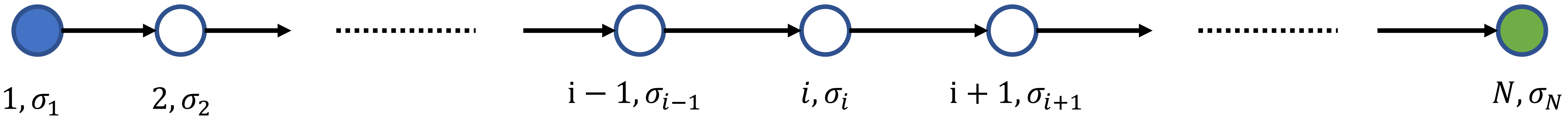
Our Result

CLS is as hard as breaking soundness of **Fiat-Shamir** when applied to the **sumcheck** protocol

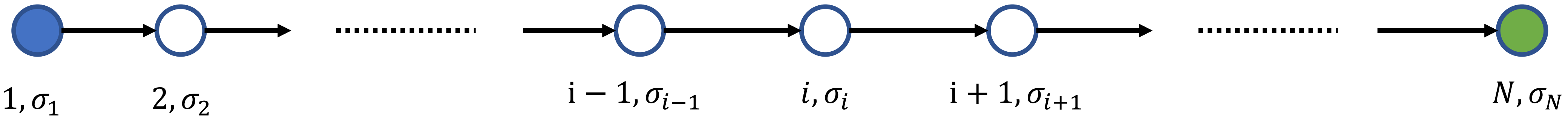
Our Construction

SVL Is No Easier Than Breaking Fiat-Shamir

Basic Idea

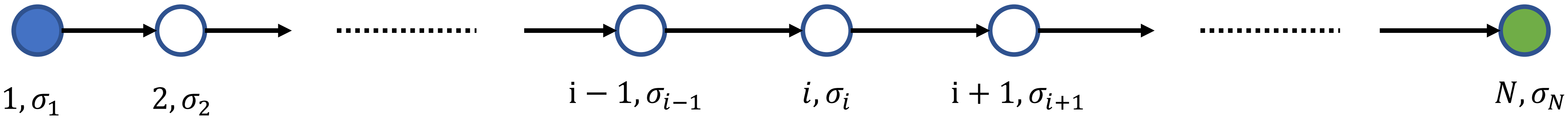

$1, \sigma_1$
$2, \sigma_2$
$\mathrm{i}-1, \sigma_{i-1}$
i, σ_i
$\mathrm{i}+1, \sigma_{i+1}$
N, σ_N

Basic Idea



Reduce to SVL from #SAT

Basic Idea



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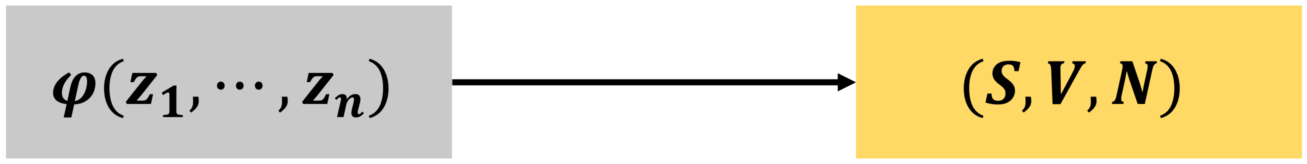
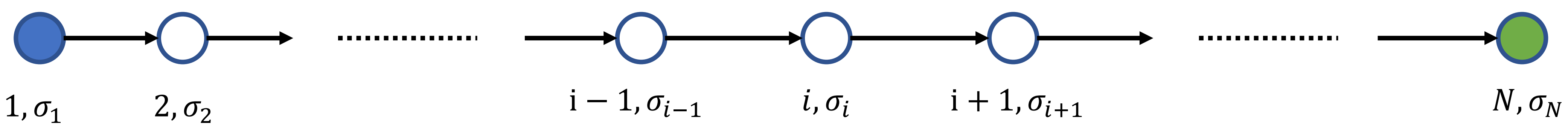


Basic Idea



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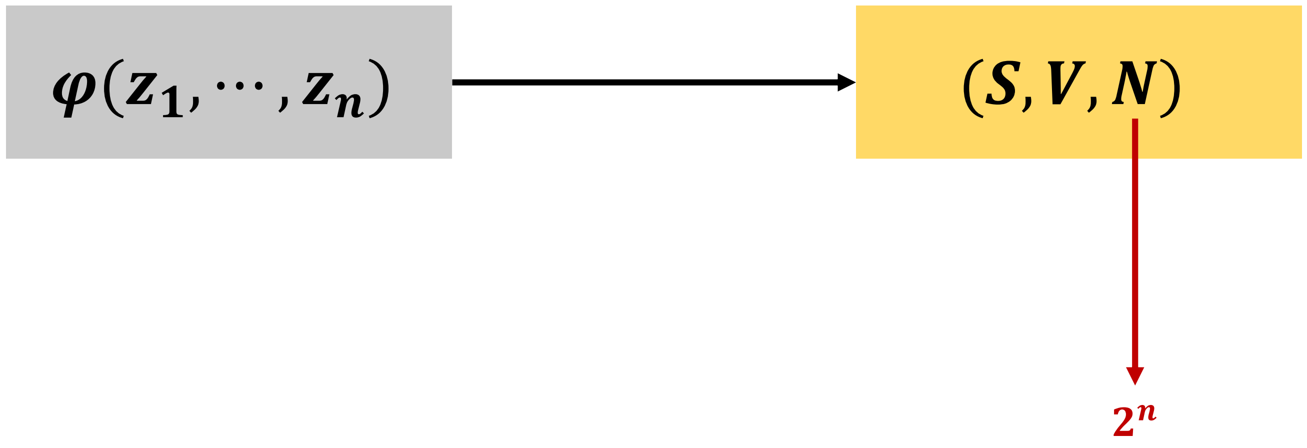
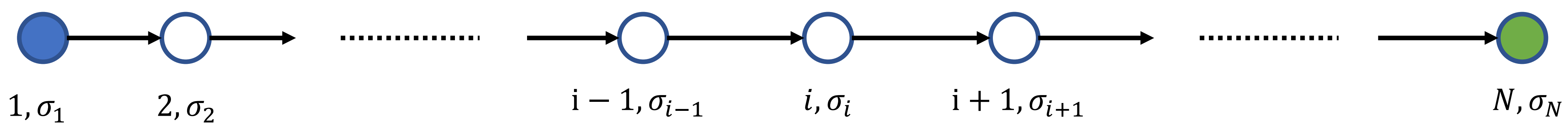
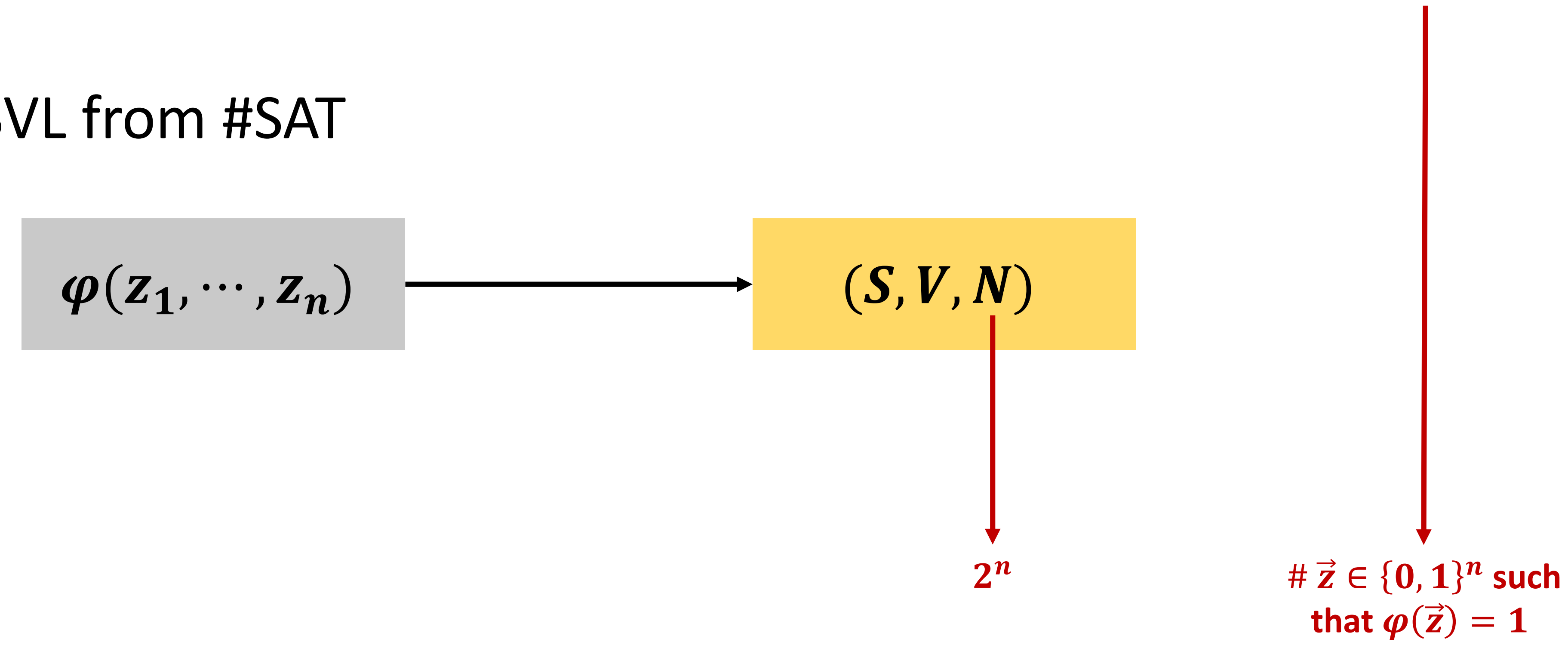


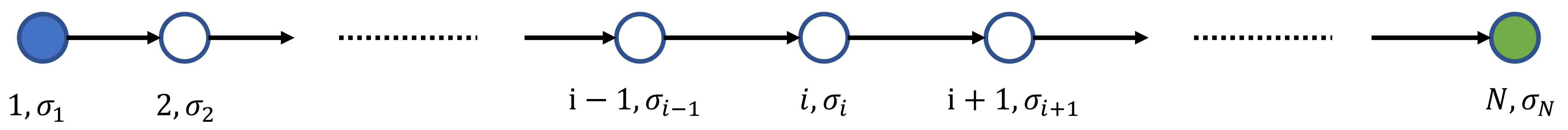
Basic Idea



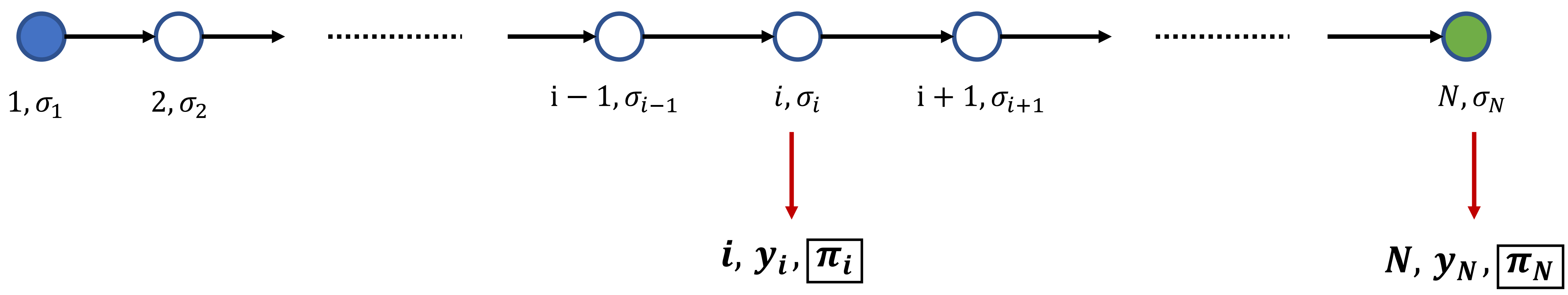
Reduce to SVL from #SAT



Basic Idea

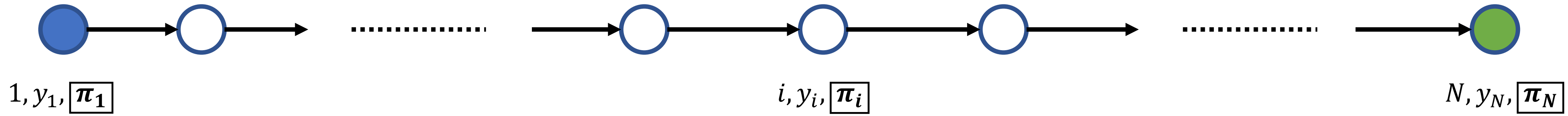

$1, \sigma_1$
$2, \sigma_2$
$\mathrm{i}-1, \sigma_{i-1}$
i, σ_i
$\mathrm{i}+1, \sigma_{i+1}$
N, σ_N

Basic Idea

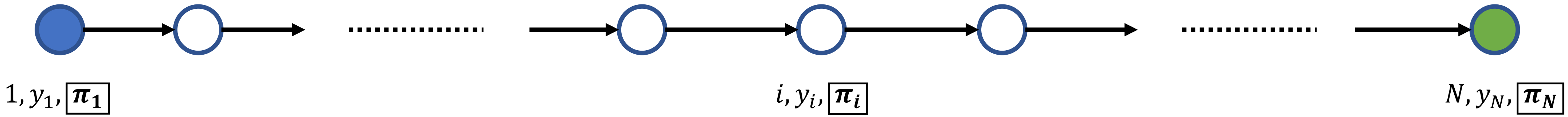


$1, \sigma_1$
$2, \sigma_2$
$\mathrm{i} - 1, \sigma_{i-1}$
i, σ_i
$\mathrm{i} + 1, \sigma_{i+1}$
N, σ_N
$\boldsymbol{i}, \boldsymbol{y}_{\boldsymbol{i}}, \boxed{\boldsymbol{\pi}_{\boldsymbol{i}}}$
$\boldsymbol{N}, \boldsymbol{y}_{\boldsymbol{N}}, \boxed{\boldsymbol{\pi}_{\boldsymbol{N}}}$

Basic Idea

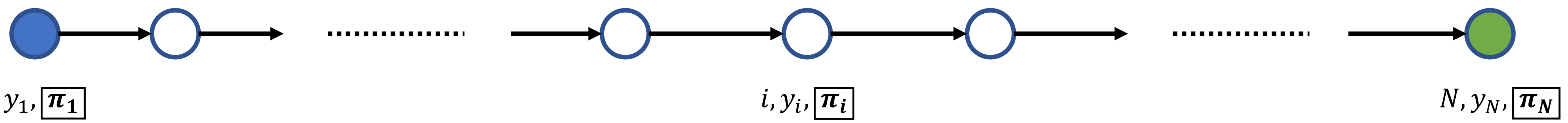

$1, y_1, \boxed{\boldsymbol{\pi_1}}$
$i, y_i, \boxed{\boldsymbol{\pi_i}}$
$N, y_N, \boxed{\boldsymbol{\pi_N}}$

Basic Idea



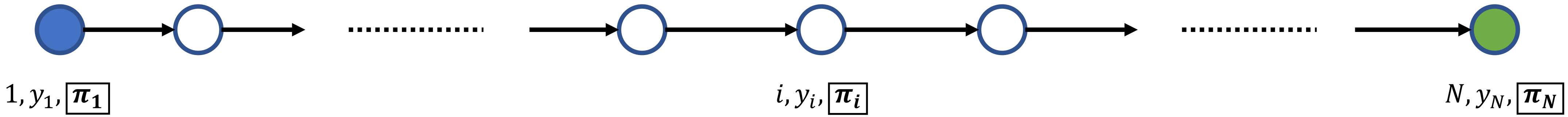
$\boldsymbol{V}\left(\boldsymbol{i},\, \boldsymbol{y_i}, \boxed{\boldsymbol{\pi_i}}\right)$ = ACCEPT $\iff$ y_i is the # of $\vec{z} \leq i$ such that $\varphi(\vec{z}) = 1$

Basic Idea

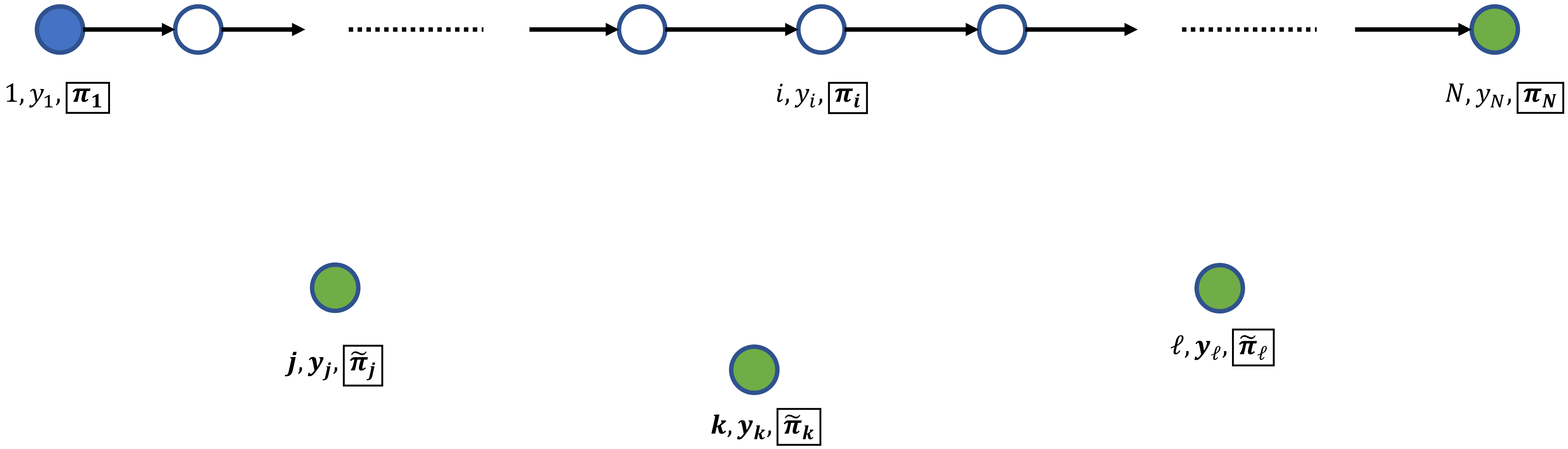


$$\boldsymbol{S\left(i,\ y_i, \boxed{\pi_i}\right) = i + 1,\ y_{i+1}, \boxed{\pi_{i+1}}}$$

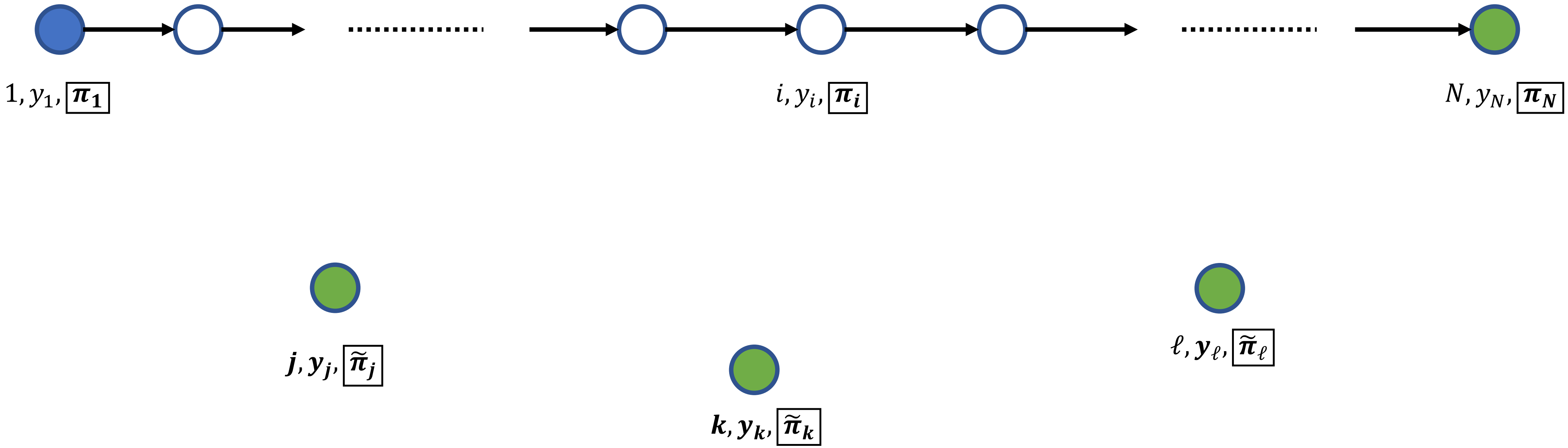
Relaxed SVL (rSVL)



Relaxed SVL (rSVL)



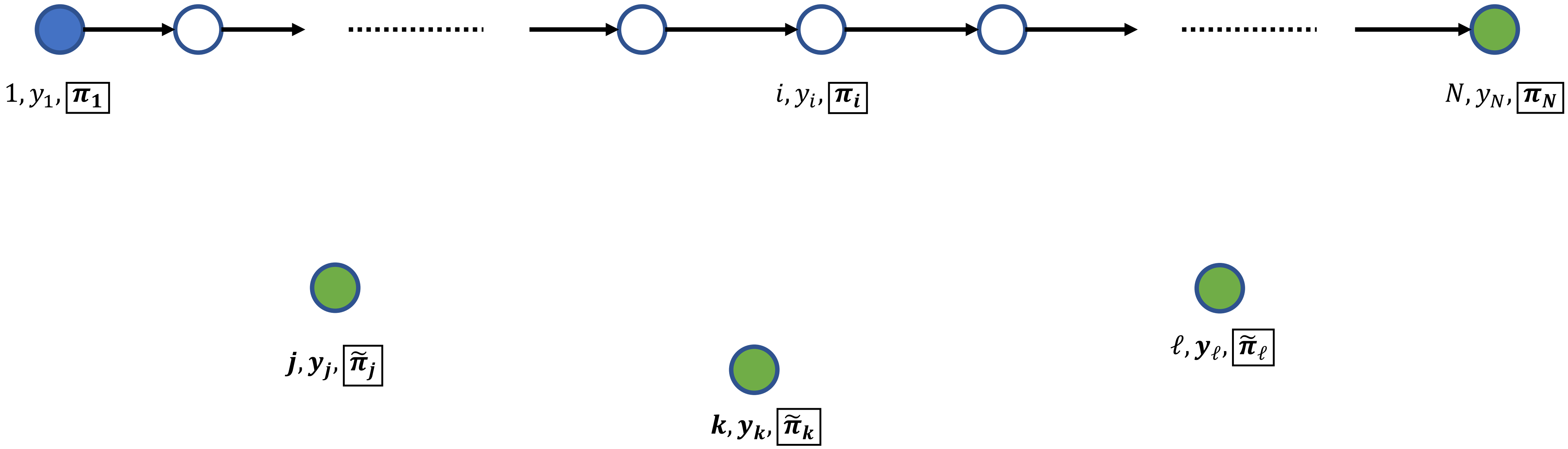
Relaxed SVL (rSVL)



Should be hard to find ``off-path'' $\boldsymbol{j}, \boldsymbol{y}_{\boldsymbol{j}}, \boxed{\widetilde{\boldsymbol{\pi}}_{\boldsymbol{j}}}$ such that

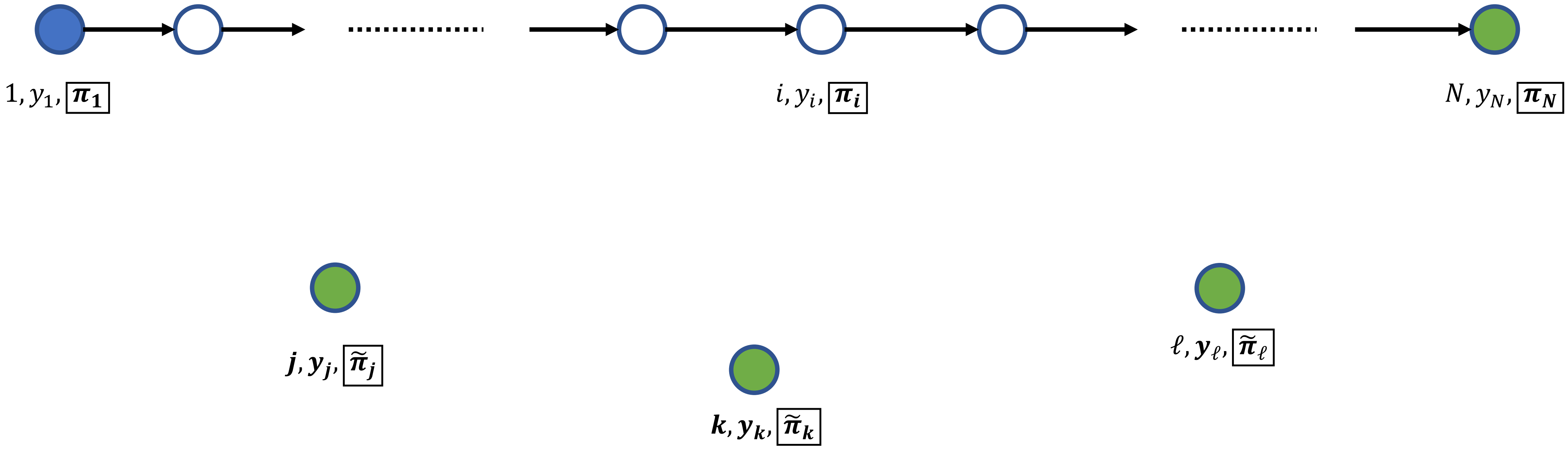
$$\boldsymbol{V}\left(\boldsymbol{j}, \boldsymbol{y}_{\boldsymbol{j}}, \boxed{\widetilde{\boldsymbol{\pi}}_{\boldsymbol{j}}}\right) = \texttt{ACCEPT}$$

Relaxed SVL (rSVL)



Solving an instance of rSVL

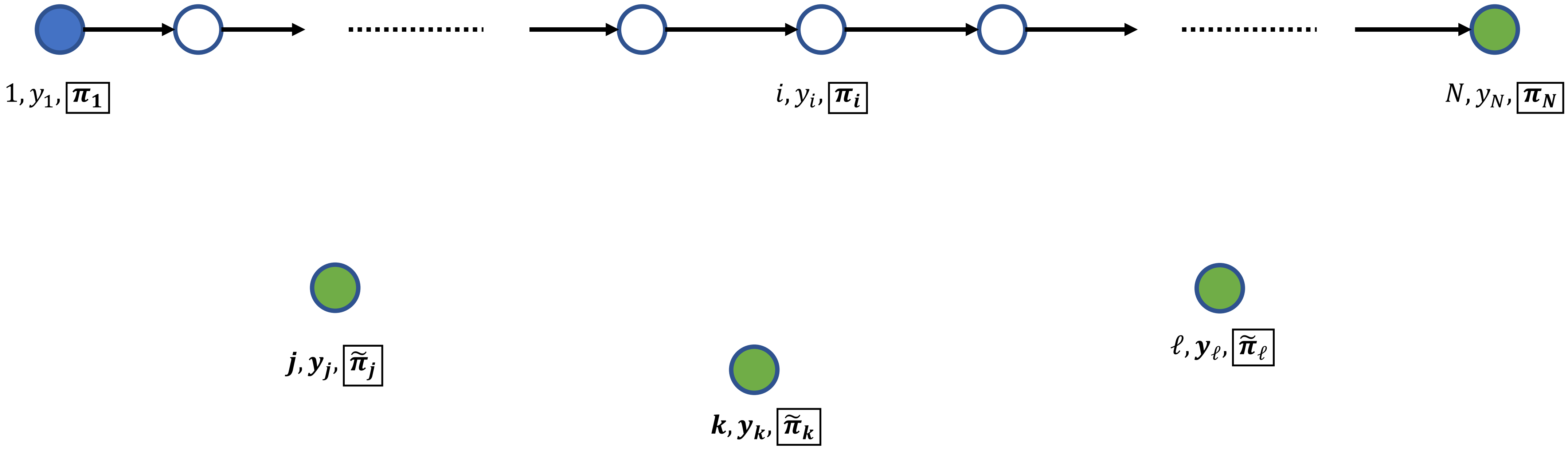
Relaxed SVL (rSVL)



Solving an instance of rSVL

solve #SAT instance φ

Relaxed SVL (rSVL)

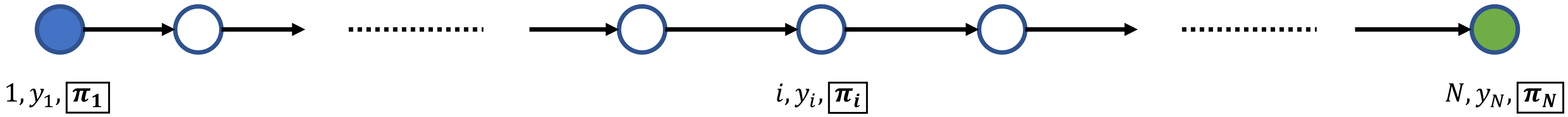


Solving an instance of rSVL

solve #SAT instance φ

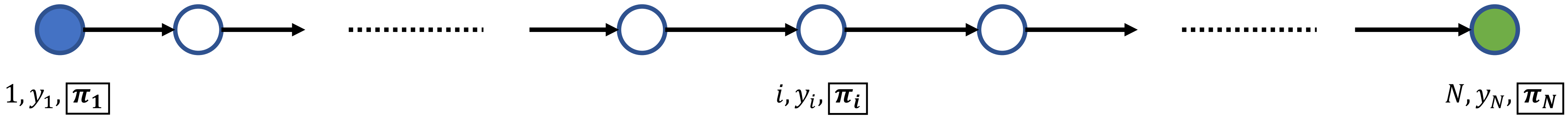
break (computational) soundness of $\boxed{\boldsymbol{\pi}}$

Challenges



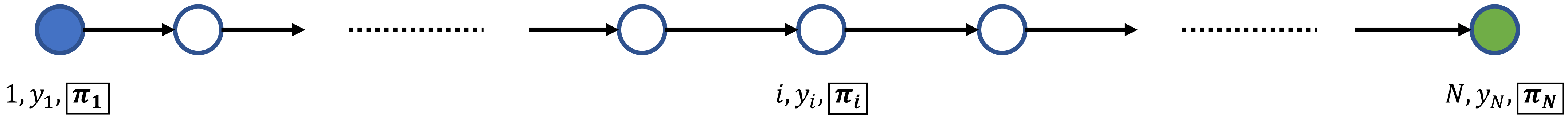
$1, y_1, \boxed{\boldsymbol{\pi_1}}$
$i, y_i, \boxed{\boldsymbol{\pi_i}}$
$N, y_N, \boxed{\boldsymbol{\pi_N}}$

Challenges



Several Challenges:

Challenges



Several Challenges:

Proof size has to be polynomial

Sumcheck Protocol [Lund-Fortnow-Karloff-Nisan'90]

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Arithmetization

Sumcheck Protocol [Lund-Fortnow-Karloff-Nisan'90]

Arithmetization

$$\boldsymbol{\varphi}(\boldsymbol{z_1}, \cdots, \boldsymbol{z_n}) \longrightarrow \boldsymbol{f}\colon \mathbb{F}^{\boldsymbol{n}} \to \mathbb{F}$$

Sumcheck Protocol [Lund-Fortnow-Karloff-Nisan'90]

Arithmetization

$$\boldsymbol{\varphi}(\boldsymbol{z}_1, \cdots, \boldsymbol{z}_n) \longrightarrow \boldsymbol{f}\colon \mathbb{F}^{\boldsymbol{n}} \to \mathbb{F}$$

$$\text{degree}(f) = d = 4$$

Sumcheck Protocol [Lund-Fortnow-Karloff-Nisan'90]

Arithmetization

$$\boldsymbol{\varphi(z_1, \cdots, z_n)} \longrightarrow \boldsymbol{f} \colon \mathbb{F}^{\boldsymbol{n}} \to \mathbb{F}$$

$\text{degree}(f) = d = 4$

#SAT $\leq$ #3SAT4

Sumcheck Protocol [Lund-Fortnow-Karloff-Nisan’90]

Arithmetization

$$\boldsymbol{\varphi(z_1, \cdots, z_n)} \longrightarrow \boldsymbol{f} \colon \mathbb{F}^{\boldsymbol{n}} \to \mathbb{F}$$

$\text{degree}(f) = d = 4$

#SAT $\leq$ #3SAT4

Number of $\vec{z} \in \{0,1\}^n$ such that $\varphi(\vec{z}) = 1$ is

$$y = \sum_{\vec{z} \in \{0,1\}} f(\vec{z})$$

Sumcheck Protocol

Sumcheck

The sum $\sum_{z\in\{0,1\}^n} f(z)$ is some value y

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Prove it!

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Prove it!

$$\tilde{g}_1(x) \coloneqq \sum_{z_2, \cdots, z_n \in \{0,1\}} f(x, z_2, \cdots, z_n)$$

Sumcheck

The sum $\sum_{z\in\{0,1\}^n} f(z)$ is some value y

Prove it!

$$\tilde{g}_1(x) \coloneqq \sum_{z_2,\cdots,z_n\in\{0,1\}} f(x, z_2, \cdots, z_n) \xrightarrow{\tilde{g}_1(x)}$$

Sumcheck

The sum $\sum_{z \in \{0,1\}^n} f(z)$ is some value y

Prove it!

$$\tilde{g}_1(x) := \sum_{z_2, \cdots, z_n \in \{0,1\}} f(x, z_2, \cdots, z_n) \xrightarrow{\tilde{g}_1(x)}$$

$$\tilde{g}_1(0) + \tilde{g}_1(1) \stackrel{?}{=} y$$

Sumcheck

The sum $\sum_{z\in\{0,1\}^n} f(z)$ is some value y

Prove it!

$$\tilde{g}_1(x) \coloneqq \sum_{z_2,\cdots,z_n\in\{0,1\}} f(x, z_2, \cdots, z_n) \xrightarrow{\tilde{g}_1(x)}$$

$\{\tilde{g}_1(0), \cdots, \tilde{g}_1(d)\}$

$$\tilde{g}_1(0) + \tilde{g}_1(1) \stackrel{?}{=} y$$

Sumcheck

The sum $\sum_{z\in\{0,1\}^n} f(z)$ is some value y

Prove it!

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$$\xrightarrow[\{\tilde{g}_1(0),\cdots,\tilde{g}_1(d)\}]{\tilde{g}_1(x)}$$

$$\tilde{g}_1(0) + \tilde{g}_1(1) \stackrel{?}{=} y$$

$$\beta_1 \leftarrow_R \mathbb{F}$$

Sumcheck

The sum $\sum_{z\in\{0,1\}^n} f(z)$ is some value y

Prove it!

$$\tilde{g}_1(x) \coloneqq \sum_{z_2,\cdots,z_n\in\{0,1\}} f(x, z_2, \cdots, z_n)$$

$\tilde{g}_1(x)$

$\{\tilde{g}_1(0), \cdots, \tilde{g}_1(d)\}$

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β_1

Sumcheck

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$\{\tilde{g}_1(0), \cdots, \tilde{g}_1(d)\}$

$$\tilde{g}_1(0) + \tilde{g}_1(1) \stackrel{?}{=} y$$

$$\beta_1 \leftarrow_R \mathbb{F}$$

β_1

$$y_1 \coloneqq \tilde{g}_1(\beta_1)$$

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Sumcheck

The sum $\sum_{z\in\{0,1\}^n} f(z)$ is some value y

Prove it!

$$\tilde{g}_1(x) \coloneqq \sum_{z_2,\cdots,z_n\in\{0,1\}} f(x, z_2, \cdots, z_n)$$

$\tilde{g}_1(x)$

$\{\tilde{g}_1(0), \cdots, \tilde{g}_1(d)\}$

$$\tilde{g}_1(0) + \tilde{g}_1(1) \stackrel{?}{=} y$$

$$\beta_1 \leftarrow_R \mathbb{F}$$

β_1

$$y_1 \coloneqq \tilde{g}_1(\beta_1)$$

$$\tilde{g}_2(x) \coloneqq \sum_{z_3,\cdots,z_n\in\{0,1\}} f(\beta_1, x, z_3, \cdots, z_n)$$

$$y_1 \coloneqq \tilde{g}_1(\beta_1)$$

Sumcheck

The sum $\sum_{z\in\{0,1\}^n} f(z)$ is some value y

Prove it!

$$\tilde{g}_1(x) \coloneqq \sum_{z_2,\cdots,z_n\in\{0,1\}} f(x, z_2, \cdots, z_n)$$

$$\xrightarrow[\{\tilde{g}_1(0),\cdots,\tilde{g}_1(d)\}]{\tilde{g}_1(x)}$$

$$\tilde{g}_1(0) + \tilde{g}_1(1) \stackrel{?}{=} y$$

$$\beta_1 \leftarrow_R \mathbb{F}$$

$$\xleftarrow{\beta_1}$$

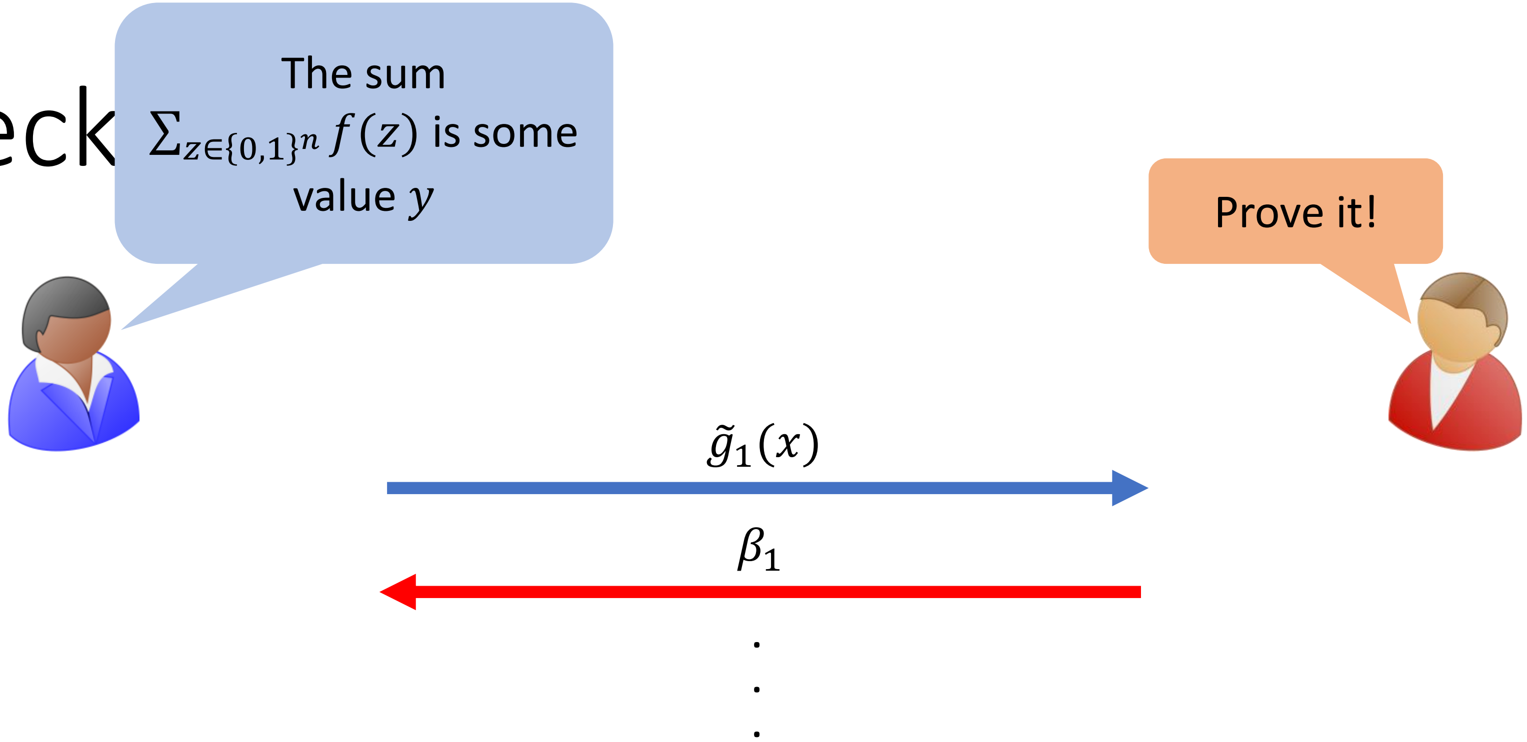
$$y_1 \coloneqq \tilde{g}_1(\beta_1)$$

$$\tilde{g}_2(x) \coloneqq \sum_{z_3,\cdots,z_n\in\{0,1\}} f(\beta_1, x, z_3, \cdots, z_n)$$

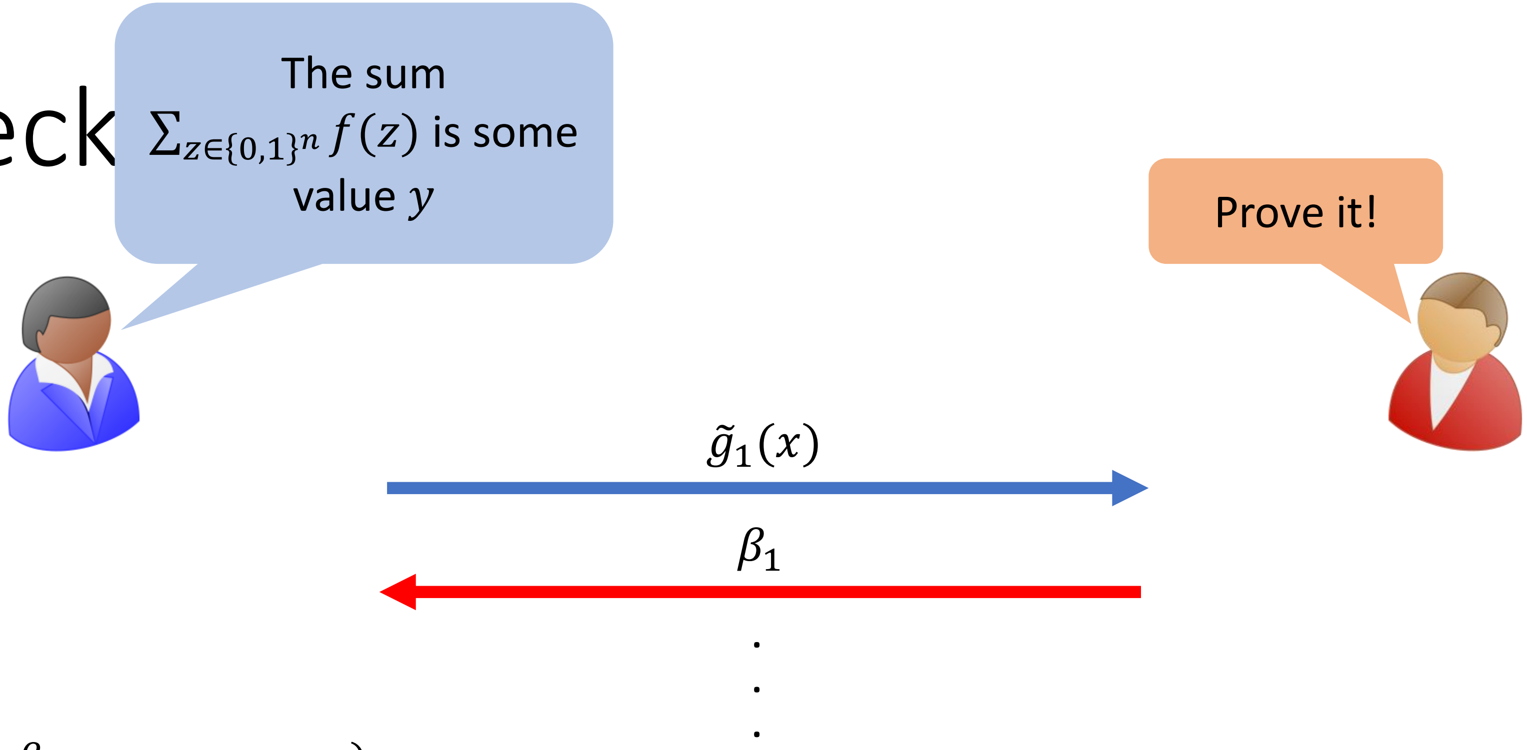
$$y_1 \coloneqq \tilde{g}_1(\beta_1)$$

$$\xrightarrow{\tilde{g}_2(x)}$$

Sumcheck


The sum $\Sigma_{z\in\{0,1\}^n} f(z)$ is some value y
Prove it!
$\tilde{g}_1(x)$
β_1
.
.
.

Sumcheck



$$\tilde{g}_j(x) \coloneqq \sum_{z_{j+1},\cdots,z_n\in\{0,1\}} f(\beta_1,\cdots,\beta_{j-1},x,z_{j+1},\cdots,z_n)$$

Sumcheck

The sum $\Sigma_{z\in\{0,1\}^n} f(z)$ is some value y

Prove it!

$\tilde{g}_1(x)$

β_1

$\vdots$

$\tilde{g}_j(x)$

$$\tilde{g}_j(x) \coloneqq \sum_{z_{j+1},\cdots,z_n\in\{0,1\}} f(\beta_1,\cdots,\beta_{j-1},x,z_{j+1},\cdots,z_n)$$

Sumcheck

The sum $\sum_{z\in\{0,1\}^n} f(z)$ is some value y

Prove it!

$\tilde{g}_1(x)$ →

← β_1

⋮

$\tilde{g}_j(x)$ →

$$\tilde{g}_j(x) \coloneqq \sum_{z_{j+1},\cdots,z_n\in\{0,1\}} f(\beta_1,\cdots,\beta_{j-1},x,z_{j+1},\cdots,z_n)$$

$$\tilde{g}_j(0)+\tilde{g}_j(1) \stackrel{?}{=} \tilde{g}_{j-1}(\beta_{j-1})$$

Sumcheck

The sum $\sum_{z \in \{0,1\}^n} f(z)$ is some value y

Prove it!

$\tilde{g}_1(x)$

β_1

$\vdots$

$\tilde{g}_j(x)$

$$\tilde{g}_j(x) \coloneqq \sum_{z_{j+1},\cdots,z_n \in \{0,1\}} f(\beta_1, \cdots, \beta_{j-1}, x, z_{j+1}, \cdots, z_n)$$

$$\tilde{g}_j(0) + \tilde{g}_j(1) \stackrel{?}{=} \tilde{g}_{j-1}(\beta_{j-1})$$

$$\beta_j \leftarrow_R \mathbb{F}$$

Sumcheck

The sum $\sum_{z \in \{0,1\}^n} f(z)$ is some value y

Prove it!

$\tilde{g}_1(x)$

β_1

$\vdots$

$$\tilde{g}_j(x) \coloneqq \sum_{z_{j+1},\cdots,z_n \in \{0,1\}} f(\beta_1, \cdots, \beta_{j-1}, x, z_{j+1}, \cdots, z_n)$$

$\tilde{g}_j(x)$

$$\tilde{g}_j(0) + \tilde{g}_j(1) \stackrel{?}{=} \tilde{g}_{j-1}(\beta_{j-1})$$

β_j

$$\beta_j \leftarrow_R \mathbb{F}$$

Sumcheck

The sum $\sum_{z\in\{0,1\}^n} f(z)$ is some value y

Prove it!

$\tilde{g}_1(x)$ →

← β_1

⋮

$$\tilde{g}_j(x) \coloneqq \sum_{z_{j+1},\cdots,z_n\in\{0,1\}} f(\beta_1,\cdots,\beta_{j-1},x,z_{j+1},\cdots,z_n)$$

$\tilde{g}_j(x)$ →

$$\tilde{g}_j(0)+\tilde{g}_j(1) \stackrel{?}{=} \tilde{g}_{j-1}(\beta_{j-1})$$

← β_j

$$\beta_j \leftarrow_R \mathbb{F}$$

$$y_j \coloneqq \tilde{g}_j(\beta_j)$$

***j*-th claim**

Sumcheck

The sum $\Sigma_{z\in\{0,1\}^n} f(z)$ is some value y

Prove it!

$\tilde{g}_1(x)$ →

← β_1

⋮

$$\tilde{g}_j(x) \coloneqq \sum_{z_{j+1},\cdots,z_n\in\{0,1\}} f(\beta_1,\cdots,\beta_{j-1},x,z_{j+1},\cdots,z_n)$$

$\tilde{g}_j(x)$ →

$$\tilde{g}_j(0)+\tilde{g}_j(1) \overset{?}{=} \tilde{g}_{j-1}(\beta_{j-1})$$

$$\beta_j \leftarrow_R \mathbb{F}$$

← β_j

$$y_j \coloneqq \tilde{g}_j(\beta_j)$$

***j*-th claim**

⋮

Sumcheck

The sum $\sum_{z\in\{0,1\}^n} f(z)$ is some value y

Prove it!

$\tilde{g}_1(x)$

β_1

$\vdots$

$$\tilde{g}_j(x) \coloneqq \sum_{z_{j+1},\cdots,z_n\in\{0,1\}} f(\beta_1,\cdots,\beta_{j-1},x,z_{j+1},\cdots,z_n)$$

$\tilde{g}_j(x)$

$$\tilde{g}_j(0)+\tilde{g}_j(1)\stackrel{?}{=}\tilde{g}_{j-1}(\beta_{j-1})$$

β_j

$\beta_j \leftarrow_R \mathbb{F}$

$y_j \coloneqq \tilde{g}_j(\beta_j)$

***j*-th claim**

$\vdots$

β_{n-1}

$\tilde{g}_n(x)$

Sumcheck

The sum $\sum_{z\in\{0,1\}^n} f(z)$ is some value y

Prove it!

$\tilde{g}_1(x)$

β_1

$\vdots$

$$\tilde{g}_j(x) := \sum_{z_{j+1},\cdots,z_n\in\{0,1\}} f(\beta_1,\cdots,\beta_{j-1},x,z_{j+1},\cdots,z_n)$$

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⋮

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size $N = 2^n$
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***j*-th claim**

⋮

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size $N/2^j$ claim

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size $N/2^j$ claim

Recall

$$\tilde{g}_j(x) := \sum_{z_{j+1},\cdots,z_n\in\{0,1\}} f(\beta_1,\cdots,\beta_{j-1},x,z_{j+1},\cdots,z_n)$$

Soundness

Soundness: if the j-th claim is **false** then $\forall \tilde{g}_{j+1}(x)$ the $(j+1)$-th claim is **also false**

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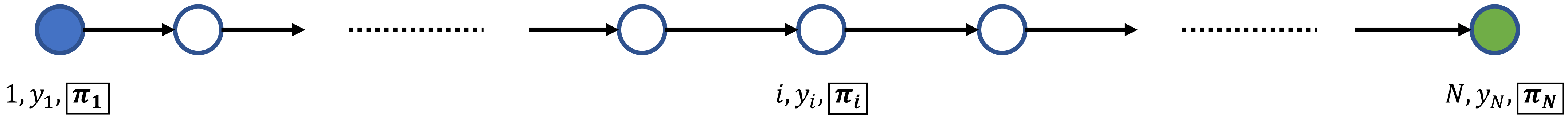
Both with high probability over β_{j+1} - Schwartz-Zippel.

Basic Idea



Several Challenges:

Proof size has to be polynomial

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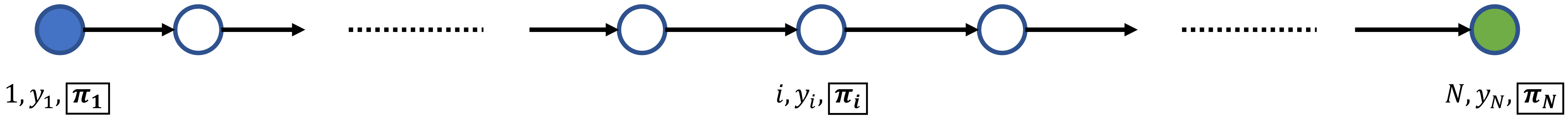

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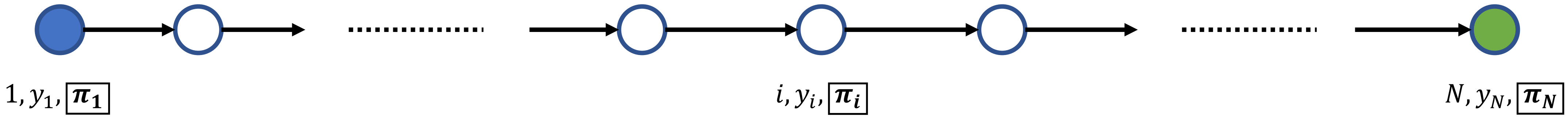

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[Fiat-Shamir'86] Transformation

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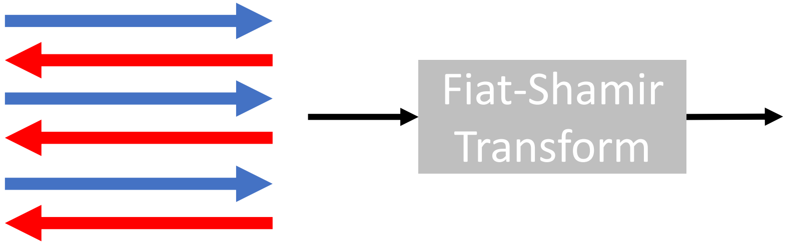

[Fiat-Shamir'86] Transformation

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Replaced by a hash function h

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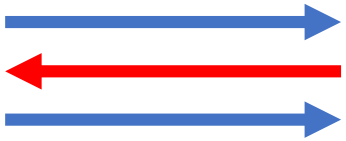
$=h($



$)$

Fiat-Shamir for Sumcheck

$$\xleftarrow{\quad\beta_j\quad} \;=\; h\left(\begin{array}{c} \xrightarrow{\quad\tilde{g}_1(x)\quad} \\ \xleftarrow{\quad\beta_1\quad} \\ \vdots \\ \xrightarrow{\quad\tilde{g}_j(x)\quad} \end{array} \right)$$

Fiat-Shamir for Sumcheck

Assumption

Resulting non-interactive (determinstic) argument is (adaptively) unambiguously sound

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Resulting non-interactive (determinstic) argument is (adaptively) unambiguously sound

Given h, no poly-time prover can find accepting proof:

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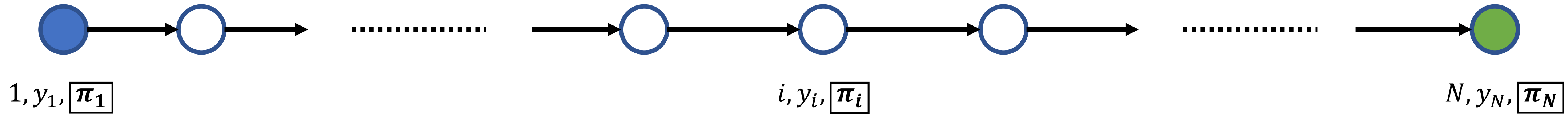
True if h is a random oracle.

Basic Idea



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Sumcheck Protocol is interactive

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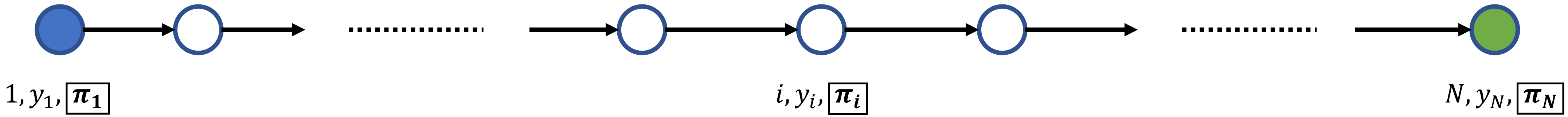

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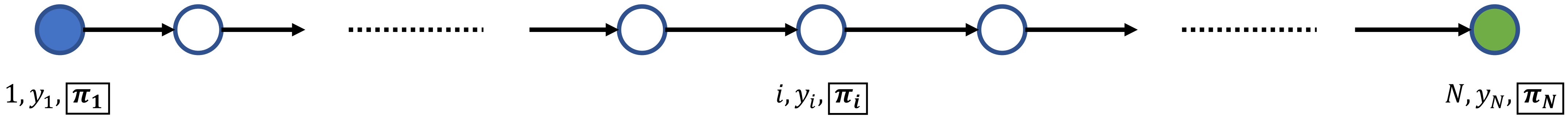

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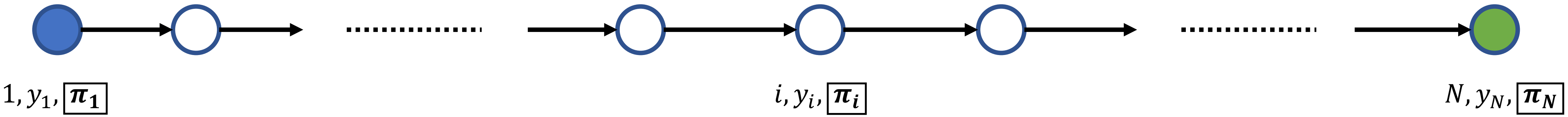
Fiat-Shamir Transform

Computing $\boldsymbol{S}\left(\boldsymbol{i}, \boldsymbol{y_i}, \boxed{\boldsymbol{\pi_i}}\right)$

Basic Idea



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Incremental Proof Updates

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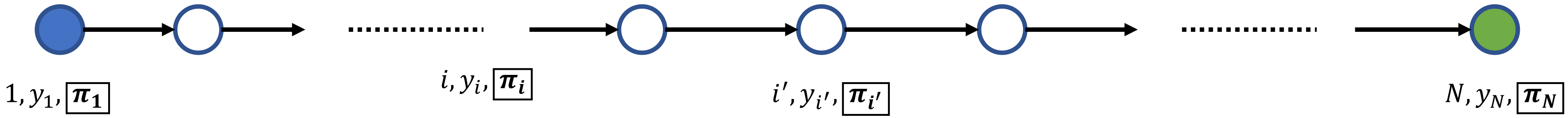

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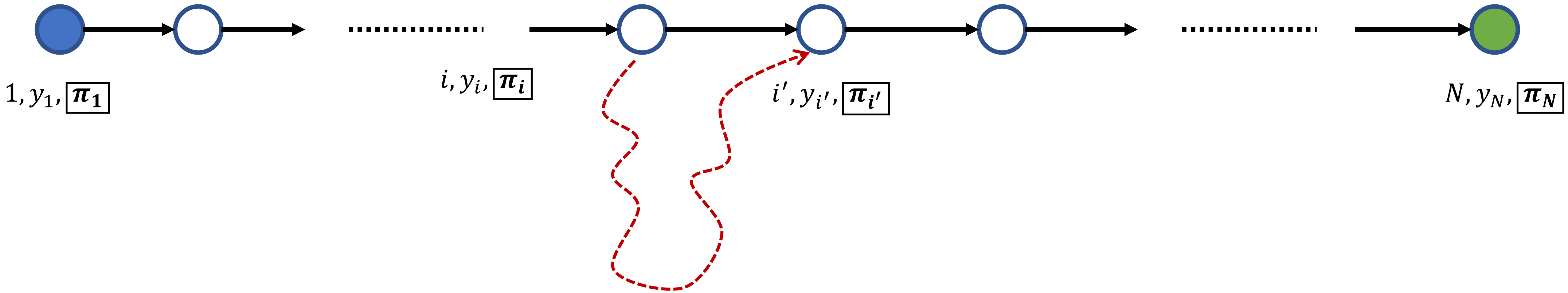

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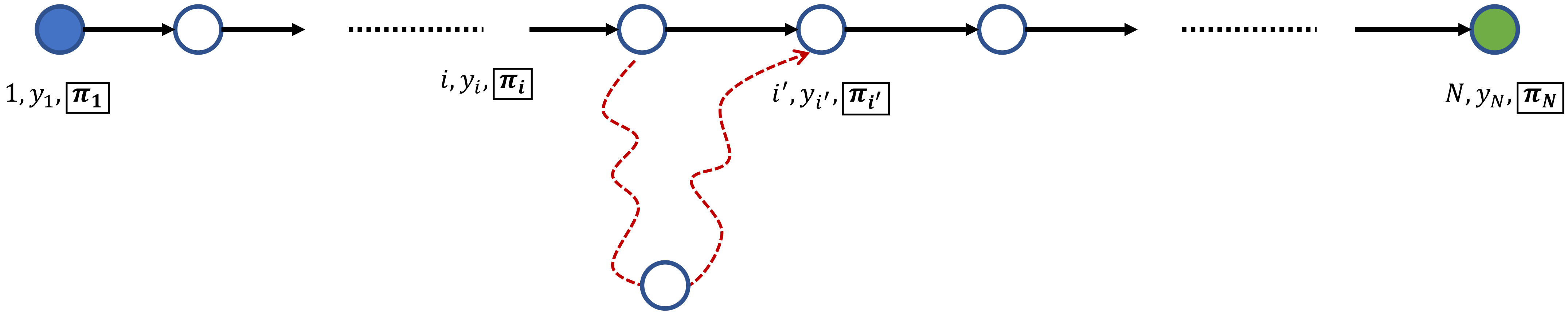

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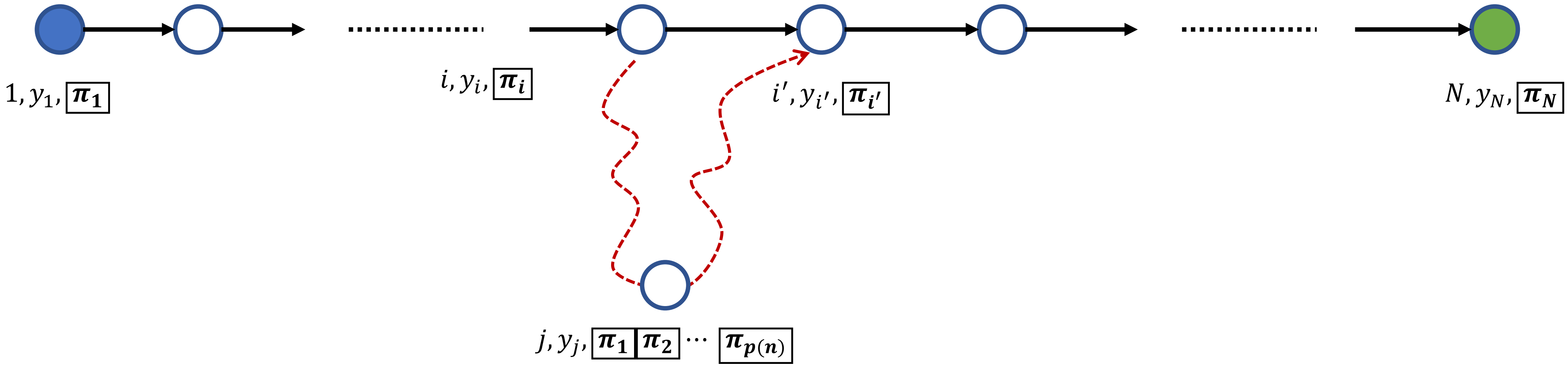

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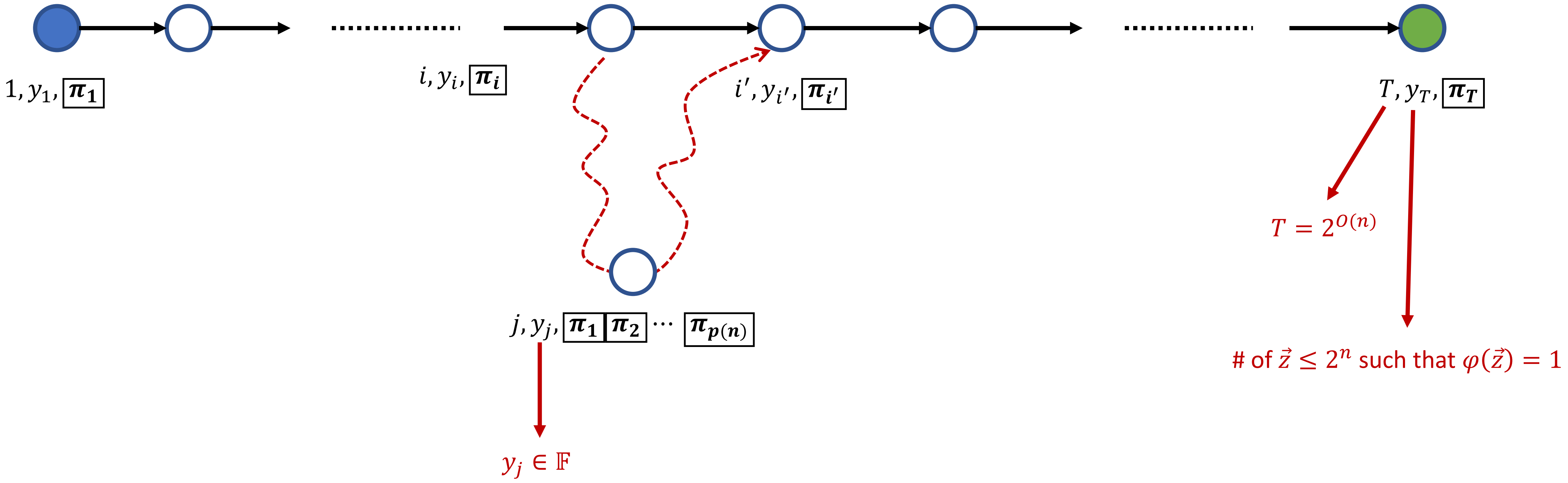
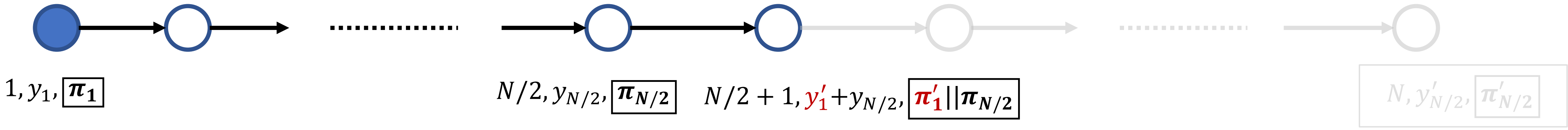


Naïve Recursive Construction



Construction for $N/2 \rightarrow$ to construction for N

First $N/2$ assignments: Do recursively

Second N/2 assignments: Add second proof $y_i, \boxed{\boldsymbol{\pi_i}}$

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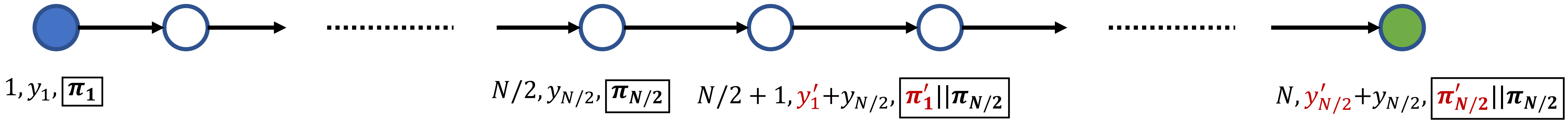

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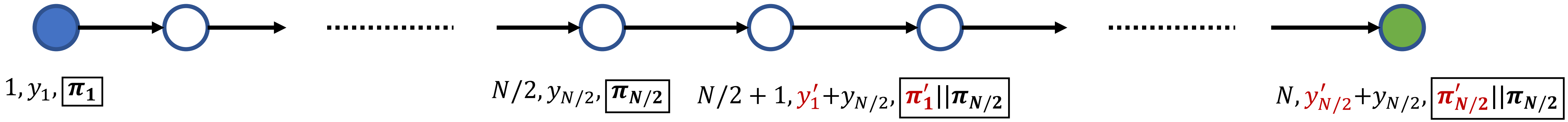

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Proof size: $\boldsymbol{P(N) = 2\ P(N/2)}$

 # Steps: $\boldsymbol{T(N) = 2\ T(N/2)}$

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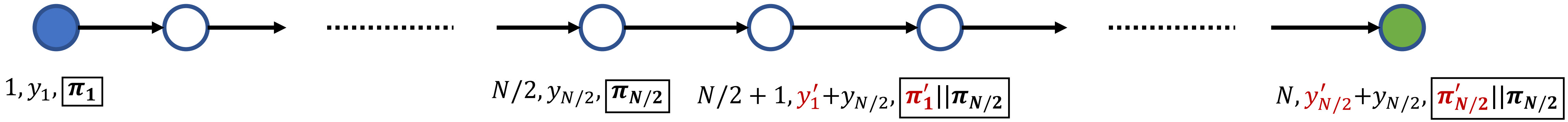

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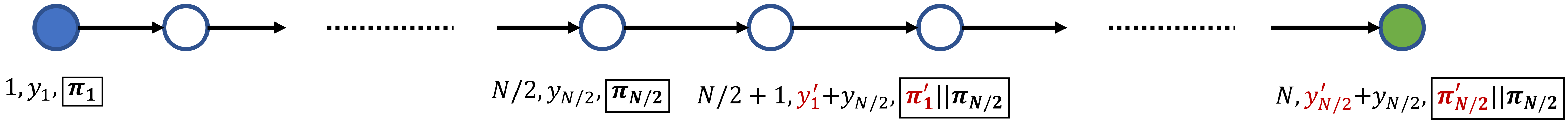
Merge Proofs [Valiant'06]



Merge proofs for $y_{N/2}, \boxed{\boldsymbol{\pi}_{N/2}}$ and $y'_{N/2}, \boxed{\boldsymbol{\pi}'_{N/2}}$

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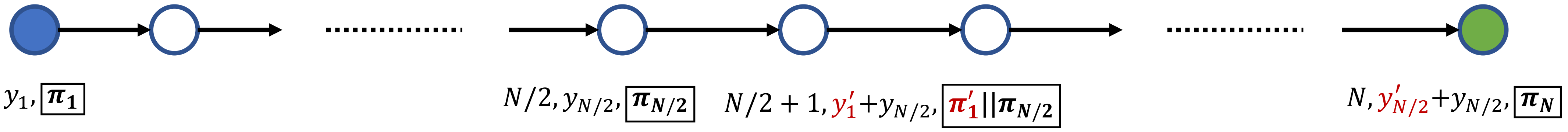

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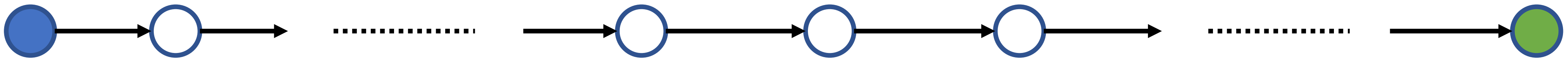

$1, y_1, \boxed{\boldsymbol{\pi}_1}$

$N/2, y_{N/2}, \boxed{\boldsymbol{\pi}_{N/2}}$

$N/2 + 1, y'_1 + y_{N/2}, \boxed{\boldsymbol{\pi}'_1 || \boldsymbol{\pi}_{N/2}}$

$N, y'_{N/2} + y_{N/2}, \boxed{\boldsymbol{\pi}'_{N/2} || \boldsymbol{\pi}_{N/2}}$

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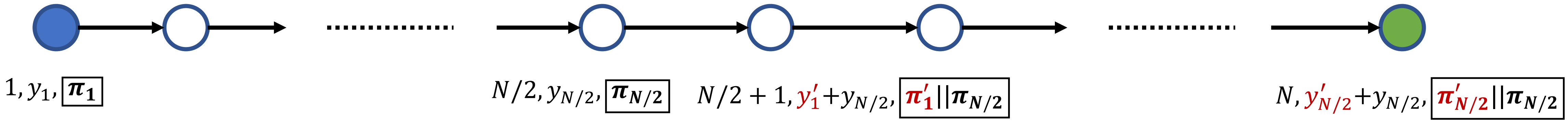
Proof size: $P(N) = \boldsymbol{P(N/2)}$

 # Steps: $T(N) = 2\,T\,(N/2) + \mathbf{1}$

Merge Proofs [Valiant'06]



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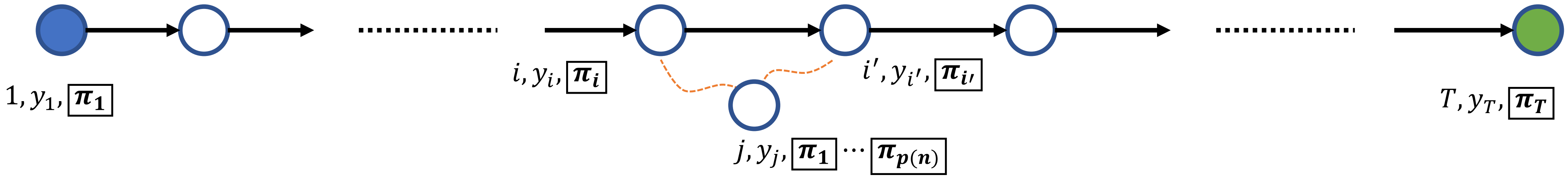
Proof size: $P(N) = \boldsymbol{P(N/2)}$

 # Steps: $\boldsymbol{T}(\boldsymbol{N}) = \boldsymbol{2\,T\,(N/2) + 1}$

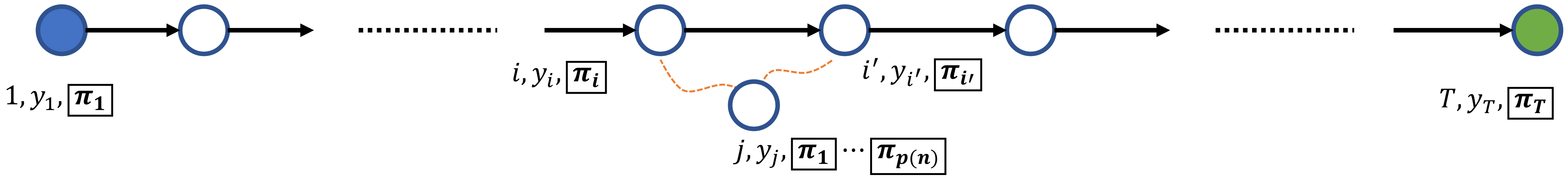
Requires "super-extractable" SNARKs

New Idea: Incremental Merge



Merge via (long) incrementally verifiable computation.

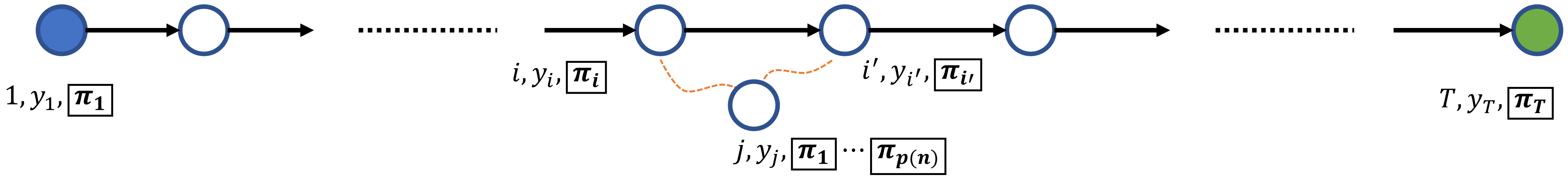
New Idea: Incremental Merge



Merge via (long) incrementally verifiable computation.

How long?

New Idea: Incremental Merge



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How long? $\boldsymbol{O(T(N/2))}$

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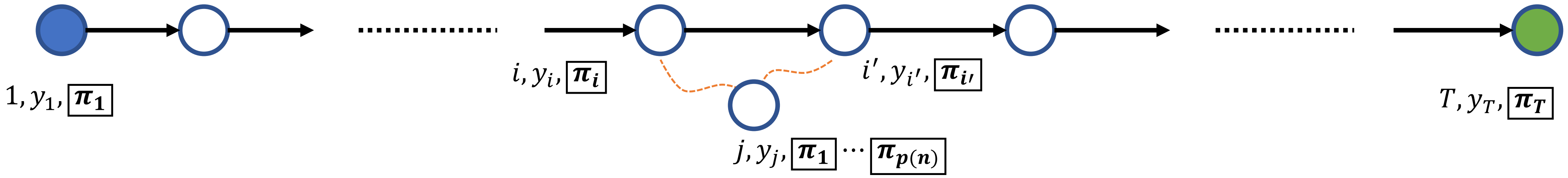


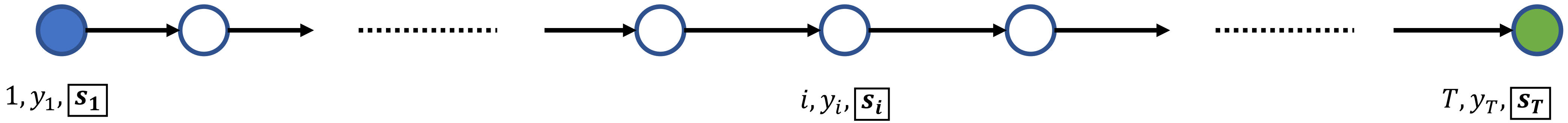
Merge via (long) incrementally verifiable computation.

How long? $\boldsymbol{O(T(N/2))}$

Proof size: $\boldsymbol{P(N) = P(N/2) + poly(n)}$

 # Steps: $\boldsymbol{T(N) = d\,T\,(N/2) + poly(n)}$

New Idea: Incremental Merge



How do you efficiently compute

$$\boldsymbol{s}\left(\boldsymbol{i},\ \boldsymbol{y_i}, \boxed{\boldsymbol{\pi_i}}\right) = \boldsymbol{i} + \mathbf{1},\ \boldsymbol{y_{i+1}}, \boxed{\boldsymbol{\pi_{i+1}}}$$

Proof Merging for (Fiat-Shamir) sumcheck

$$\sum_{z_{j+1},\cdots,z_n\in\{0,1\}} f(\beta_1,\cdots,\beta_{j-1},\beta_j,z_{j+1},\cdots,z_n) = y_j$$

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$$\xrightarrow{\tilde{g}_{j+1}(x)}$$

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$$\tilde{g}_{j+1}(x)$$

$$\beta_{j+1} = h(\alpha_1, \beta_1, \cdots, \alpha_{j+1})$$

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$j+1$-th claim

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size $N/2^{j+1}$ claim

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$$y_{j+1} \coloneqq \tilde{g}_{j+1}(\beta_{j+1})$$

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$\{\tilde{g}_{j+1}(0),\cdots,\tilde{g}_{j+1}(d)\}$

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$\{\tilde{g}_{j+1}(0),\cdots,\tilde{g}_{j+1}(d)\}$

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$\beta_{j+1} = h(\alpha_1,\beta_1,\cdots,\alpha_{j+1})$

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size $N/2^{j+1}$ claim

$(d+2)$ size $N/2^{j+1}$ claims

Proving the j-th claim

$$\left\{ \begin{array}{c} \tilde{g}_{j+1}(0), \boxed{\boldsymbol{\pi_0}} \\ \tilde{g}_{j+1}(1), \boxed{\boldsymbol{\pi_1}} \\ \vdots \\ \tilde{g}_{j+1}(d), \boxed{\boldsymbol{\pi_d}} \end{array} \right.$$

Proving the j-th claim

$$\left\{\begin{array}{l} \tilde{g}_{j+1}(0), \boxed{\boldsymbol{\pi_0}} \\ \tilde{g}_{j+1}(1), \boxed{\boldsymbol{\pi_1}} \\ \vdots \\ \tilde{g}_{j+1}(d), \boxed{\boldsymbol{\pi_d}} \end{array}\right.$$

size $N/2^{j+1}$ claims

Proving the j-th claim

$$\xrightarrow{\tilde{\boldsymbol{g}}_{\boldsymbol{j+1}}(\boldsymbol{x})} \left\{ \begin{array}{l} \tilde{g}_{j+1}(0), \boxed{\boldsymbol{\pi_0}} \\ \tilde{g}_{j+1}(1), \boxed{\boldsymbol{\pi_1}} \\ \vdots \\ \tilde{g}_{j+1}(d), \boxed{\boldsymbol{\pi_d}} \end{array} \right.$$

size $\boldsymbol{N/2^{j+1}}$ claims

Proving the j-th claim

$\tilde{\boldsymbol{g}}_{j+1}(\boldsymbol{x})$

$\tilde{g}_{j+1}(0), \boxed{\boldsymbol{\pi_0}}$

$\tilde{g}_{j+1}(1), \boxed{\boldsymbol{\pi_1}}$

$\vdots$

$\tilde{g}_{j+1}(d), \boxed{\boldsymbol{\pi_d}}$

size $N/2^{j+1}$ claims

$\boldsymbol{\beta_{j+1} = h(\alpha_1, \beta_1, \cdots, \alpha_{j+1})}$

Proving the j-th claim

$$\xrightarrow{\boldsymbol{\tilde{g}_{j+1}(x)}}$$

$$\left\{ \begin{array}{l} \tilde{g}_{j+1}(0), \boxed{\boldsymbol{\pi_0}} \\ \tilde{g}_{j+1}(1), \boxed{\boldsymbol{\pi_1}} \\ \vdots \\ \tilde{g}_{j+1}(d), \boxed{\boldsymbol{\pi_d}} \end{array} \right.$$

size $N/2^{j+1}$ claims

$$\xleftarrow{\boldsymbol{\beta_{j+1} = h(\alpha_1, \beta_1, \cdots, \alpha_{j+1})}}$$

$$\tilde{g}_{j+1}(\beta_{j+1}), \boxed{\boldsymbol{\pi_{d+1}}}$$

Proving the j-th claim

$\tilde{\boldsymbol{g}}_{\boldsymbol{j+1}}(\boldsymbol{x})$

$\tilde{g}_{j+1}(0), \boxed{\boldsymbol{\pi_0}}$

$\tilde{g}_{j+1}(1), \boxed{\boldsymbol{\pi_1}}$

$\vdots$

$\tilde{g}_{j+1}(d), \boxed{\boldsymbol{\pi_d}}$

size $N/2^{j+1}$ claims

$\boldsymbol{\beta_{j+1} = h(\alpha_1, \beta_1, \cdots, \alpha_{j+1})}$

$\tilde{g}_{j+1}(\beta_{j+1}), \boxed{\boldsymbol{\pi_{d+1}}}$

size $N/2^{j+1}$ claim

Parameters

Parameters

Proof size: $P(N) = \boldsymbol{P(N/2) + log}|\mathbb{F}|$

Parameters

Proof size: $\boldsymbol{P(N) = P(N/2) + log|\mathbb{F}|}$

$$\boldsymbol{P(0) = log|\mathbb{F}|}$$

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Proof size: $P(N) = P(N/2) + log|\mathbb{F}|$

Steps: $T(N) = (d+2)\, T\,(N/2) + poly(n)$

$$P(0) = log|\mathbb{F}|$$

$$P(n) = poly(n)$$

Parameters

Proof size: $\boldsymbol{P(N) = P(N/2) + log|\mathbb{F}|}$

 # Steps: $\boldsymbol{T(N) = (d+2)\, T\,(N/2) + poly(n)}$

$$\boldsymbol{P(0) = log|\mathbb{F}|}$$

$$\boldsymbol{P(n) = poly(n)}$$

$$\boldsymbol{T(0) = poly(log|\mathbb{F}|)}$$

Parameters

Proof size: $P(N) = P(N/2) + log|\mathbb{F}|$

 # Steps: $T(N) = (d+2)\,T\,(N/2) + poly(n)$

$P(0) = log|\mathbb{F}|$

$P(n) = poly(n)$

$T(0) = poly(log|\mathbb{F}|)$

$T(n) = 2^{O(n)}$

Open Problems

Instantiating Fiat-Shamir for sumcheck

Sampling small(ish) hard instances of NASH

Thank you. Questions?

The Five Worlds of Impagliazzo

Cryptomania:
∃ public-key crypto

Minicrypt:
∃ one-way functions ∄
public-key crypto

Pessiland:
NP is hard-on-average ∄
one-way functions

Heuristica:
$P \neq NP$
NP is easy-on-average

Algorithmica:
$P = NP$

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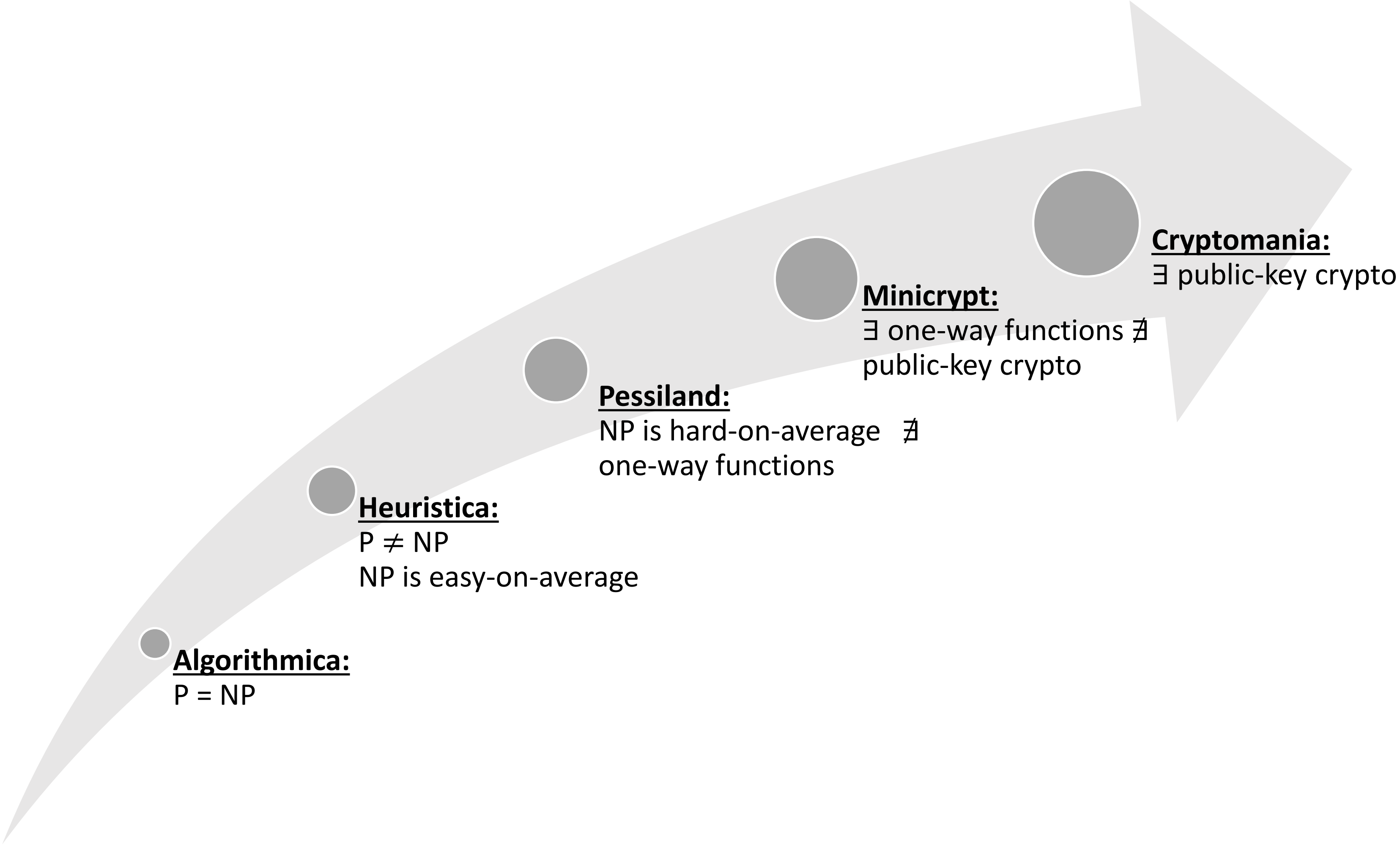

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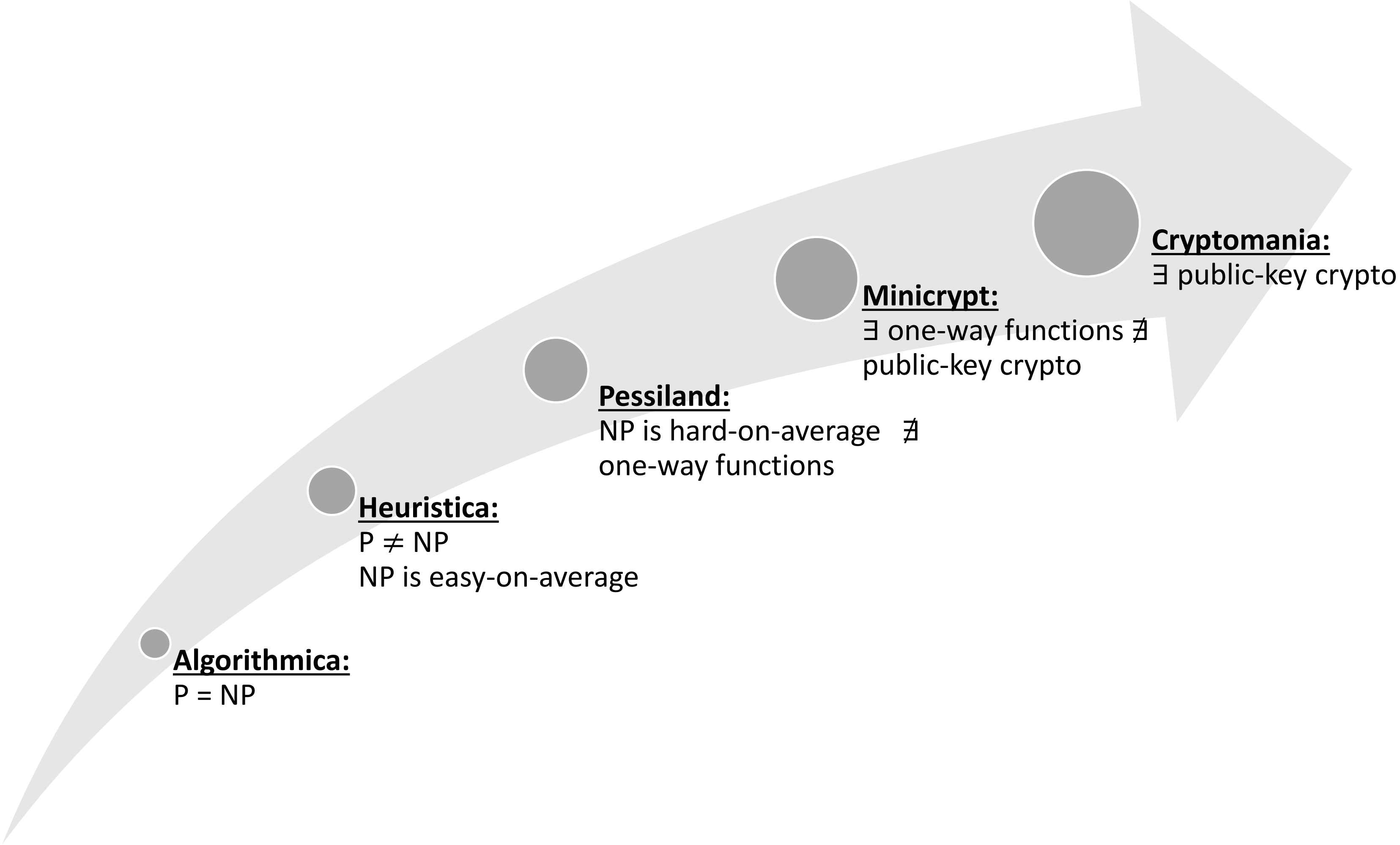

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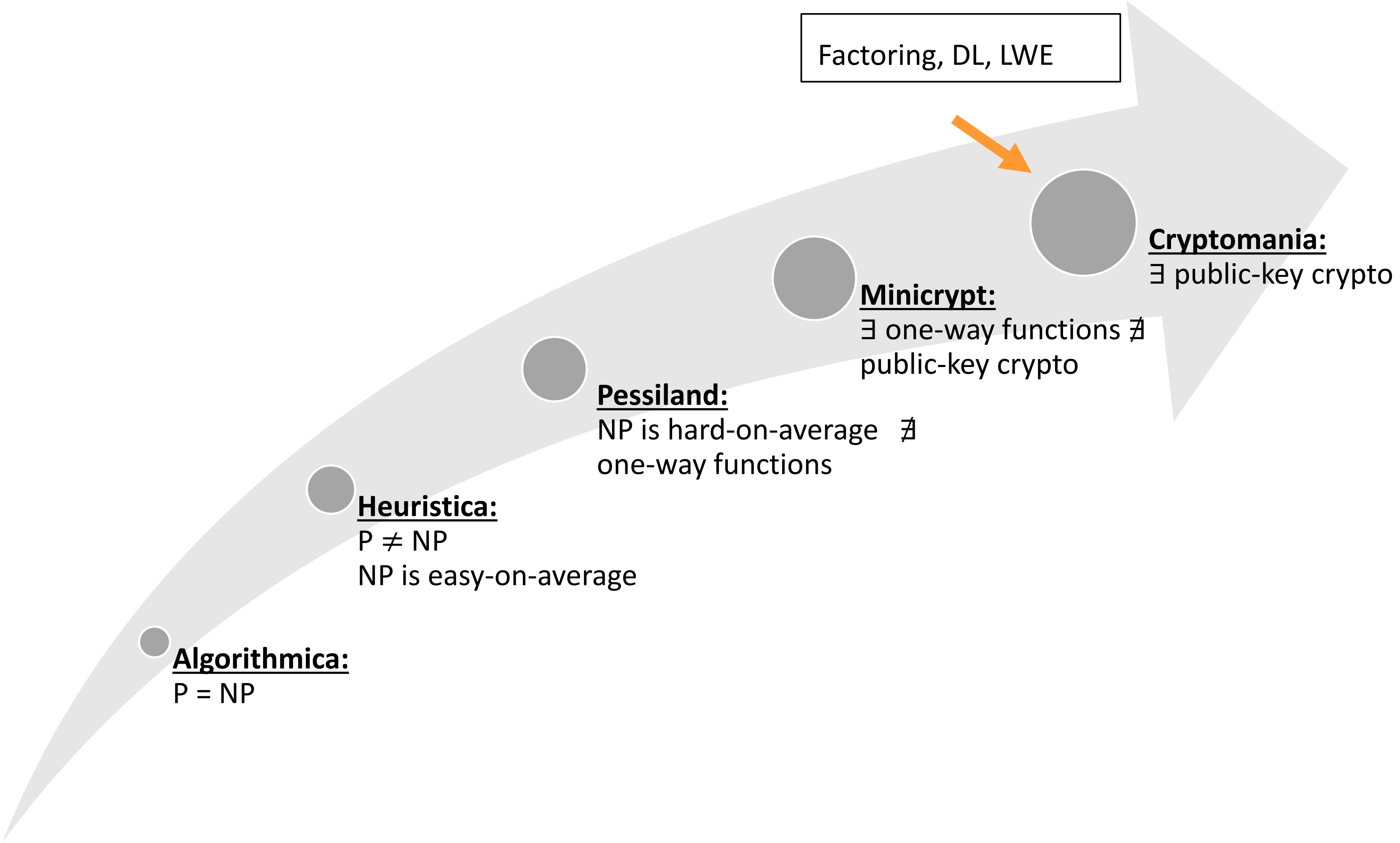

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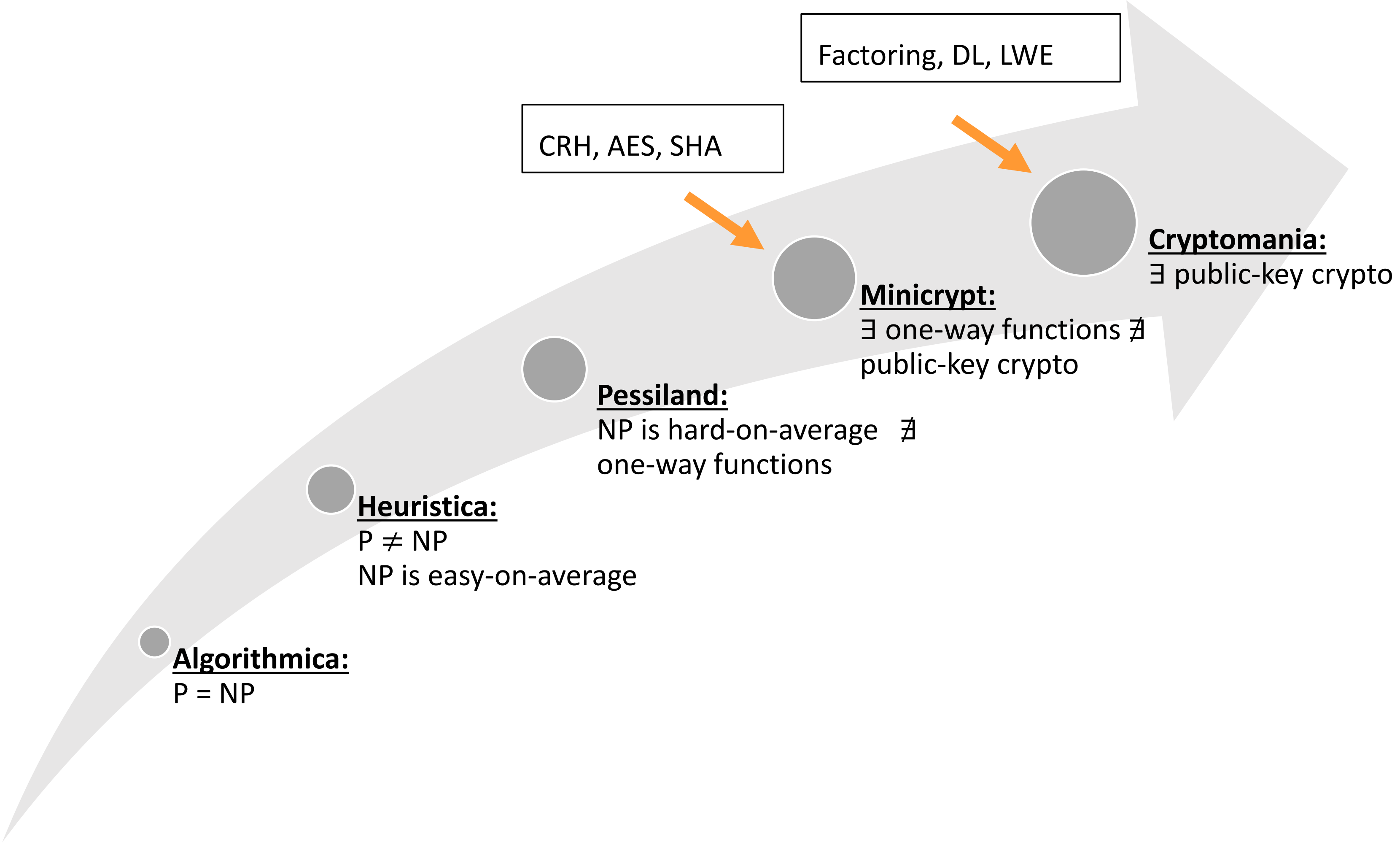

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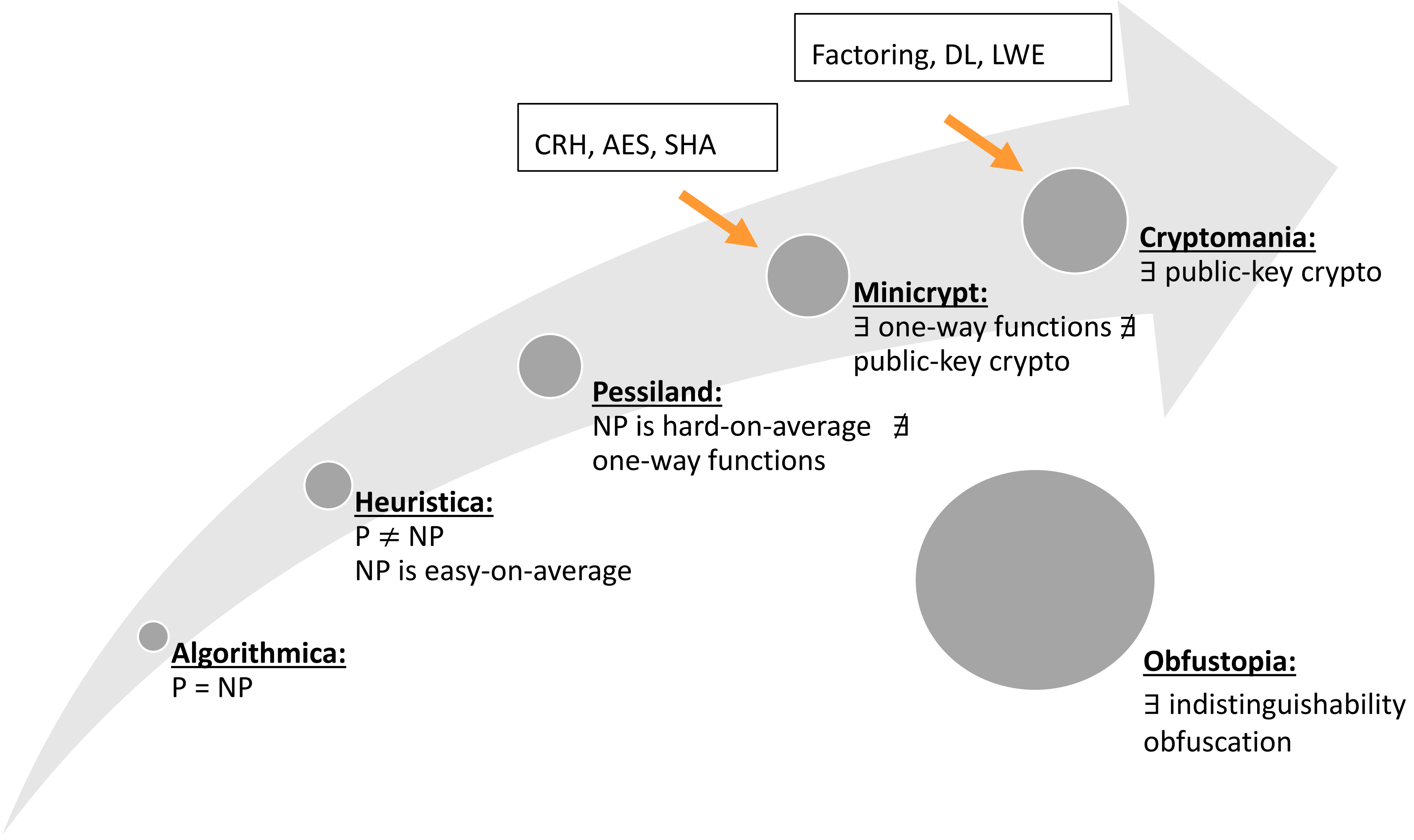

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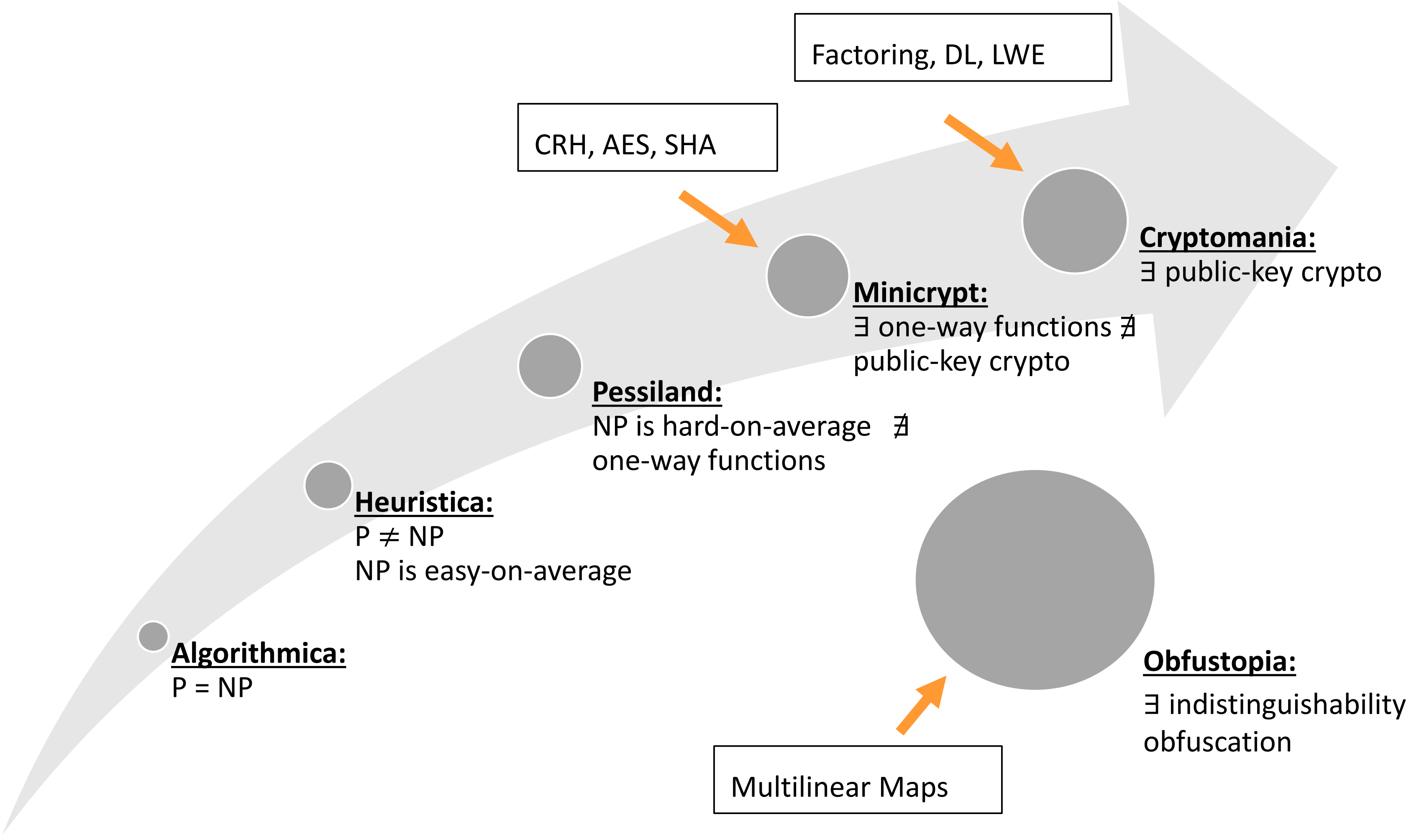

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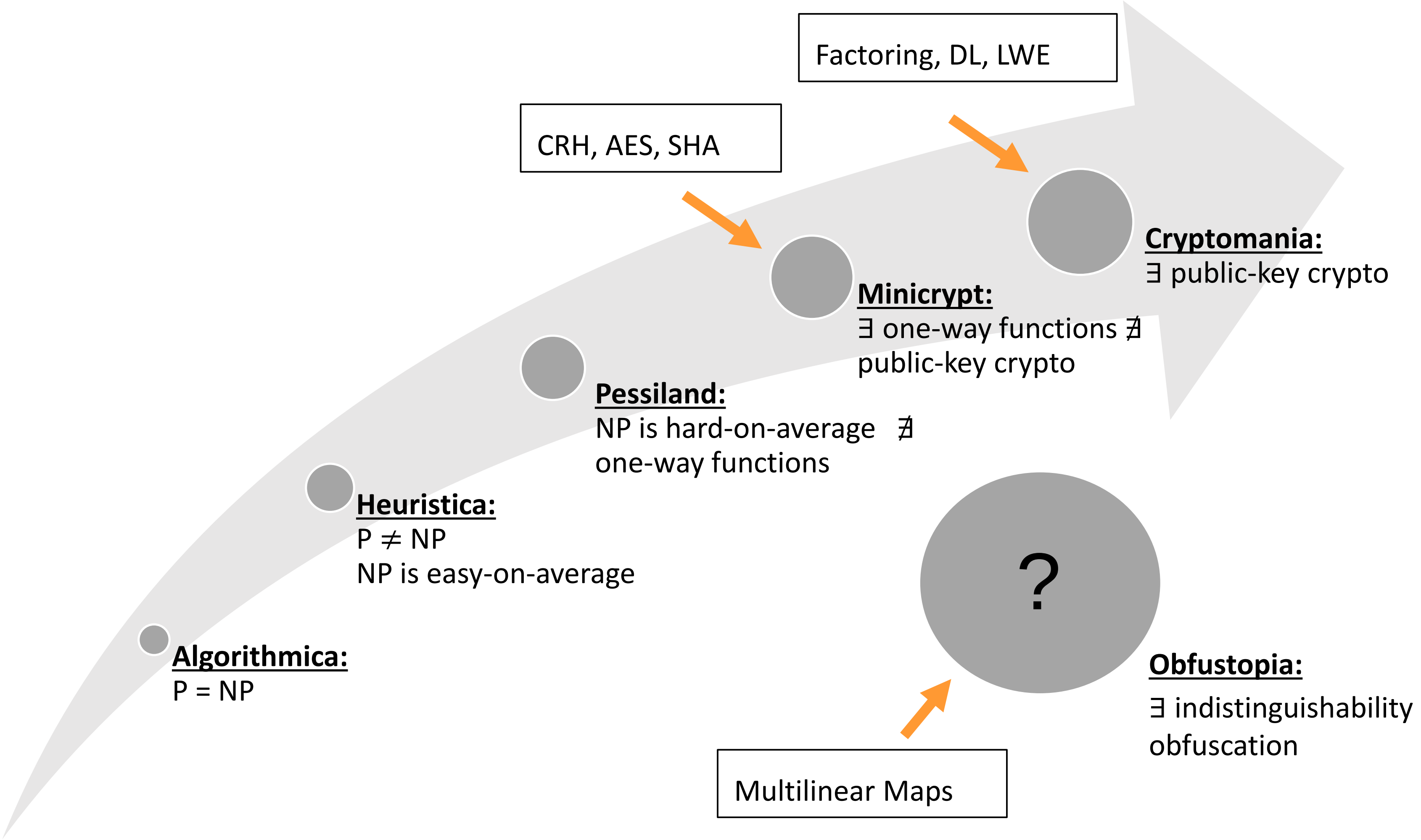

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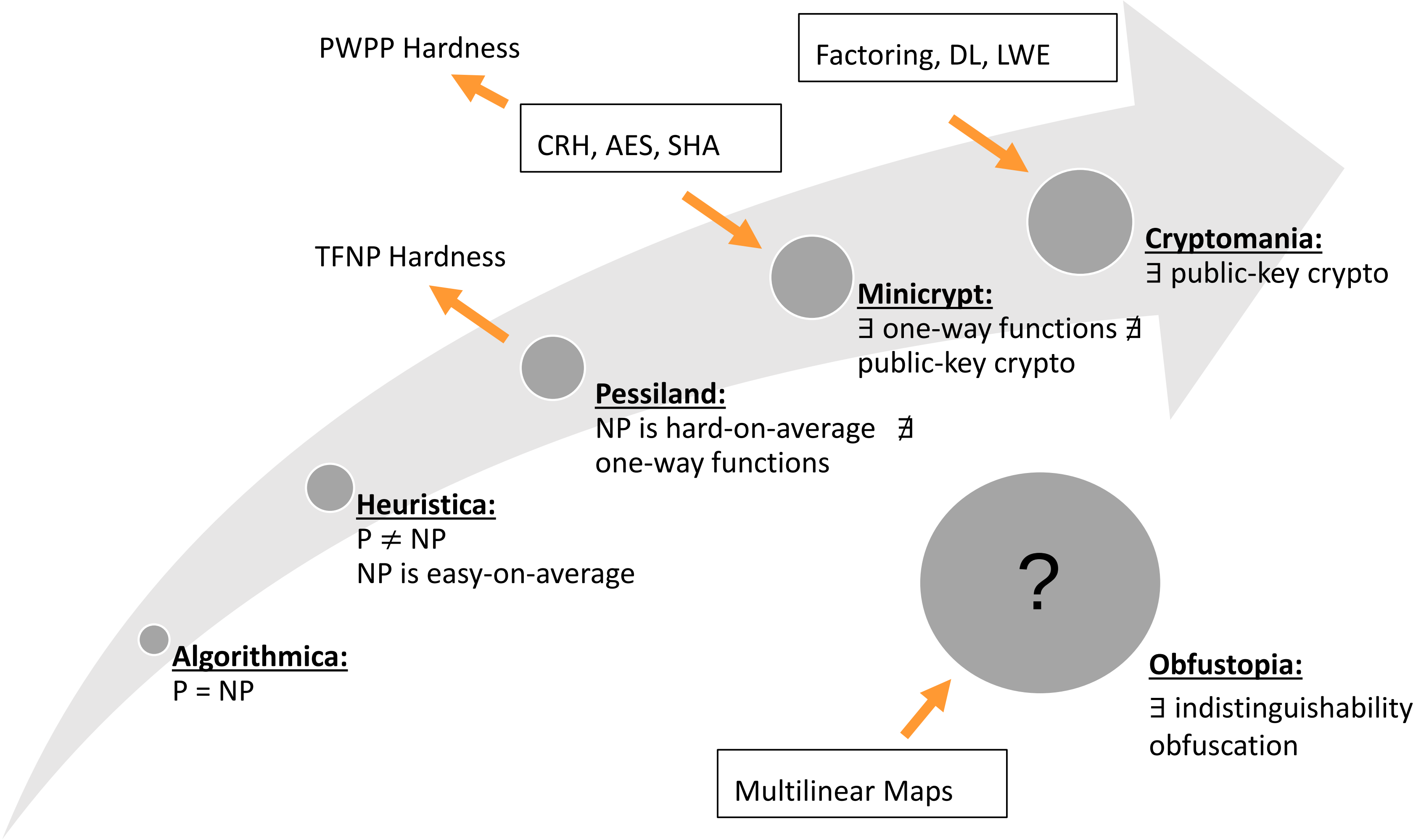

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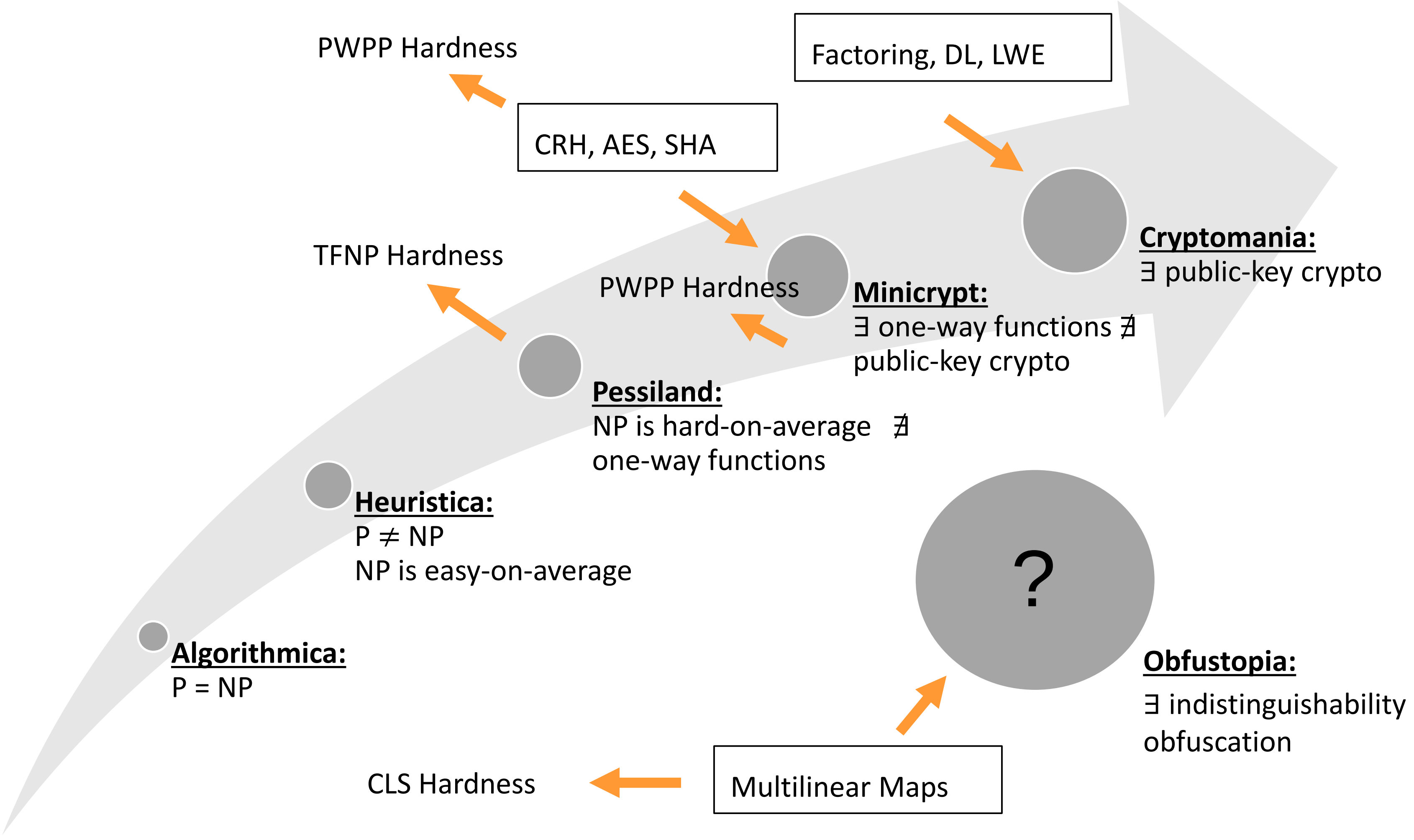

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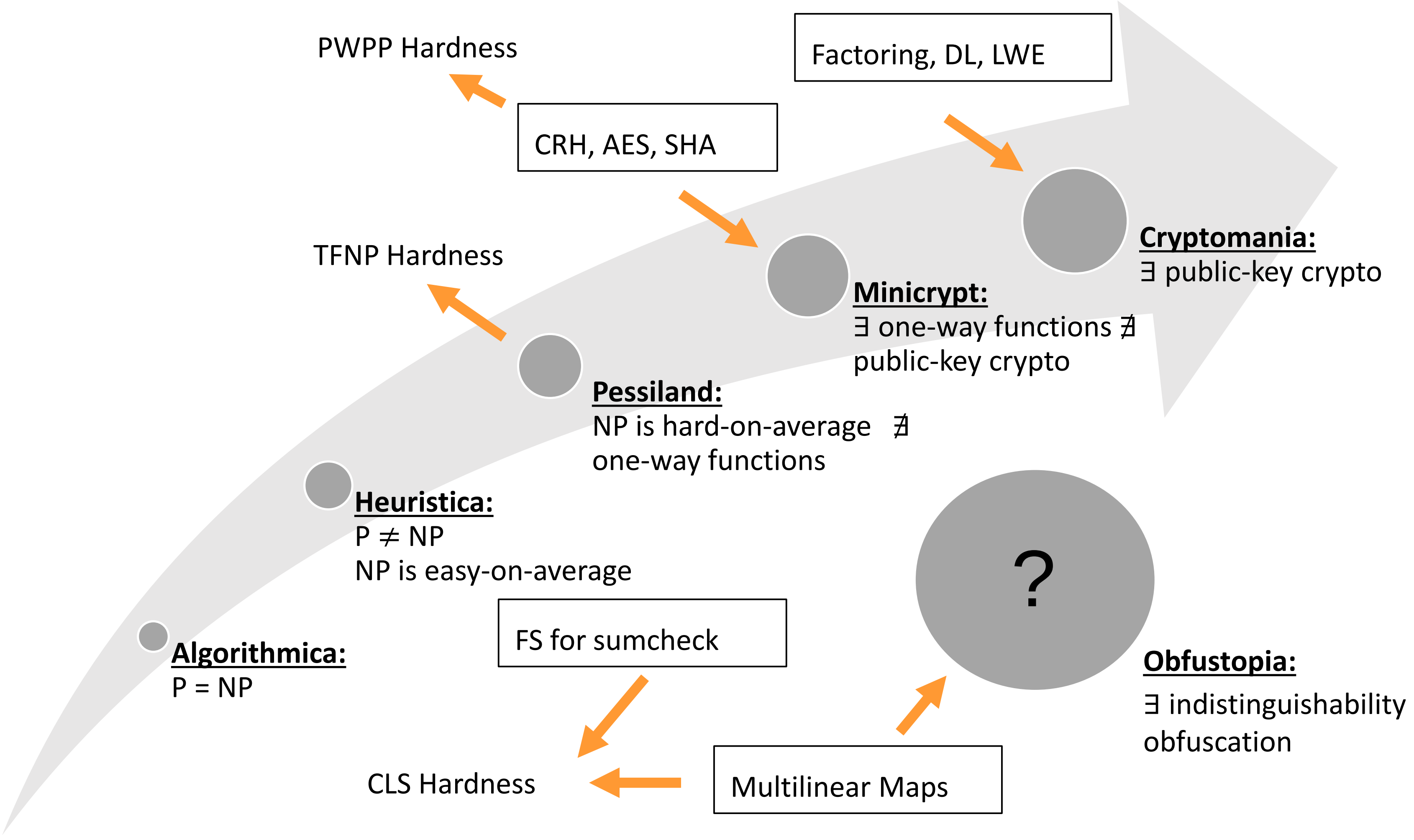

Is Crypto hardness Necessary?

[Rosen-Segev-Shachaf'17]

Black box separations

SVL hardness **not essential** for PPAD hardness

Basing PPAD hardness on OWFs:

cannot go through SVL, and

must have **exponential** #sol