

# Monotone-Policy Aggregate Signatures



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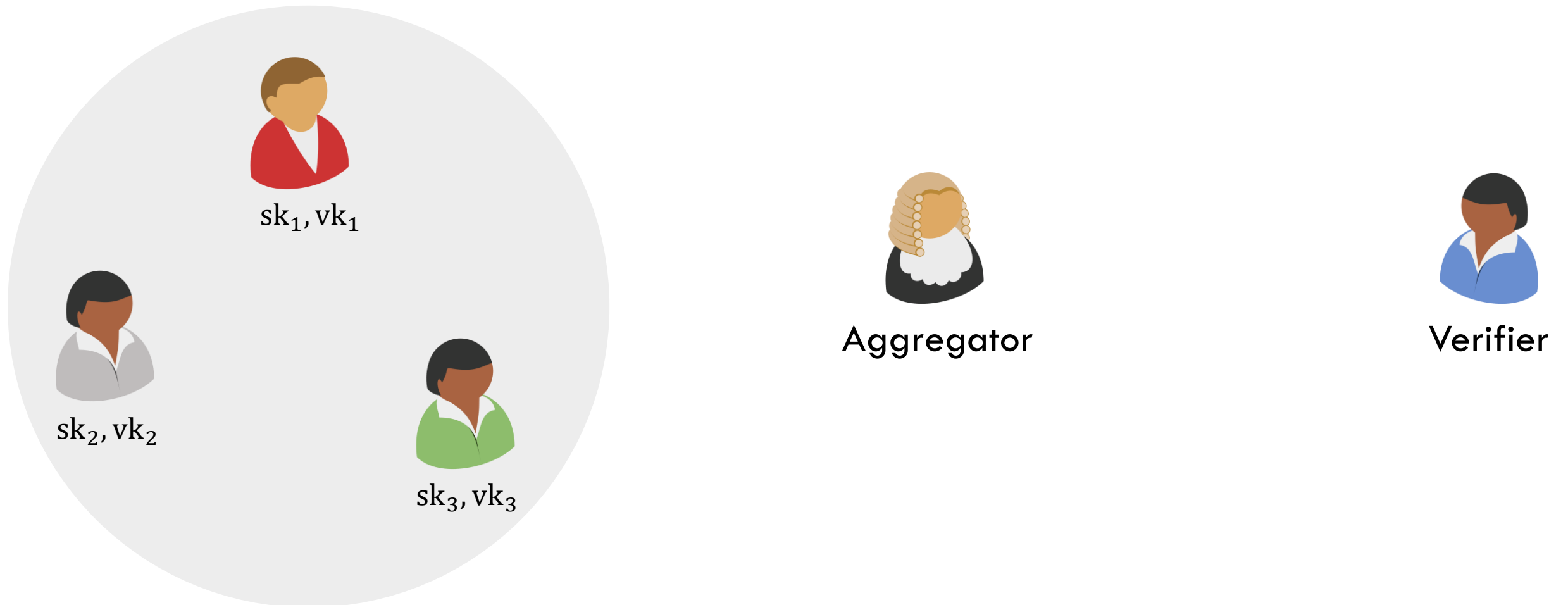
Abhishek Jain  
Johns Hopkins University  
and NTT Research



Omer Paneth  
Tel Aviv University

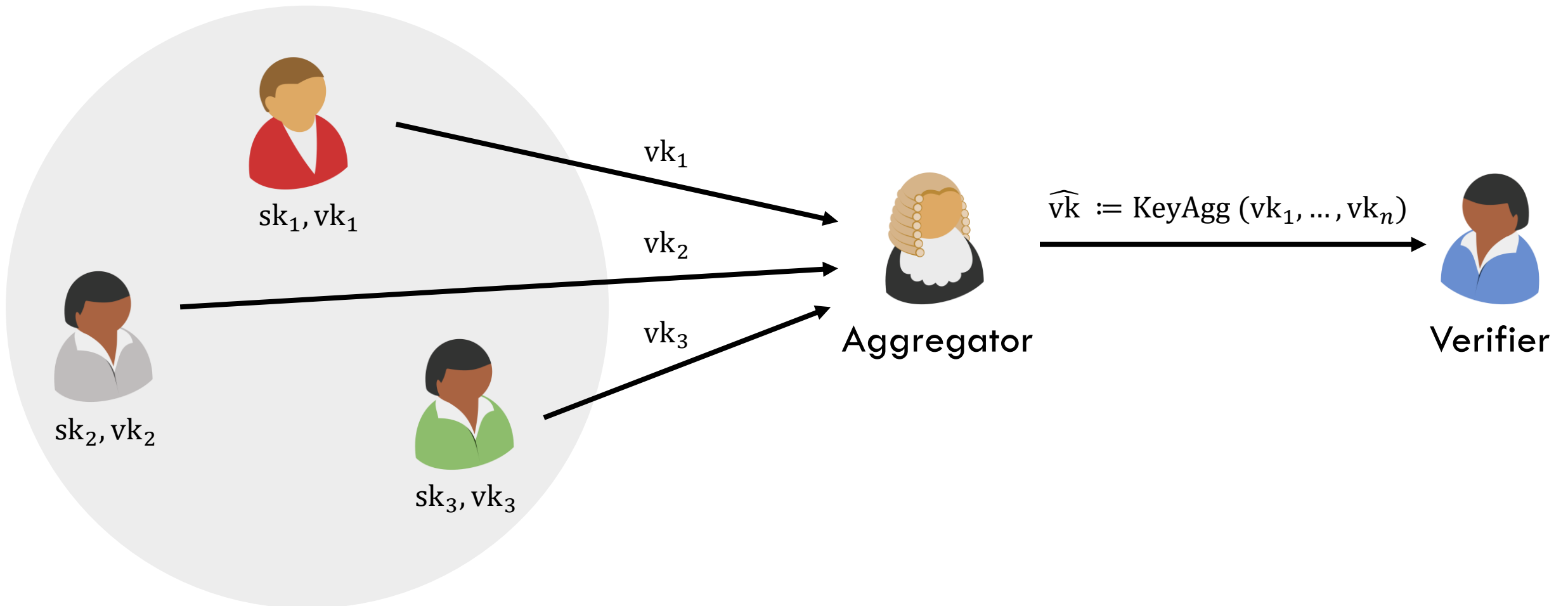
# Aggregate Signatures

[Itakura-Nakamura'83, Boneh-Gentry-Lynn-Shacham'03]



# Aggregate Signatures

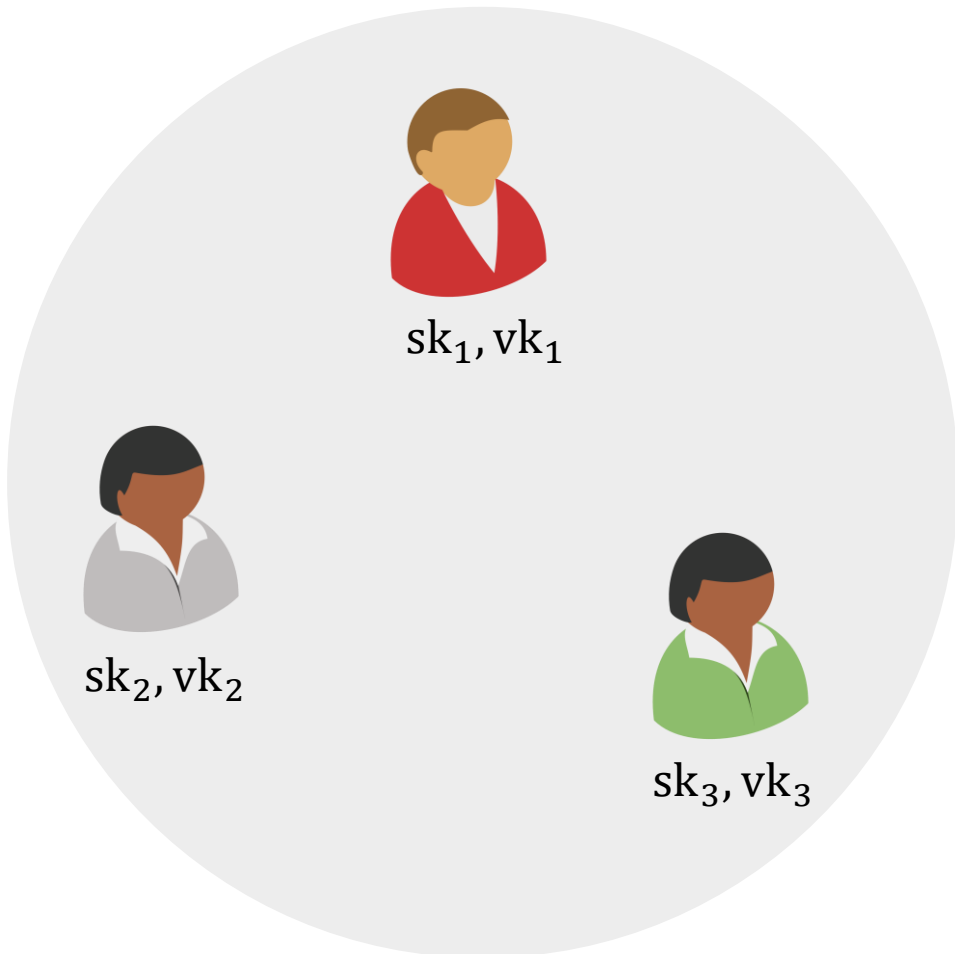
[Itakura-Nakamura'83, Boneh-Gentry-Lynn-Shacham'03]



# Aggregate Signatures

[Itakura-Nakamura'83, Boneh-Gentry-Lynn-Shacham'03]

$$\widehat{vk} := \text{KeyAgg}(vk_1, \dots, vk_n)$$



Aggregator



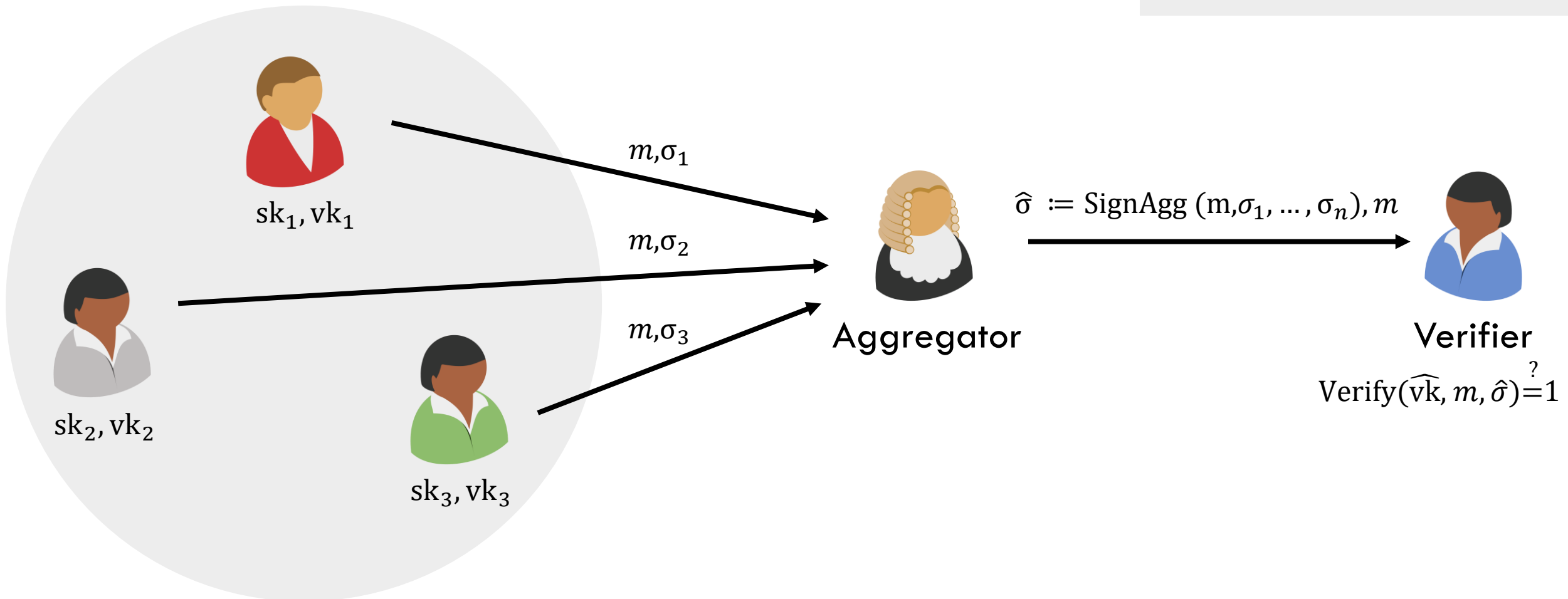
Verifier

$$\text{Verify}(\widehat{vk}, m, \hat{\sigma}) \stackrel{?}{=} 1$$

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[Itakura-Nakamura'83, Boneh-Gentry-Lynn-Shacham'03]

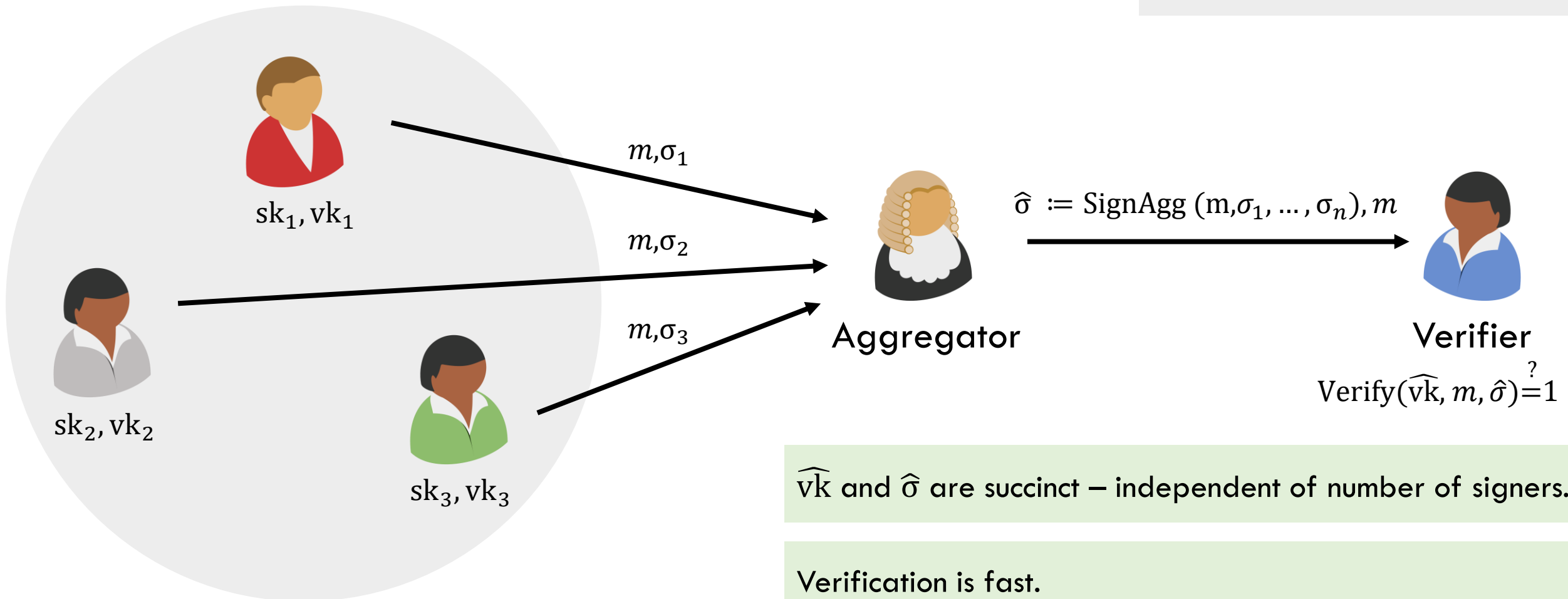
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# Aggregate Signatures

[Itakura-Nakamura'83, Boneh-Gentry-Lynn-Shacham'03]

$$\widehat{vk} := \text{KeyAgg}(vk_1, \dots, vk_n)$$



$\widehat{vk}$  and  $\hat{\sigma}$  are succinct – independent of number of signers.

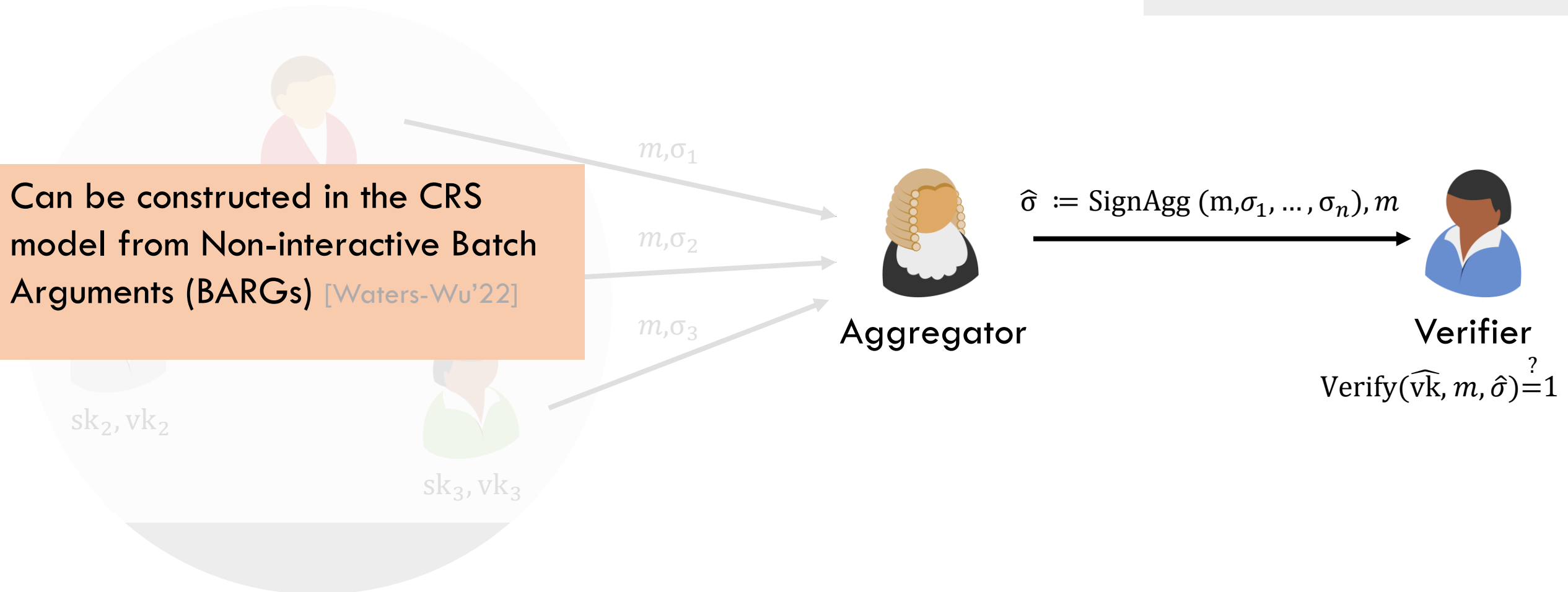
Verification is fast.

# Aggregate Signatures

[Itakura-Nakamura'83, Boneh-Gentry-Lynn-Shacham'03]

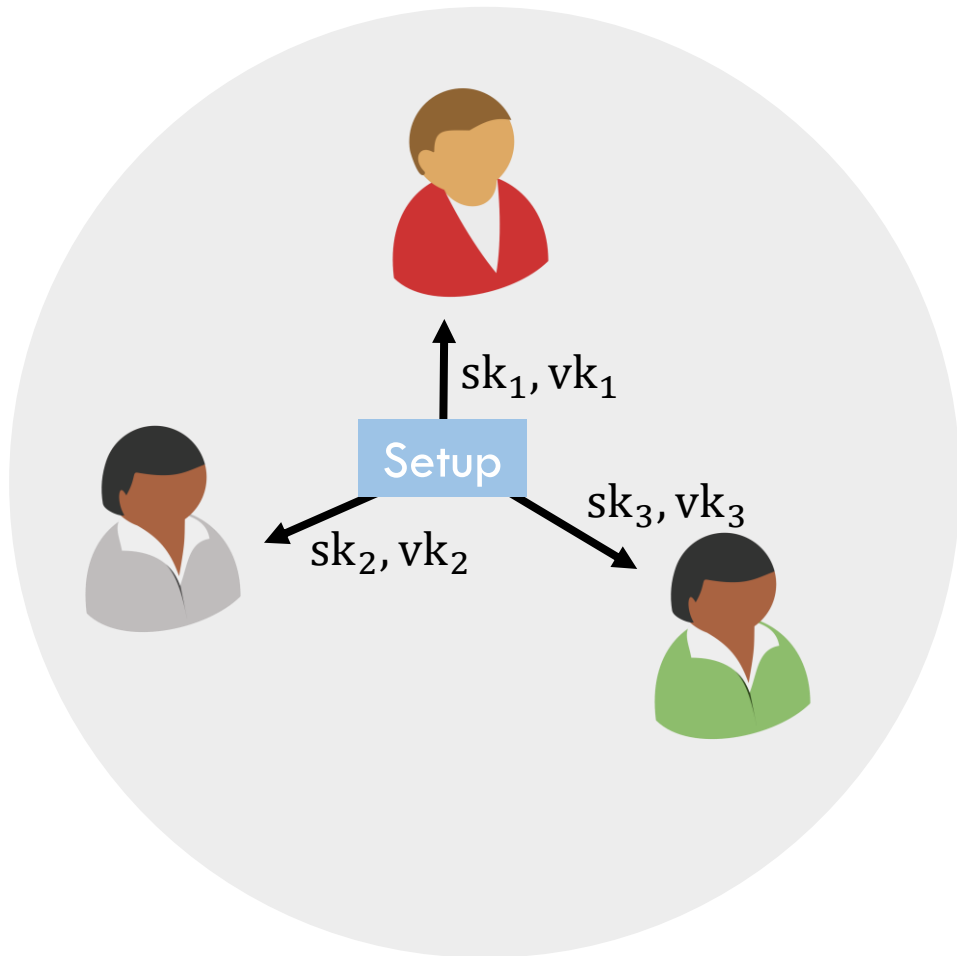
$$\widehat{vk} := \text{KeyAgg}(vk_1, \dots, vk_n)$$

Can be constructed in the CRS model from Non-interactive Batch Arguments (BARs) [Waters-Wu'22]



# Threshold Signatures

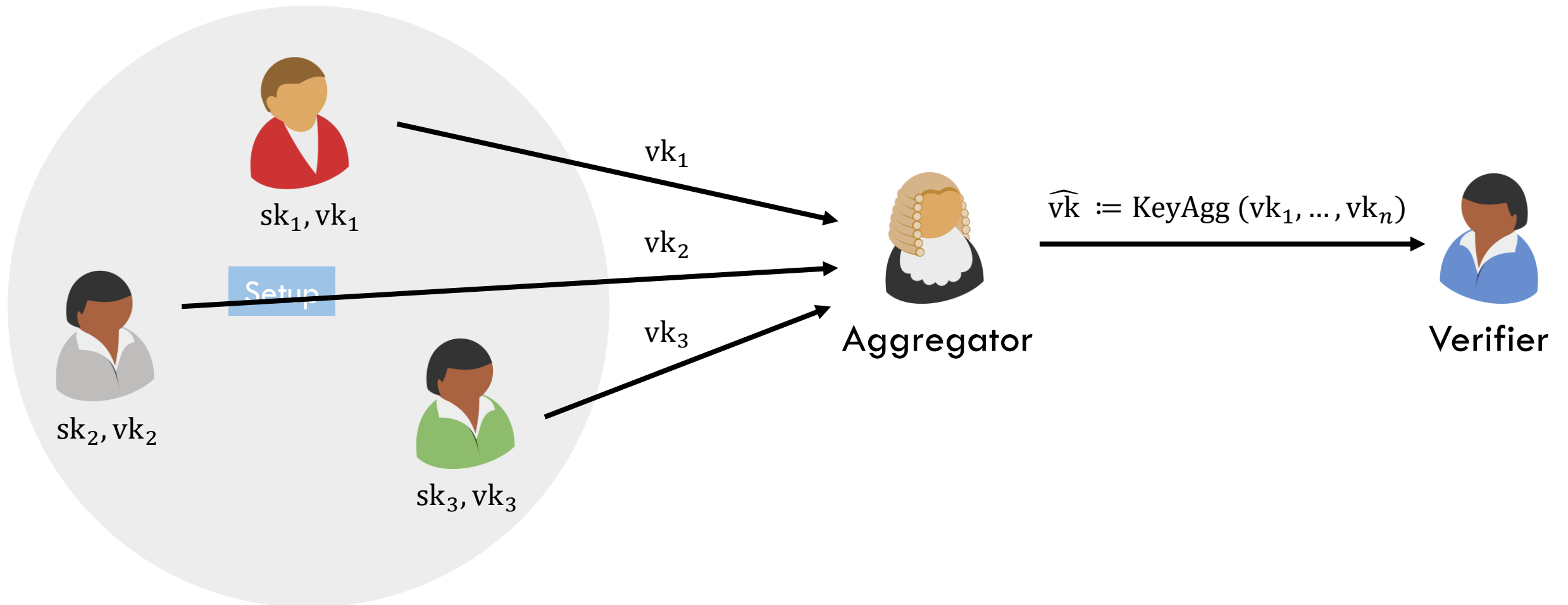
[Desmedt-Frankel'89]





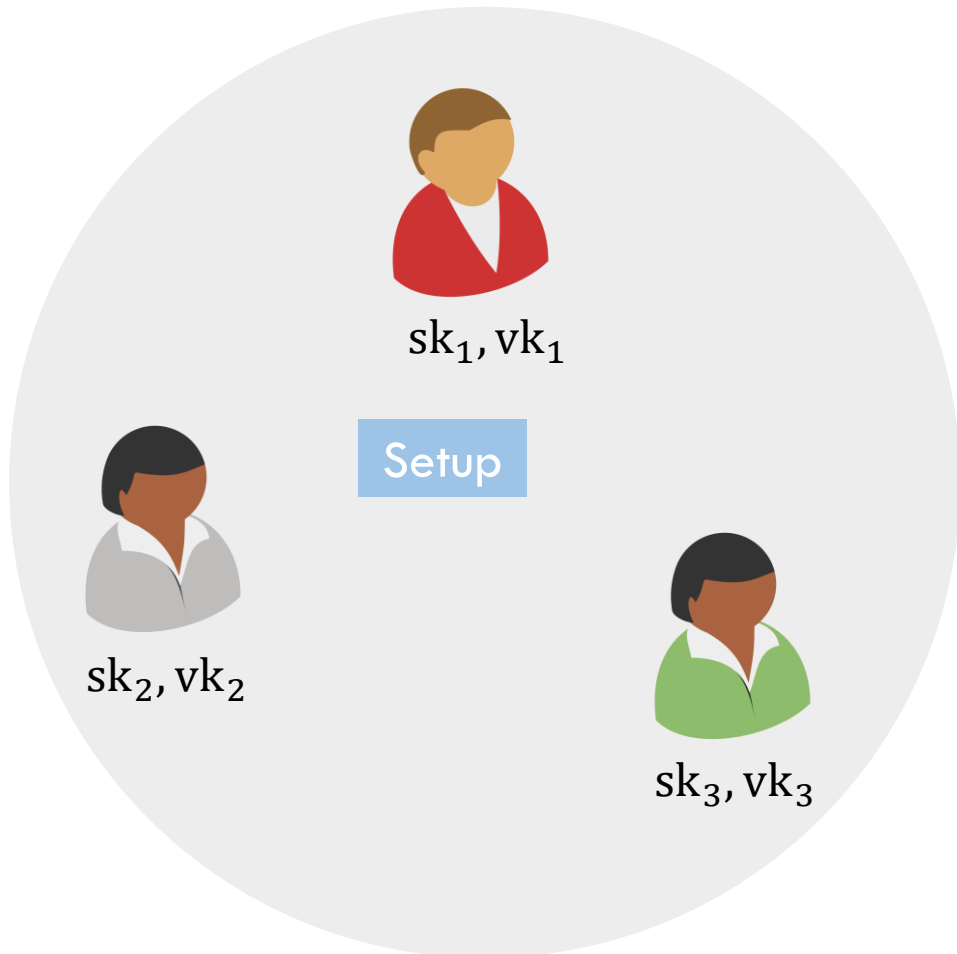
# Threshold Signatures

[Desmedt-Frankel'89]



# Threshold Signatures

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Aggregator



Verifier

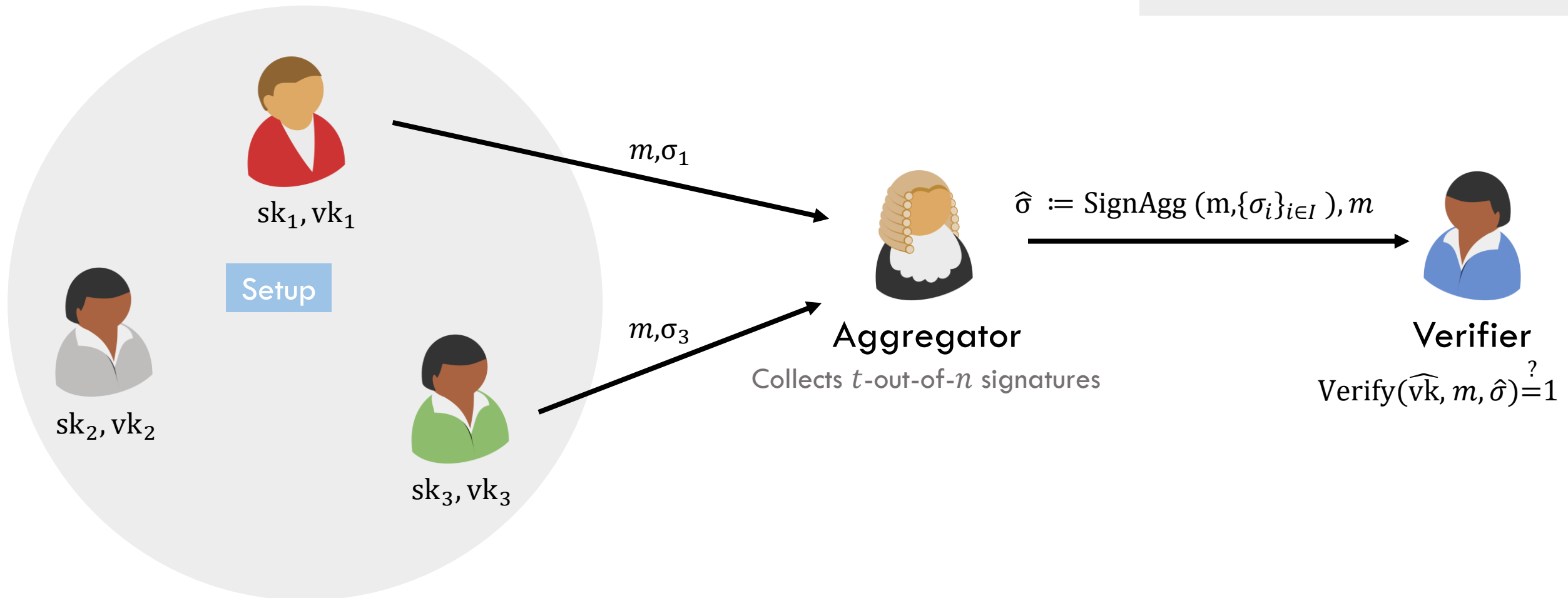
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# Threshold Signatures

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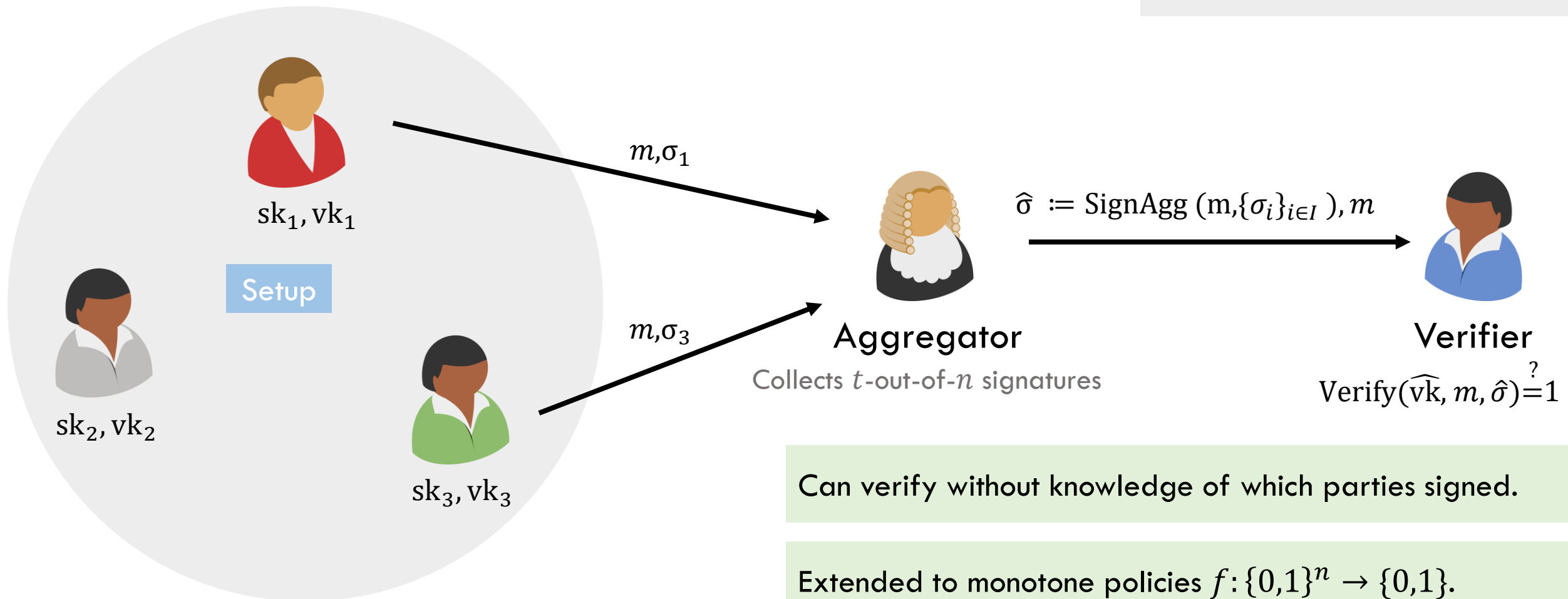
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# Threshold Signatures

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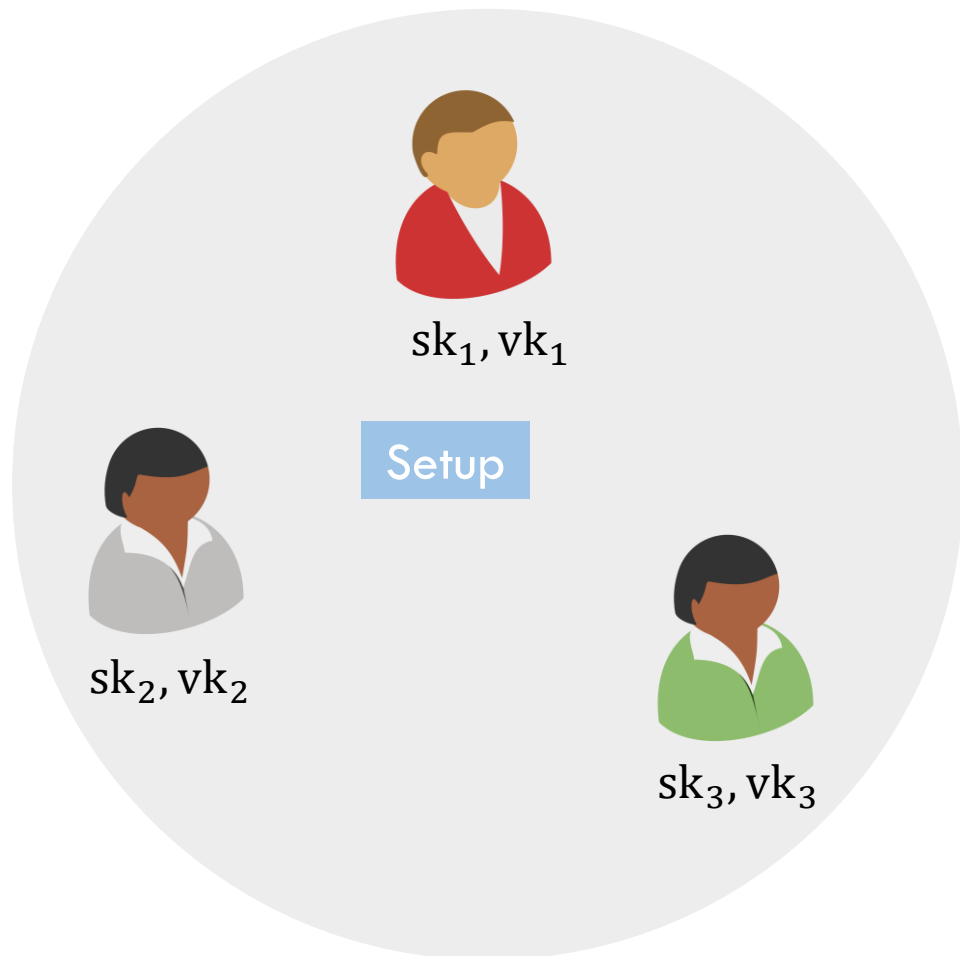
Can verify without knowledge of which parties signed.

Extended to monotone policies  $f: \{0,1\}^n \rightarrow \{0,1\}$ .

# Threshold Signatures

[Desmedt-Frankel'89]

$$\widehat{vk} := \text{KeyAgg}(vk_1, \dots, vk_n)$$



$m, \sigma_1$

Recent works [Garg-Jain-Mukherjee-Sinha-Wang-Zhang'24, Das-Camacho-Xiang-Nieto-Bünz-Ren'23] remove the need for an interactive “Setup” for specific schemes, but in idealized models.

$m, \sigma_3$

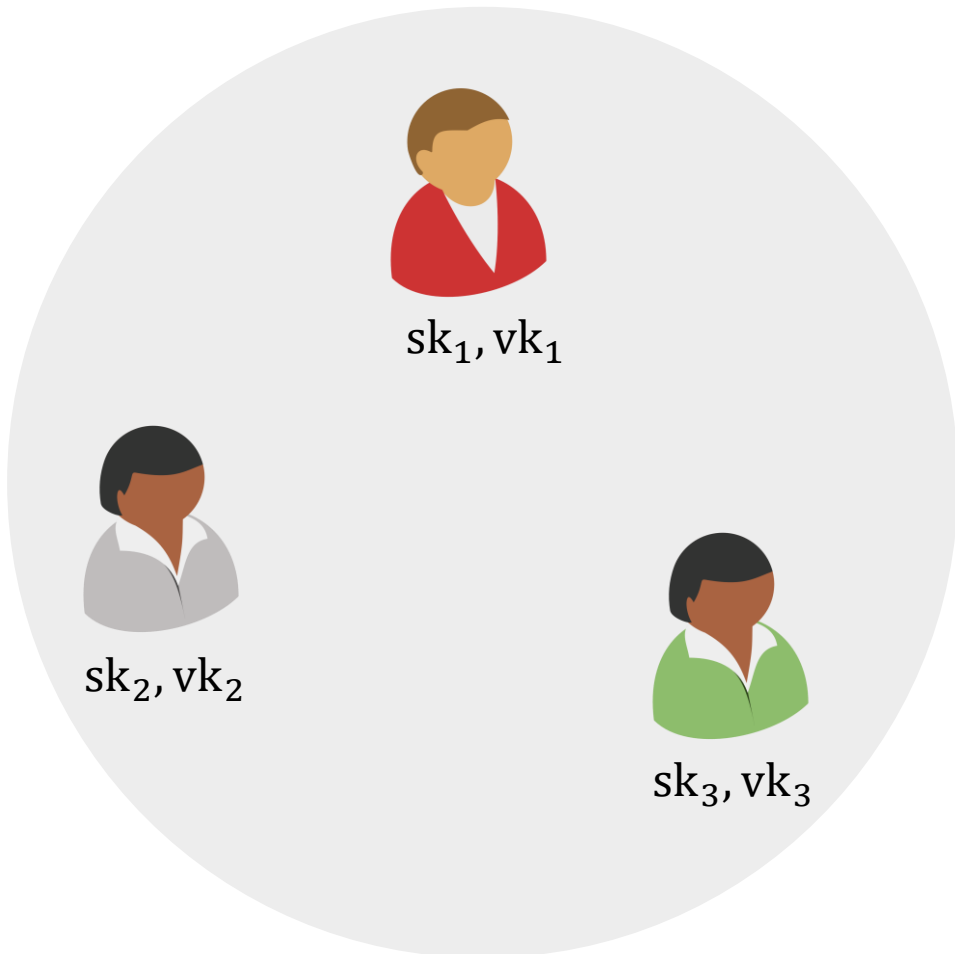
Collects a set of  $n$  signatures

$\text{Verify}(vk, m, \hat{\sigma}) = 1$

Can verify without knowledge of which parties signed.

Extended to monotone policies  $f: \{0,1\}^n \rightarrow \{0,1\}$ .

# Monotone Policy Aggregate Signatures



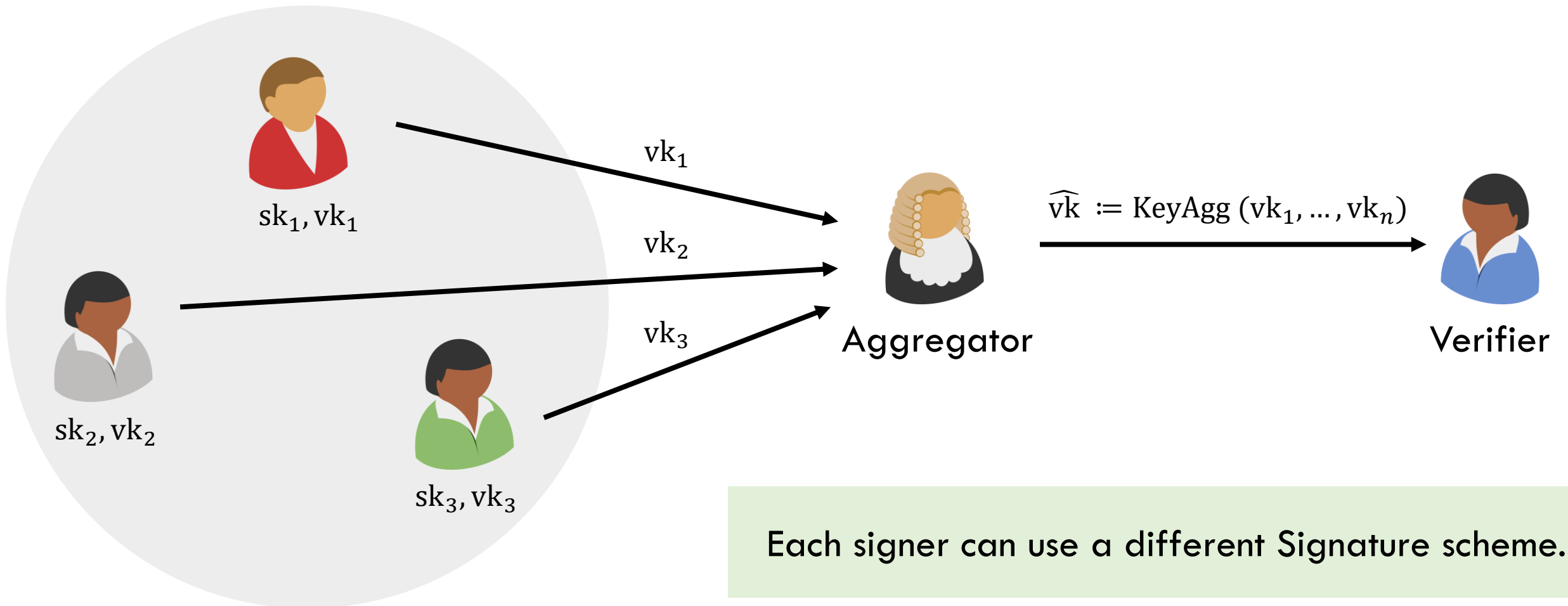
Aggregator



Verifier

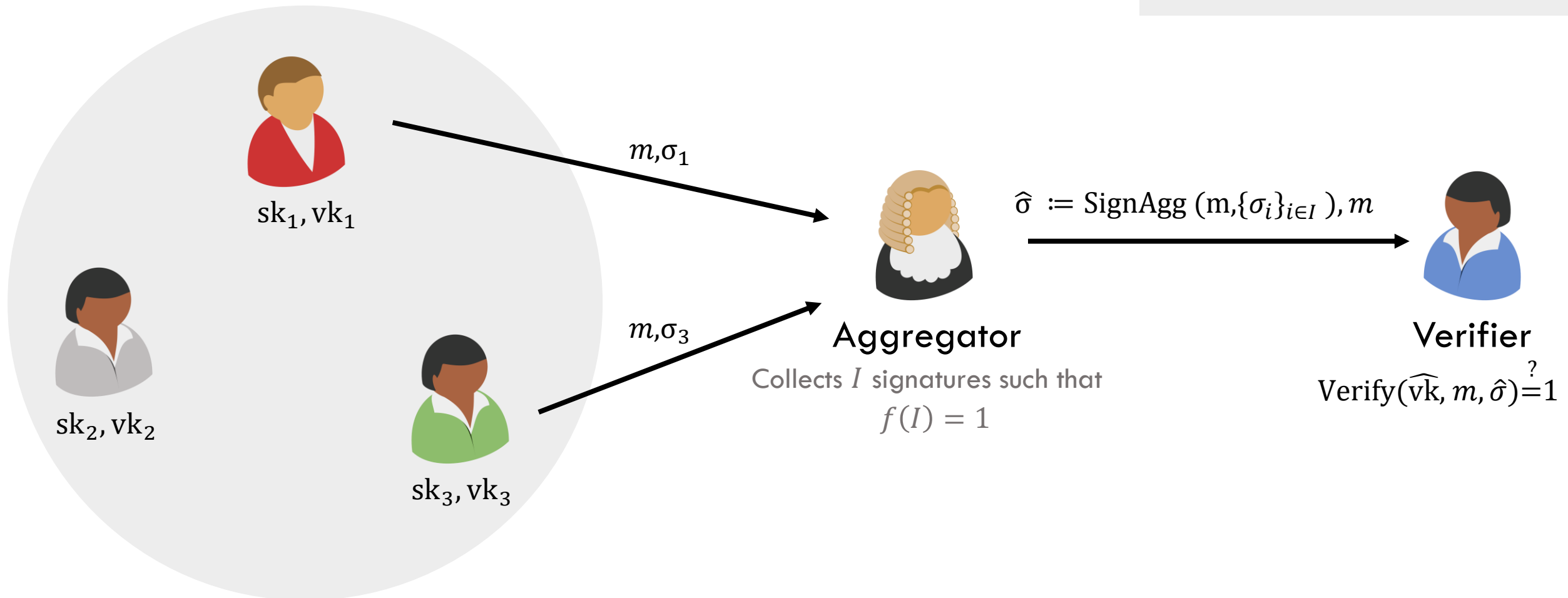
Each signer can use a different Signature scheme.

# Monotone Policy Aggregate Signatures



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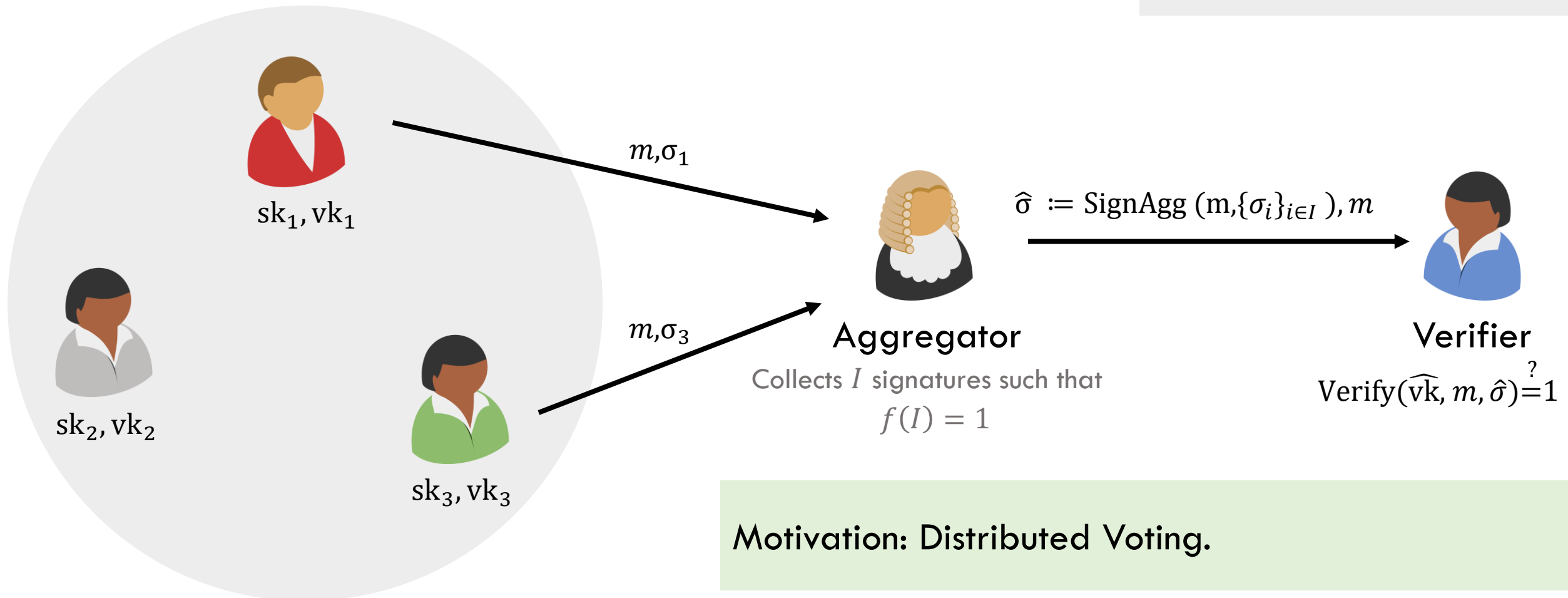
$$\widehat{vk} := \text{KeyAgg}(vk_1, \dots, vk_n)$$





# Monotone Policy Aggregate Signatures

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Motivation: Distributed Voting.

# Security Monotone Policy Aggregate Signatures

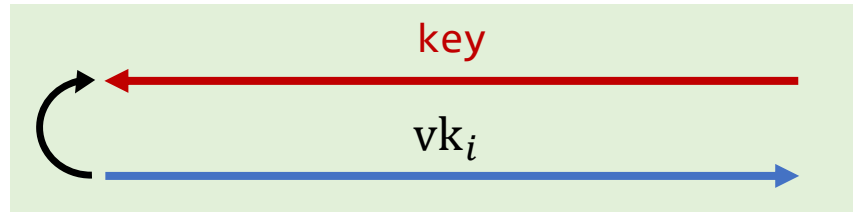
Challenger



# Security Monotone Policy Aggregate Signatures

Challenger

$sk_i, vk_i \leftarrow \text{KeyGen}()$

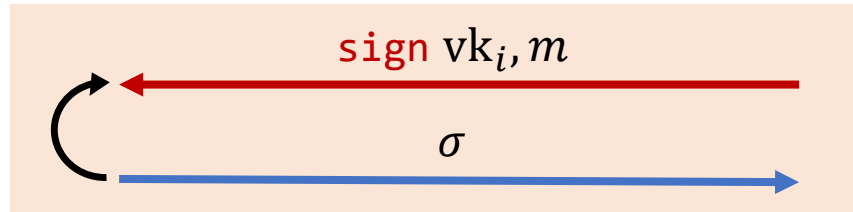
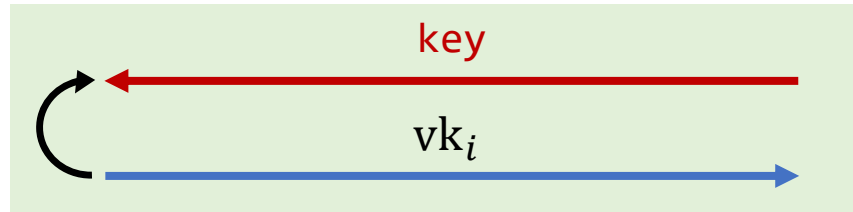


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$sk_i, vk_i \leftarrow \text{KeyGen}()$

$\sigma \leftarrow \text{Sign}(sk_i, m)$

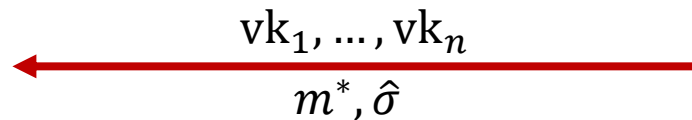
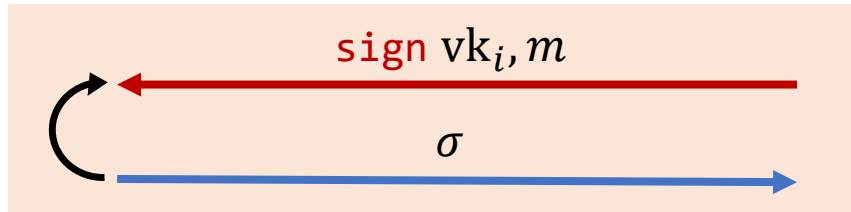
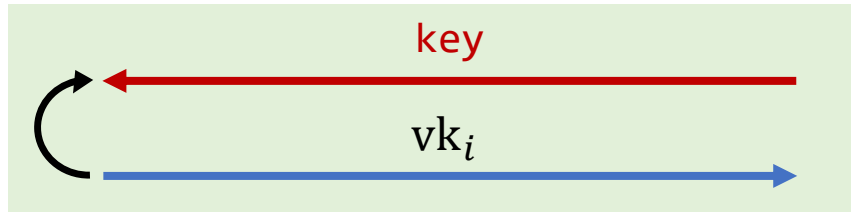


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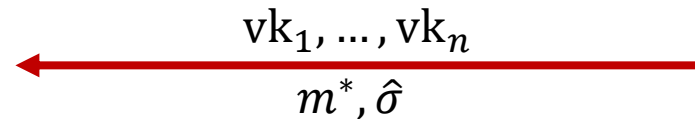
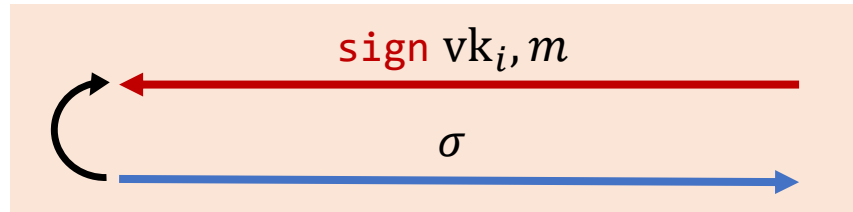
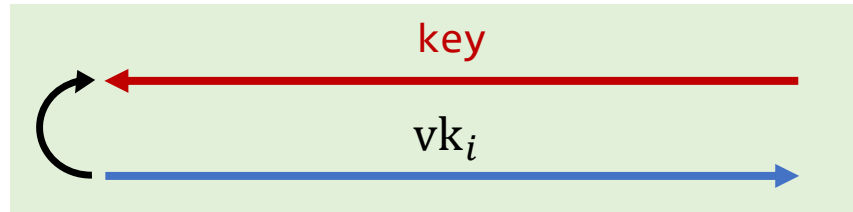
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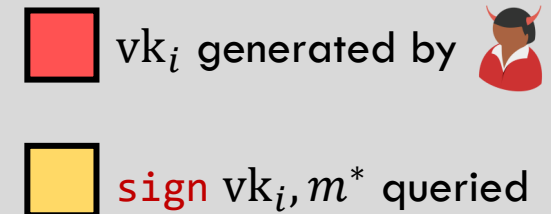
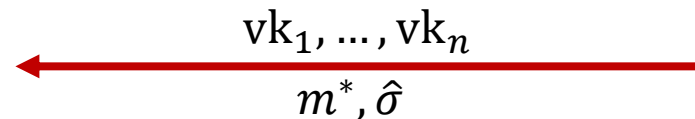
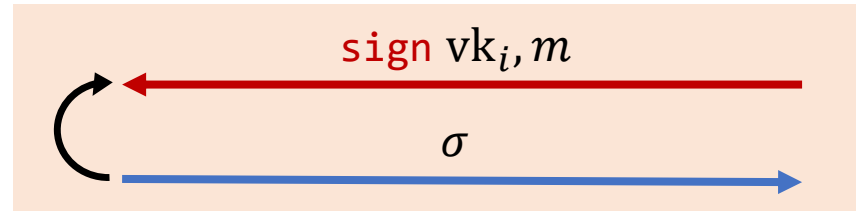
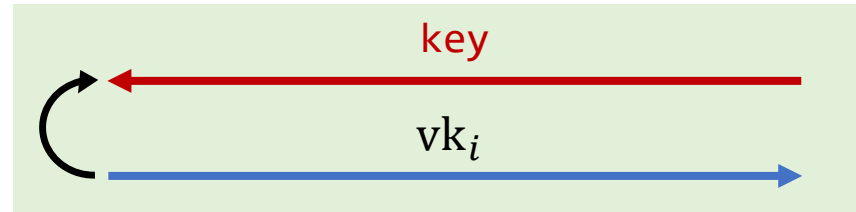
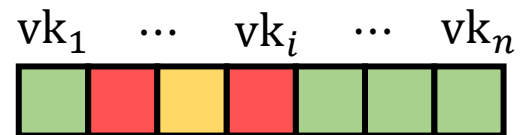
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# Security Monotone Policy Aggregate Signatures

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$sk_i, vk_i \leftarrow \text{KeyGen}()$

$\sigma \leftarrow \text{Sign}(sk_i, m)$




wins if

1.  $\text{Verify}(\widehat{vk}, m^*, \hat{\sigma}) = 1$
2.  $f(\text{0 1 1 1 0 0 0}) \neq 1$

key



$vk_i$  generated by 

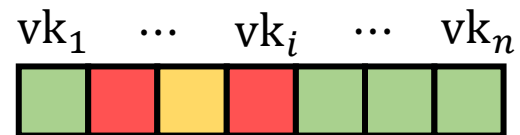


**sign**  $vk_i, m^*$  queried

$vk_1, \dots, vk_n$

$m^*, \hat{\sigma}$

$\widehat{vk} := \text{KeyAgg}(vk_1, \dots, vk_n)$





# Weak Security Monotone Policy Aggregate Signatures


Challenger

$sk_i, vk_i \leftarrow \text{KeyGen}()$


$\sigma \leftarrow \text{Sign}(sk_i, m)$



wins if

1.  $\text{Verify}(\widehat{vk}, m^*, \hat{\sigma}) = 1$
2.  $f(\text{0 1 0 1 0 0 0}) \neq 1$
3. No 




$vk_i$  generated by 



$\text{sign } vk_i, m^*$  queried

$\widehat{vk} := \text{KeyAgg}(vk_1, \dots, vk_n)$

$vk_1 \dots vk_i \dots vk_n$   


$vk_1, \dots, vk_n$

$m^*, \hat{\sigma}$



# Our Results

Non-interactive  
batch arguments.



Monotone Policy Aggregate  
Signatures for *read-once*  $O(\log n)$   
*space monotone policies.*

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BARGs

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BARGs



Monotone Policy Aggregate  
Signatures for *read-once*  $O(\log n)$   
*space monotone policies*.

Examples: threshold, weighted threshold.

Aggregation time in threshold grows with  
threshold  $t$  (not  $n$ ).

# Construction of BARGs

LWE

[C-Jain-Jin'21]

DLIN

[Waters-Wu'22]

Sub-exp DDH

+ QR

[C-Jain-Jin'21 $\alpha$ , Hulett-Jawale-Khurana-Srinivasan'22]

Sub-exp DDH

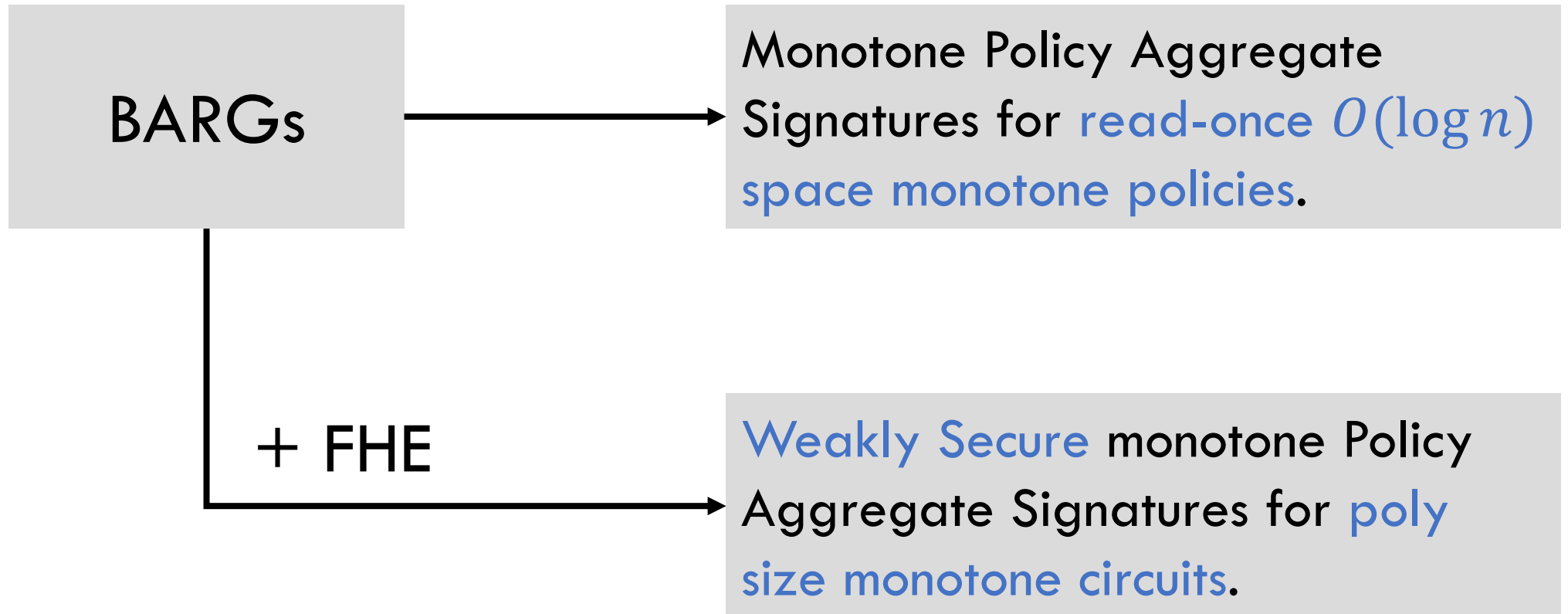
[C-Garg-Jain-Jin-Zhang'23]



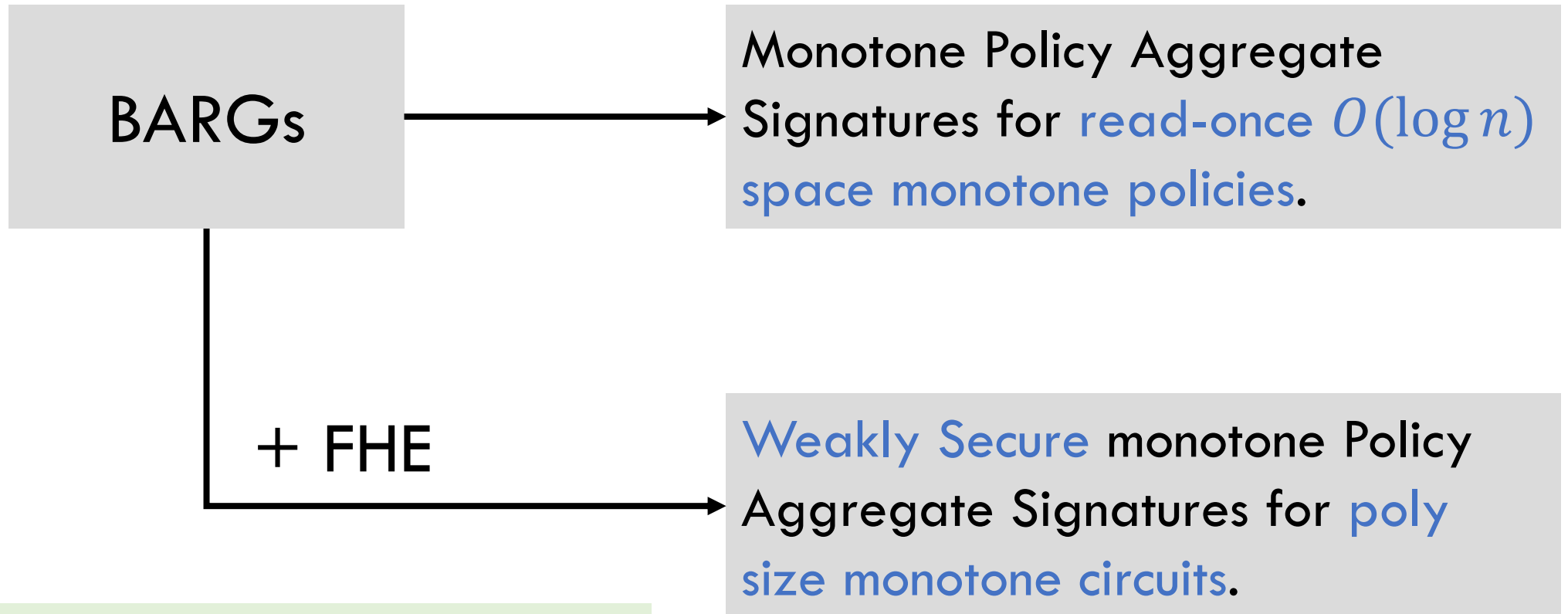
BARGs

QR – Quadratic residuosity, LWE – Learning with Error, DDH – Decisional Diffie-Hellman, DLIN – Decisional Linear Assumption over Bilinear Groups.

# Our Results



# Our Results



Recently [Nassar-Waters-Wu'24] construct static secure monotone Policy Aggregate Signatures for *poly size monotone circuits* challenge message is declared first.

# Non-Interactive Batch Arguments (or BARGs)

CRS



$x_1, \dots, x_n$   
 $w_1, \dots, w_n$



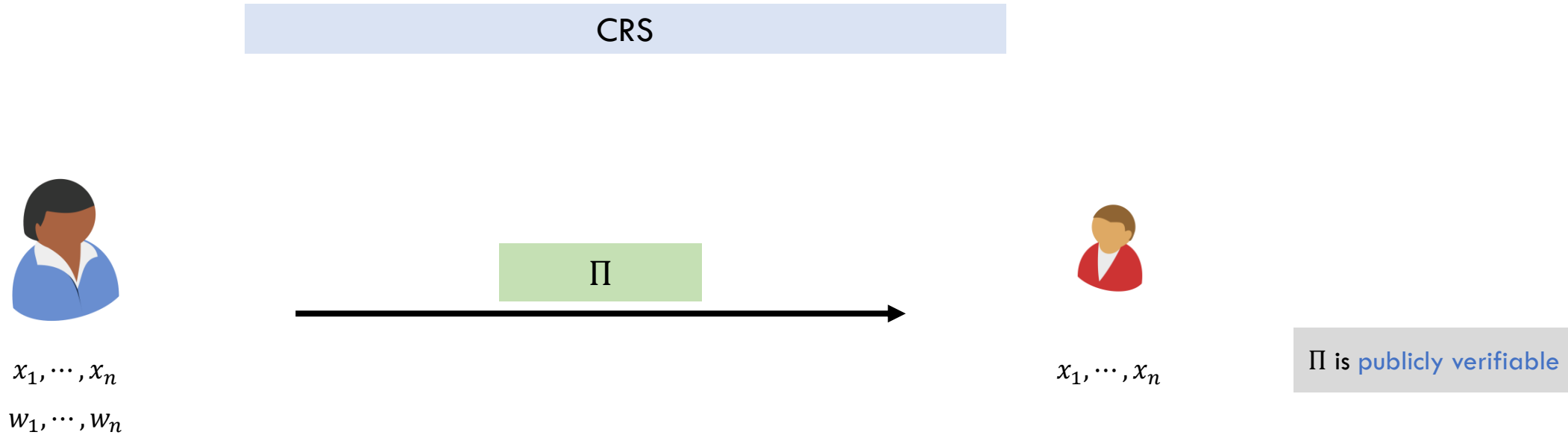
$x_1, \dots, x_n$

$$L = \{x \mid \exists w \text{ s.t. } R_L(x, w) = 1\}$$

$$\forall i \in [n], x_i \in L$$



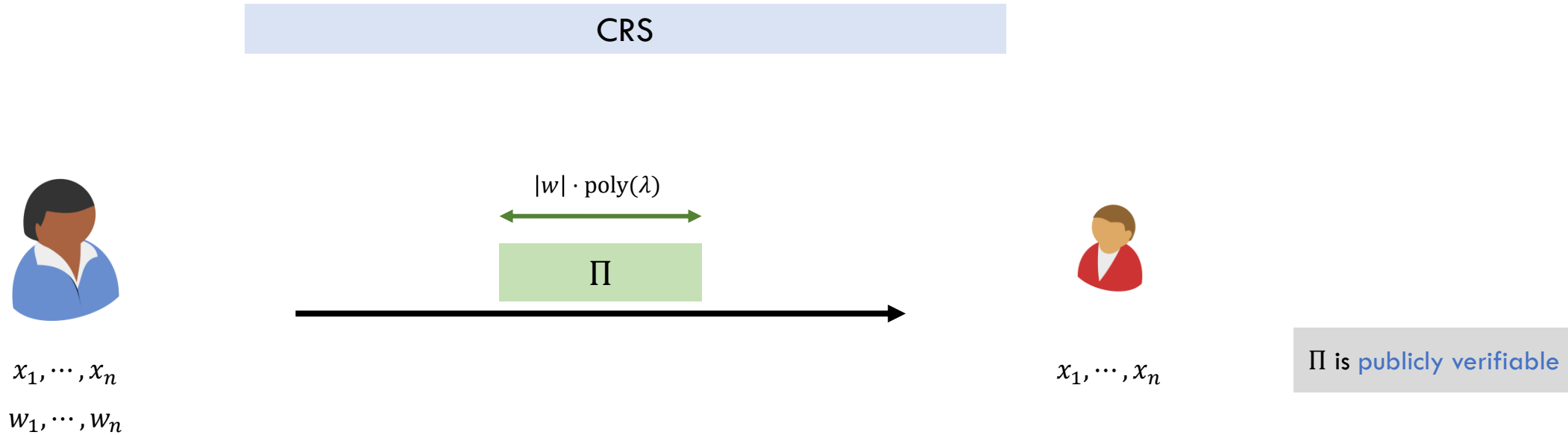
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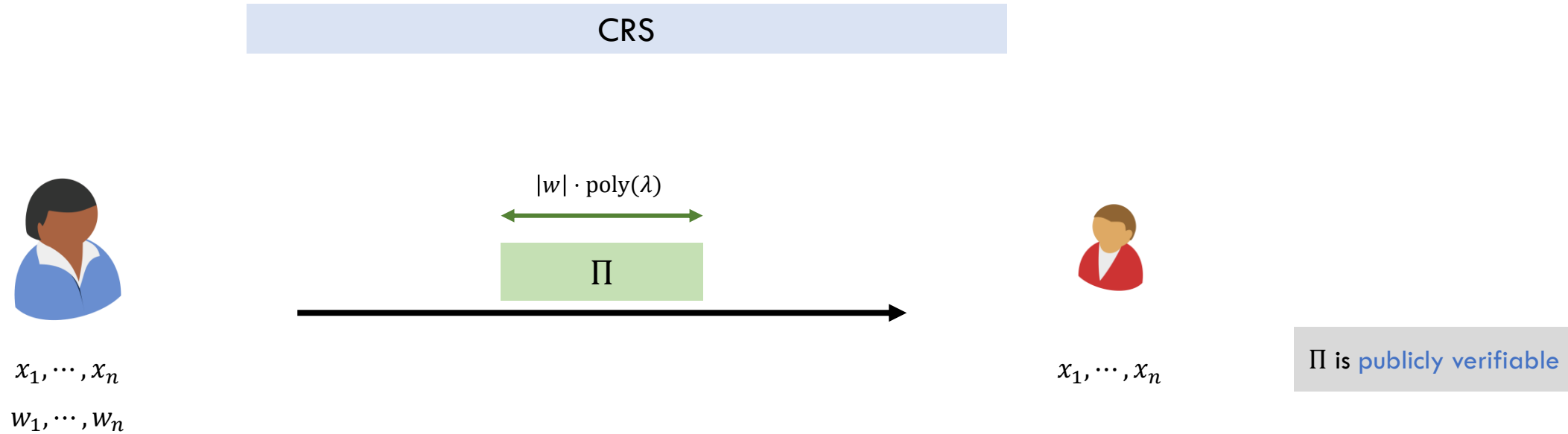
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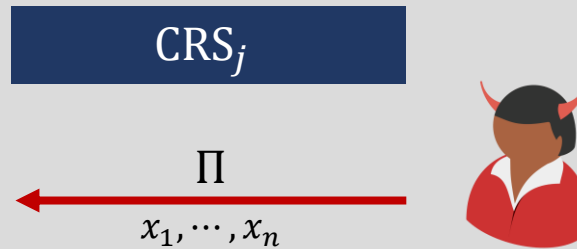
No PPT  can produce **accepting**  $\Pi$  if

$$\exists i^* \in [n], x_{i^*} \notin L$$

# somewhere extractable BARGs (seBARGs)

[C-Jain-Jin'21]

$\text{CRS}_j, \text{td} \leftarrow \text{Setup}(j)$

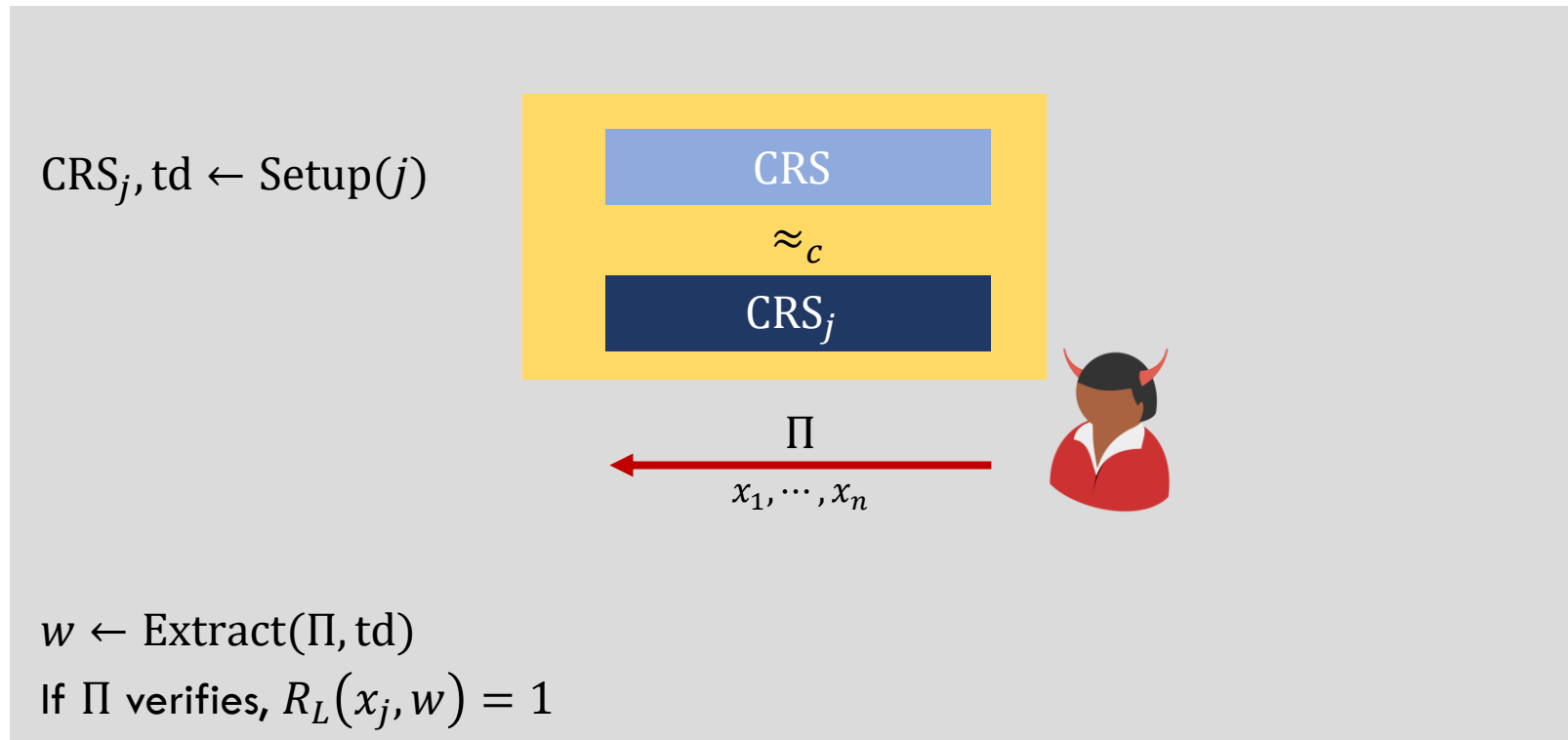


$w \leftarrow \text{Extract}(\Pi, \text{td})$

If  $\Pi$  verifies,  $R_L(x_j, w) = 1$

# somewhere extractable BARGs (seBARGs)

[C-Jain-Jin'21]



$CRS_j$  hides  $j$

# Aggregate Signatures from seBARGs

[Waters-Wu'22]

# Aggregate Signatures from seBARGs

[Waters-Wu'22]

KeyAgg (vk<sub>1</sub>, ..., vk<sub>n</sub>)

$$\widehat{vk} = \text{MerkleRoot}(vk_1, \dots, vk_n)$$

# Aggregate Signatures from seBARGs

[Waters-Wu'22]

KeyAgg ( $vk_1, \dots, vk_n$ )

$$\widehat{vk} = \text{MerkleRoot}(vk_1, \dots, vk_n)$$

SignAgg ( $m, \sigma_1, \dots, \sigma_n$ )

$$x_i = i, m, \widehat{vk}$$

$$w_i = vk_i, \sigma_i, \pi_i$$

$$\hat{\sigma} = \text{BARG}(x_1, \dots, x_n, w_1, \dots, w_n)$$

Statement:  $i, m, \widehat{vk}$

Witness:  $vk_i, \sigma_i, \pi_i$

s.t.  $\pi_i$  is a valid opening to  $vk_i$  AND  $\text{Verify}(vk_i, m, \sigma_i) = 1$



# Aggregate Signatures from seBARGs

[Waters-Wu'22]

KeyAgg ( $vk_1, \dots, vk_n$ )

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Statement:  $i, m, \widehat{vk}$

Witness:  $vk_i, \sigma_i, \pi_i$

The  $n$  statements have a **succinct representation** -  $m, \widehat{vk}, n$  and thus the BARG proof can be **verified quickly** without dependence on  $n$ .

s.t.  $\pi_i$  is a valid opening to  $vk_i$  AND  $\text{Verify}(vk_i, m, \sigma_i) = 1$

# Security Monotone Policy Aggregate Signatures



Challenger


CRS

key

sign

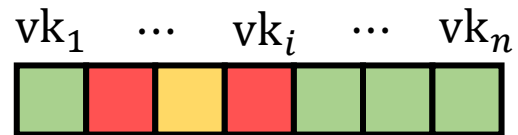


  $vk_i$  generated by 

  $sign\ vk_i, m^*$  queried

$vk_1, \dots, vk_n$   
 $m^*, \hat{\sigma}$

$\widehat{vk} := \text{KeyAgg}(vk_1, \dots, vk_n)$



wins if

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# Security ~~Monotone Policy~~ Aggregate Signatures

Challenger

CRS



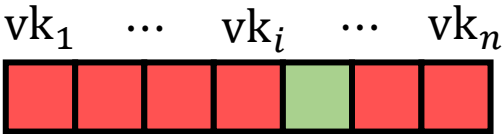
key


sign

  $vk_i$  generated by 

$vk_1, \dots, vk_n$   
 $m^*, \hat{\sigma}$

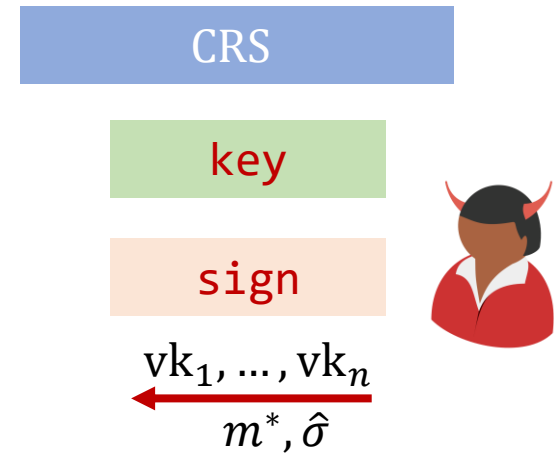
$$\widehat{vk} := \text{KeyAgg}(vk_1, \dots, vk_n)$$



 wins if

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2.  $f(\text{1 1 1 1 0 1 1}) \neq 1$

# Security Proof Aggregate Signatures



# Security Proof Aggregate Signatures

Signature Scheme  
Challenger

$i^* \leftarrow [n]$   
 $\text{CRS}_{i^*}, \text{td} \leftarrow \text{Setup}(i^*)$

$\text{CRS}_{i^*}$

key

sign



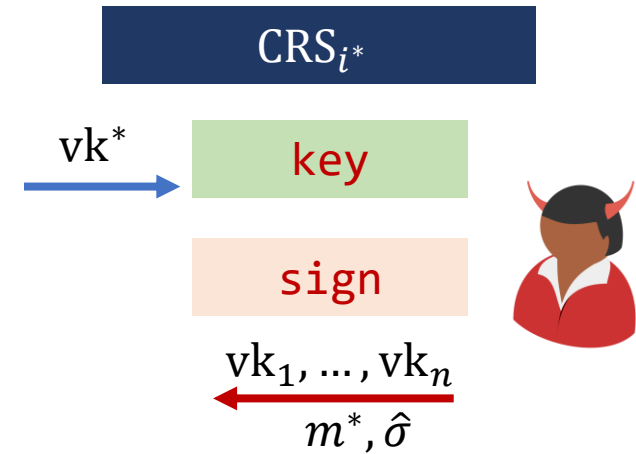
$\text{vk}_1, \dots, \text{vk}_n$   
 $\longleftarrow m^*, \hat{\sigma}$

# Security Proof Aggregate Signatures

Signature Scheme  
Challenger

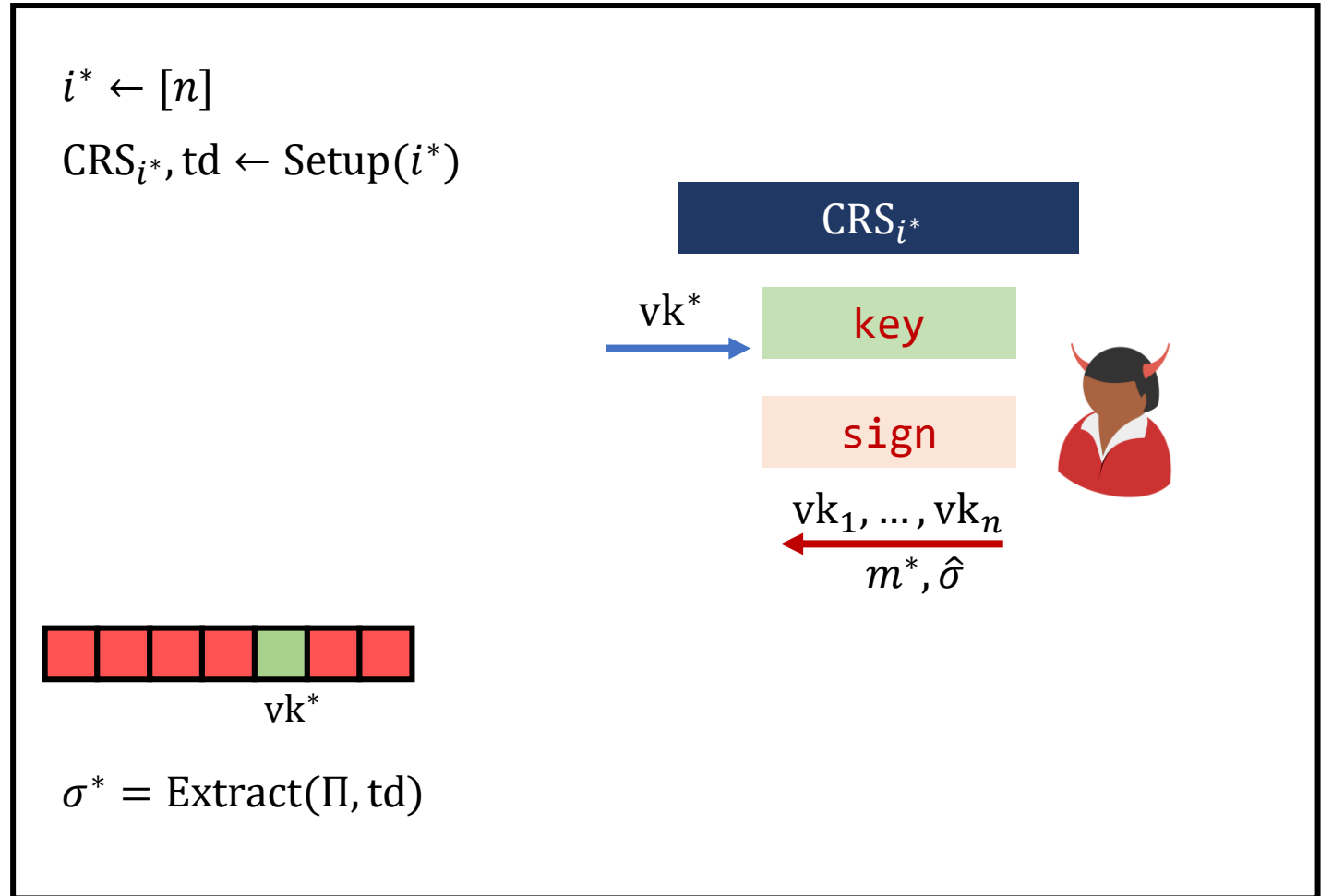
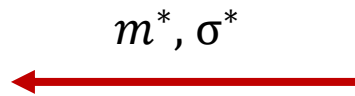


$i^* \leftarrow [n]$   
 $CRS_{i^*}, td \leftarrow Setup(i^*)$



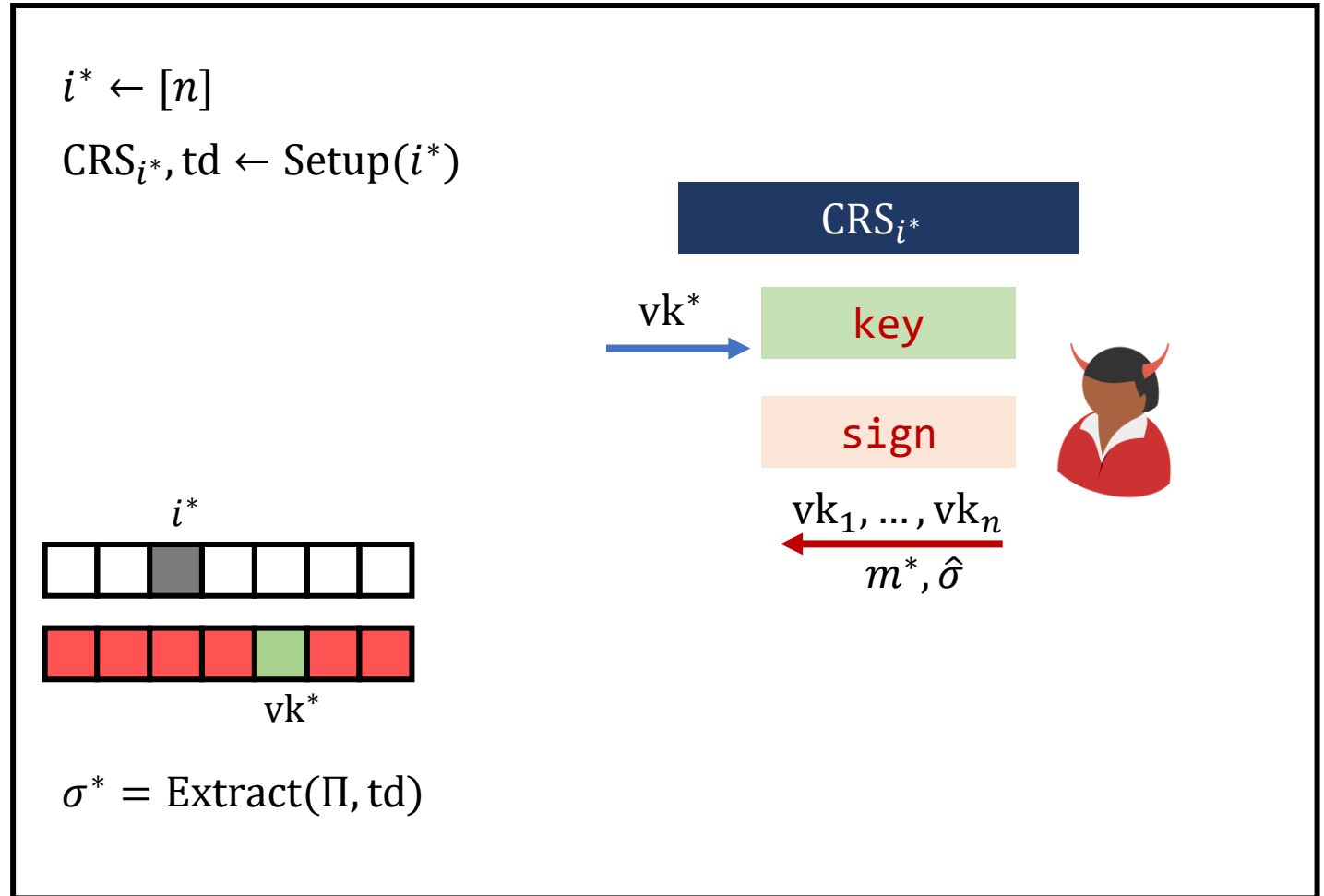
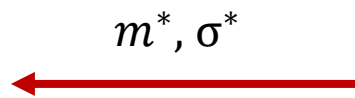
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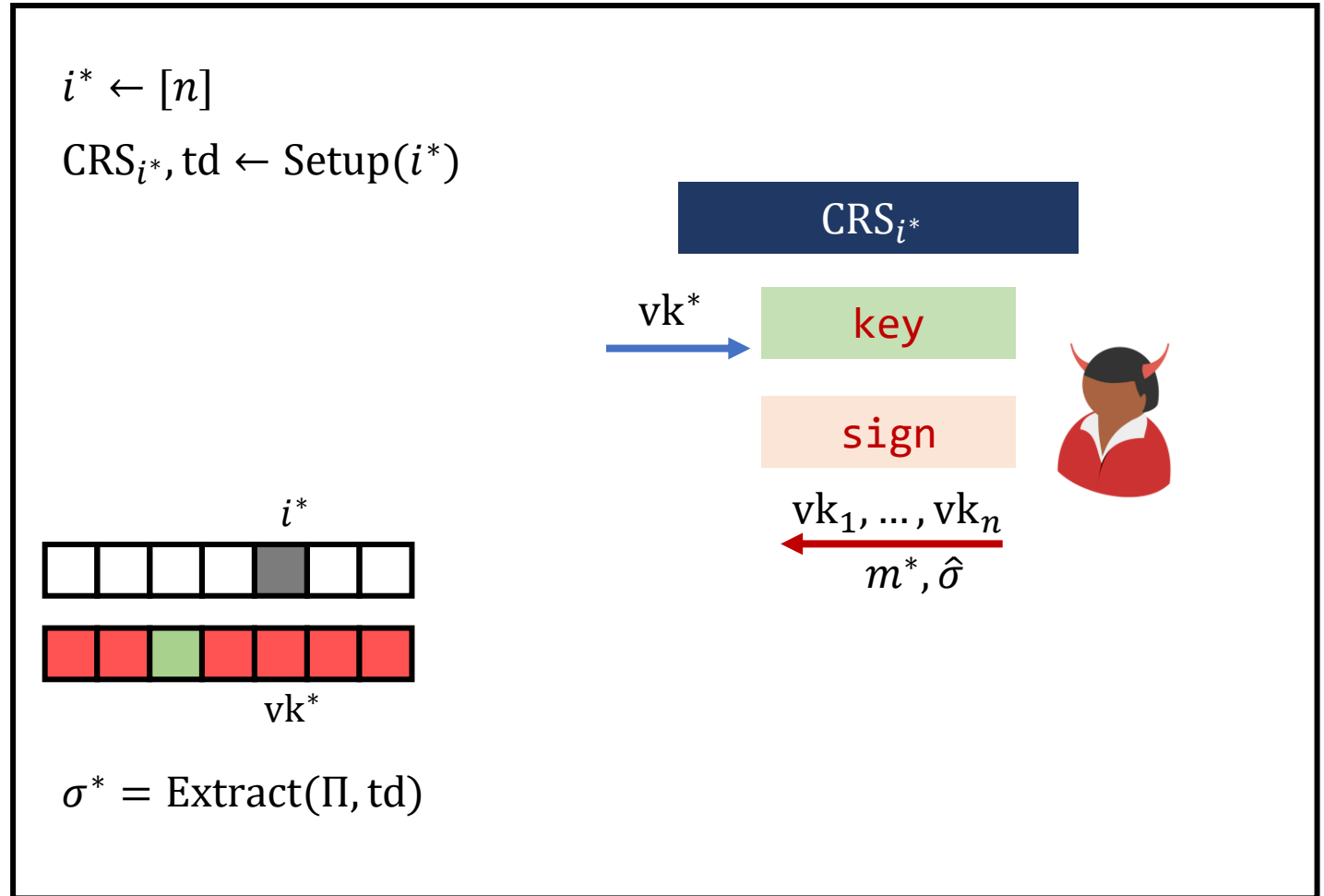
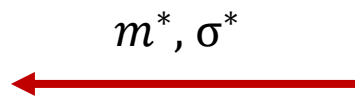
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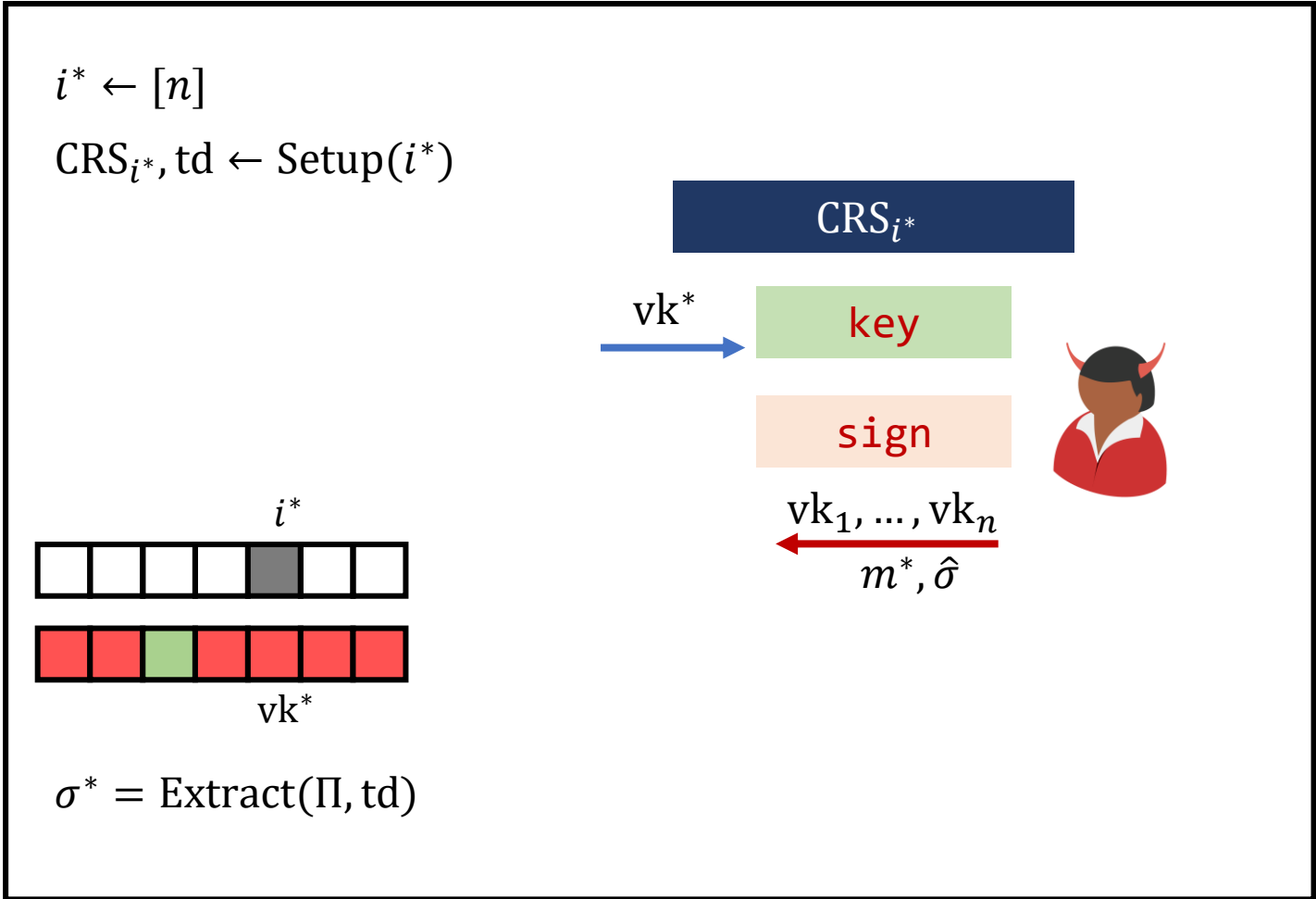


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Signature Scheme  
Challenger




Can  keep avoiding  $i^*$ ?




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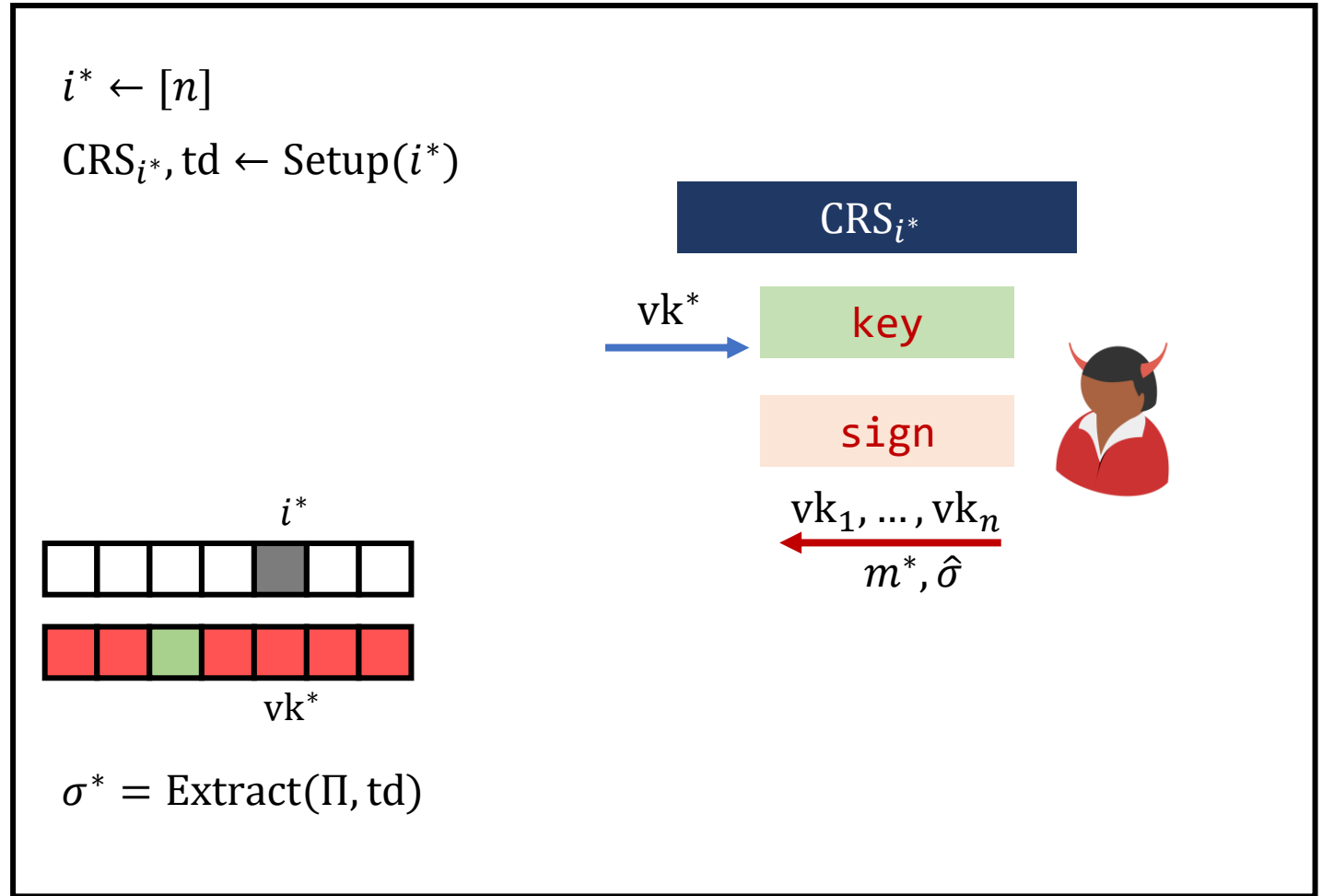
Signature Scheme  
Challenger



Can  keep avoiding  $i^*$ ?

No! Avoidance can be checked from  $i^*$  and 

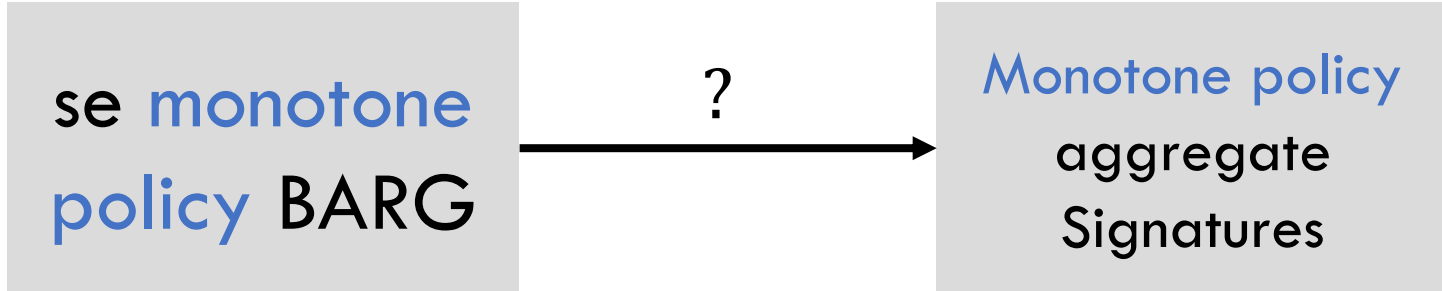
Would break CRS indistinguishability.



seBARG



Aggregate  
Signatures



# Monotone Policy BARGs (pBARGs)

Monotone Policy  
 $f: \{0,1\}^n \rightarrow \{0,1\}$



$x_1, \dots, x_n$   
 $\{w_i\}_{i \in I}$

CRS

$|w| \cdot \text{poly}(\lambda)$



$\Pi$



$x_1, \dots, x_n$

$\Pi$  is publicly verifiable

$$L = \{x \mid \exists w \text{ s.t. } R_L(x, w) = 1\}$$

$$f(b_1, \dots, b_n) = 1 \text{ where } b_i = 1 \text{ iff } i \in I$$

# somewhere extractable pBARG

[Brakerski-Brodsky-Kalai-Lombardi-Paneth'23]

Monotone Policy  
 $f: \{0,1\}^n \rightarrow \{0,1\}$

Example:

For  $t$ -out-of- $n$  threshold, **any** set  $J$  of size  $n - t + 1$  is necessary.

$J \subset [n]$  is **necessary** for  $f$  if  $f(b_1, \dots, b_n) = 0$  where  $b_i = 0$  iff  $i \in J$

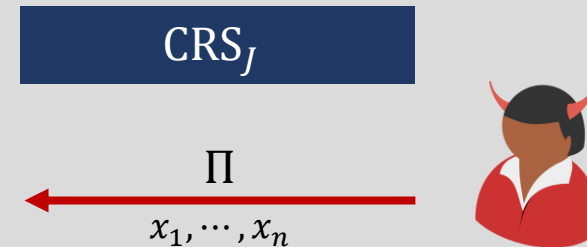
# somewhere extractable pBARG

[Brakerski-Brodsky-Kalai-Lombardi-Paneth'23]

Monotone Policy  
 $f: \{0,1\}^n \rightarrow \{0,1\}$

For any necessary set  $J$

$\text{CRS}_J, \text{td} \leftarrow \text{Setup}(J)$



$w \leftarrow \text{Extract}(\Pi, \text{td})$

If  $\Pi$  verifies,  $R_L(x_j, w) = 1$  for some  $j \in J$

$J \subset [n]$  is **necessary** for  $f$  if  $f(b_1, \dots, b_n) = 0$  where  $b_i = 0$  iff  $i \in J$



# somewhere extractable pBARG

[Brakerski-Brodsky-Kalai-Lombardi-Paneth'23]

Monotone Policy  
 $f: \{0,1\}^n \rightarrow \{0,1\}$

For any necessary set  $J$

$\text{CRS}_J, \text{td} \leftarrow \text{Setup}(J)$

CRS

$\approx_c$

$\text{CRS}_J$

$\Pi$

$x_1, \dots, x_n$



$w \leftarrow \text{Extract}(\Pi, \text{td})$

If  $\Pi$  verifies,  $R_L(x_j, w) = 1$  for some  $j \in J$

$J \subset [n]$  is **necessary** for  $f$  if  $f(b_1, \dots, b_n) = 0$  where  $b_i = 0$  iff  $i \in J$

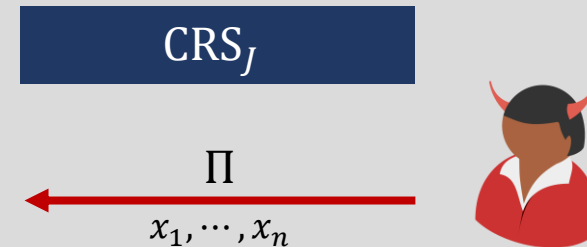
# somewhere extractable pBARG

[Brakerski-Brodsky-Kalai-Lombardi-Paneth'23]

Monotone Policy  
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$J \subset [n]$  is **necessary** for  $f$  if  $f(b_1, \dots, b_n) = 0$  where  $b_i = 0$  iff  $i \in J$

# Aggregate Signatures from pBARGs

KeyAgg ( $vk_1, \dots, vk_n$ )

$$\widehat{vk} = \text{MerkleRoot}(vk_1, \dots, vk_n)$$

SignAgg ( $m, \{\sigma_i\}_{i \in I}$ )

$$x_i = i, m, \widehat{vk}$$

$$w_i = vk_i, \sigma_i, \pi_i \text{ or } \perp \text{ if } i \notin I$$

$$\hat{\sigma} = \text{pBARG}(x_1, \dots, x_n, w_1, \dots, w_n)$$

Statement:  $i, m, \widehat{vk}$

Witness:  $vk_i, \sigma_i, \pi_i$

s.t.  $\pi_i$  is a valid opening to  $vk_i$  AND  $\text{Verify}(vk_i, m, \sigma_i) = 1$

# Security Monotone Policy Aggregate Signatures



Challenger


CRS

key

sign

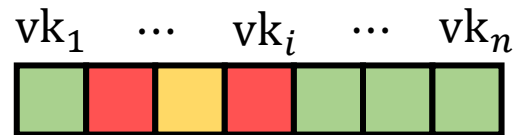


  $vk_i$  generated by 

  $sign\ vk_i, m^*$  queried

$vk_1, \dots, vk_n$   
 $m^*, \hat{\sigma}$

$\widehat{vk} := \text{KeyAgg}(vk_1, \dots, vk_n)$



wins if

1.  $\text{Verify}(\widehat{vk}, m^*, \hat{\sigma}) = 1$
2.  $f(\text{0 1 1 1 0 0 0}) \neq 1$

# Security Monotone Policy Aggregate Signatures



Challenger


CRS

key

sign



  $vk_i$  generated by 

  $sign\ vk_i, m^*$  queried

$vk_1, \dots, vk_n$   
 $m^*, \hat{\sigma}$

$\widehat{vk} := \text{KeyAgg}(vk_1, \dots, vk_n)$

$J =$  



wins if

1.  $\text{Verify}(\widehat{vk}, m^*, \hat{\sigma}) = 1$
2.  $f(\text{0 1 1 1 0 0 0}) \neq 1$

# Security Monotone Policy Aggregate Signatures



Challenger


CRS

key

sign



  $vk_i$  generated by 

  $sign\ vk_i, m^*$  queried

$vk_1, \dots, vk_n$   
 $m^*, \hat{\sigma}$

$\widehat{vk} := \text{KeyAgg}(vk_1, \dots, vk_n)$

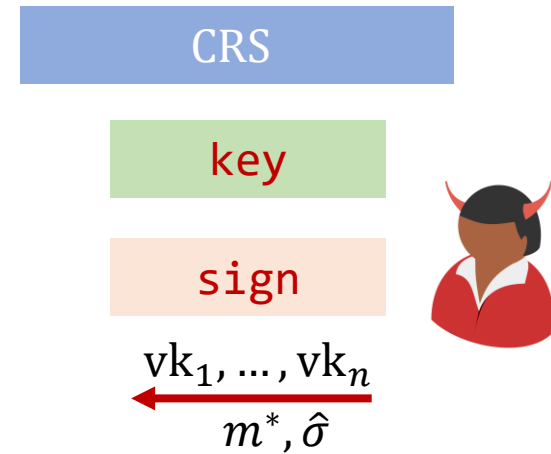
$J =$  



wins if

1.  $\text{Verify}(\widehat{vk}, m^*, \hat{\sigma}) = 1$
2.  $J$  is necessary.

# Security Proof Monotone Policy Aggregate Signatures



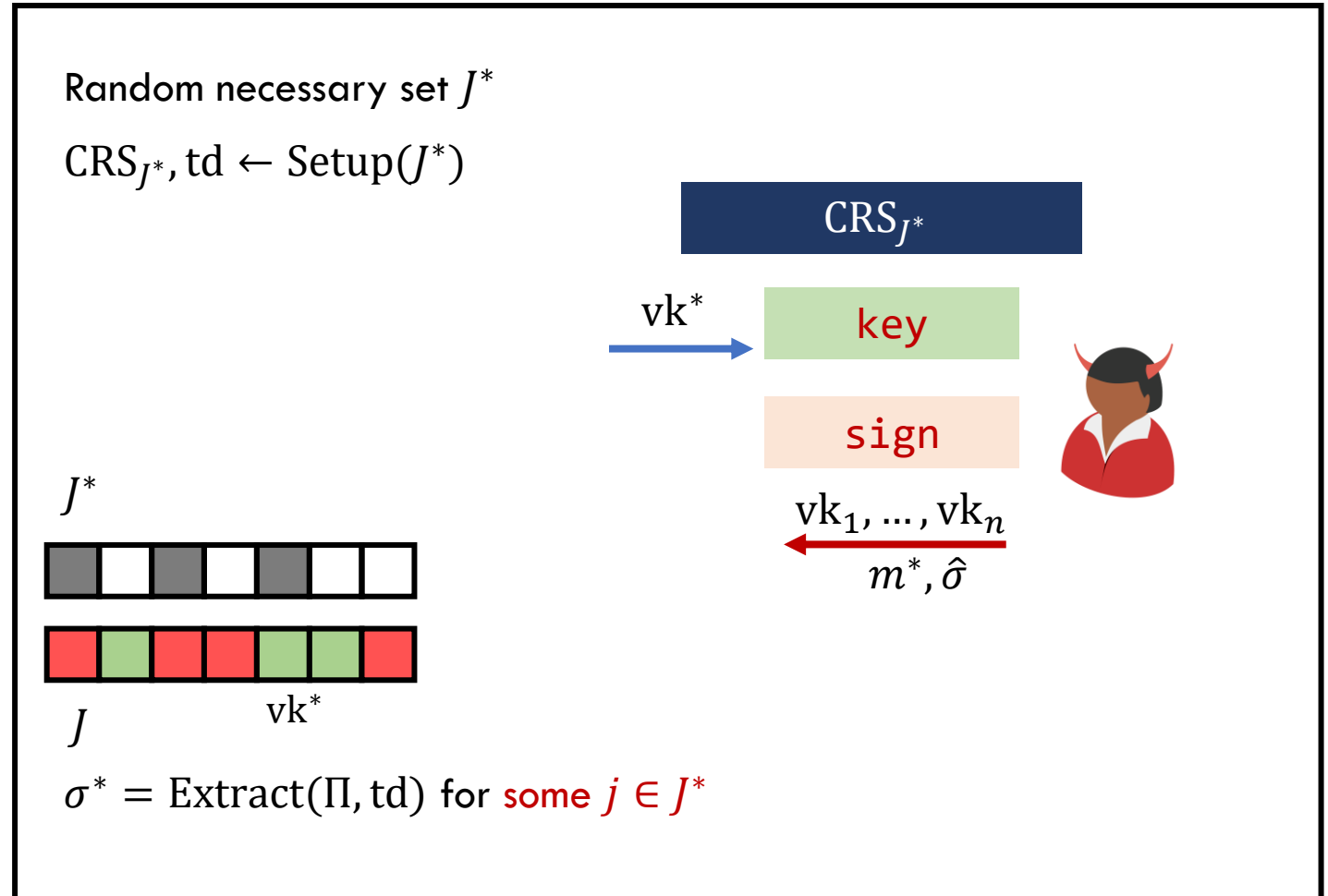

# Security Proof Monotone Policy Aggregate Signatures

Signature Scheme  
Challenger

$vk^*$



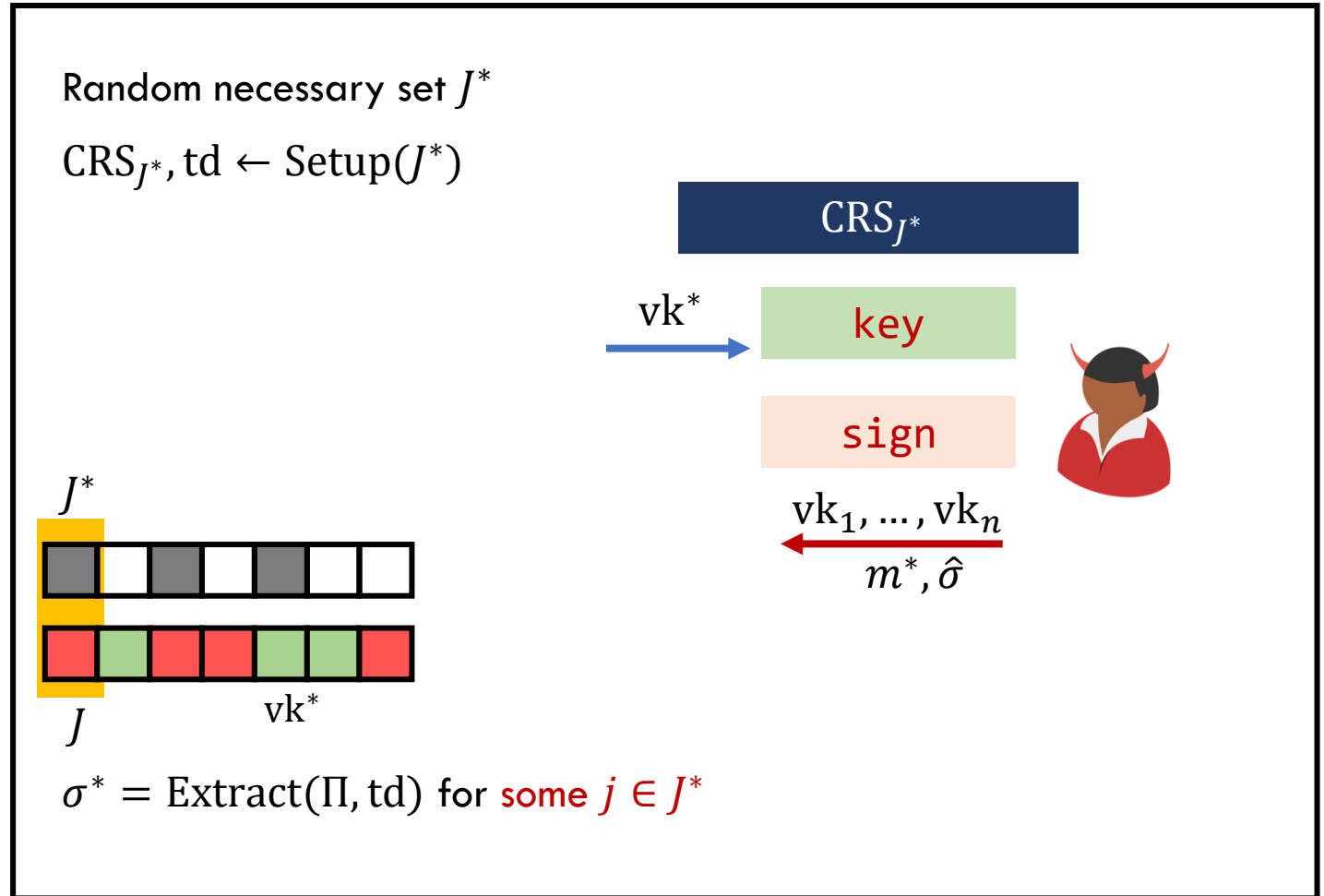
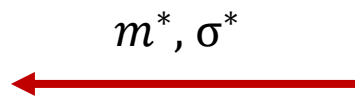
$m^*, \sigma^*$





# Security Proof Monotone Policy Aggregate Signatures

Signature Scheme  
Challenger

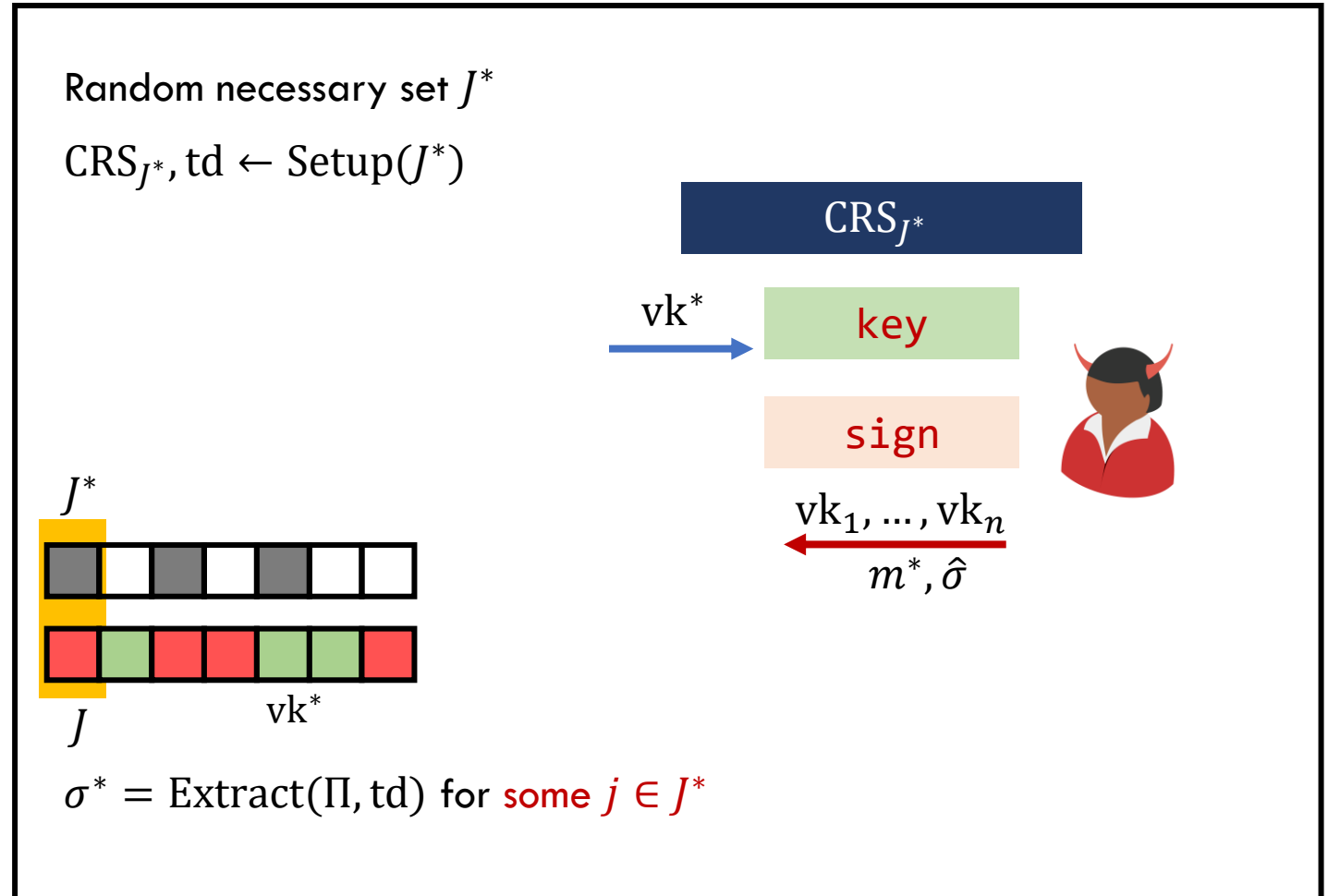
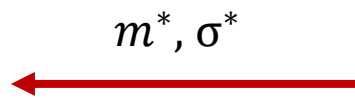


# Security Proof Monotone Policy Aggregate Signatures

Signature Scheme  
Challenger



Fails unless guessed  
 $J^*$  is the same as  
adaptively chosen  $J$ .



# Security Proof Monotone Policy Aggregate Signatures

Signature Scheme  
Challenger

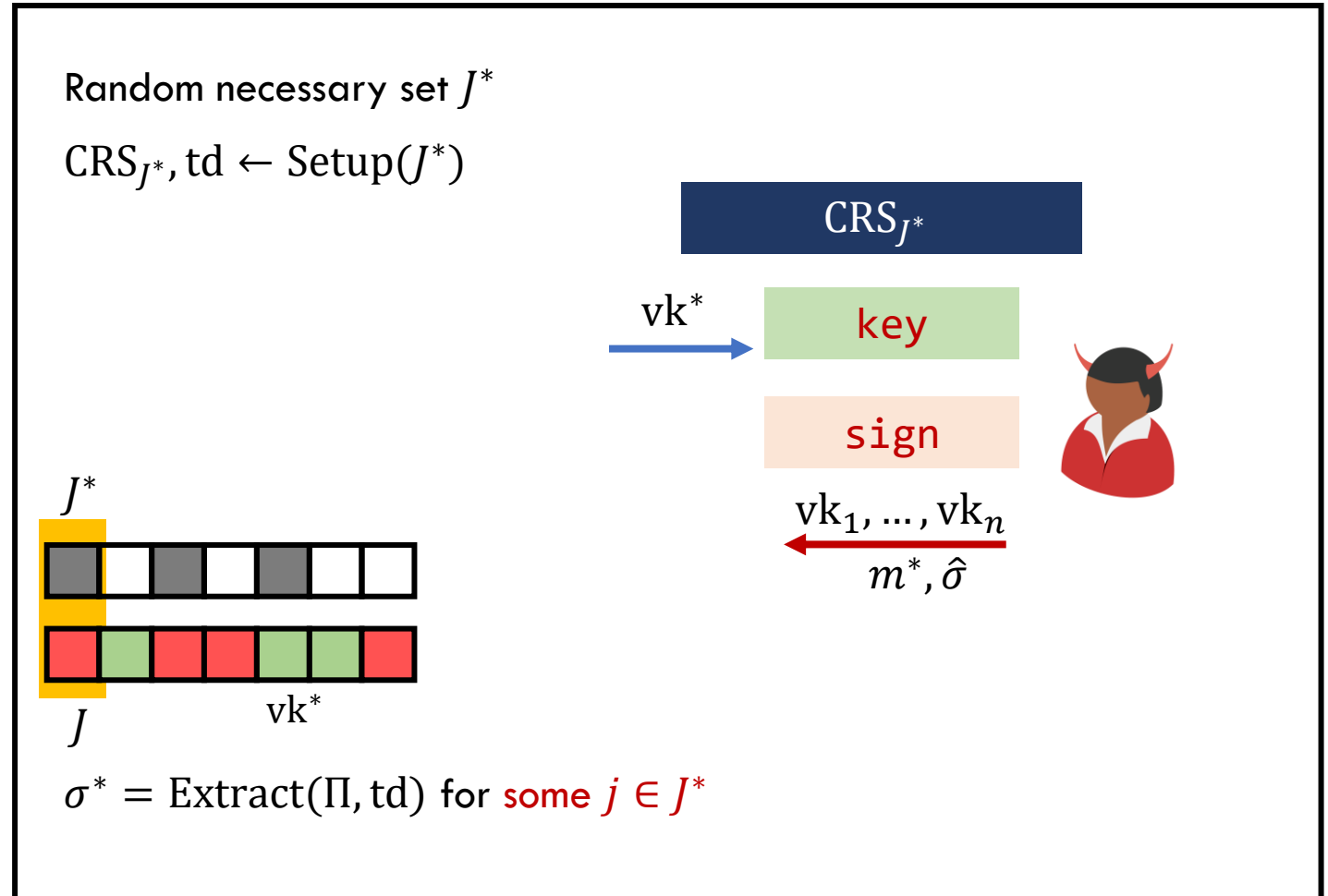
$vk^*$



Fails unless guessed  
 $J^*$  is the same as  
adaptively chosen  $J$ .

May be exponentially  
many  $J^*$ .

$m^*, \sigma^*$



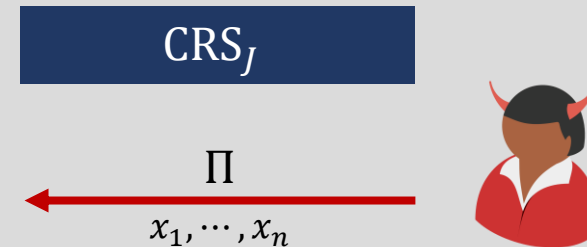
# somewhere extractable pBARG

[Brakerski-Brodsky-Kalai-Lombardi-Paneth'23]

Monotone Policy  
 $f: \{0,1\}^n \rightarrow \{0,1\}$

For any necessary set  $J$

$\text{CRS}_J, \text{td} \leftarrow \text{Setup}(J)$



$w \leftarrow \text{Extract}(\Pi, \text{td})$

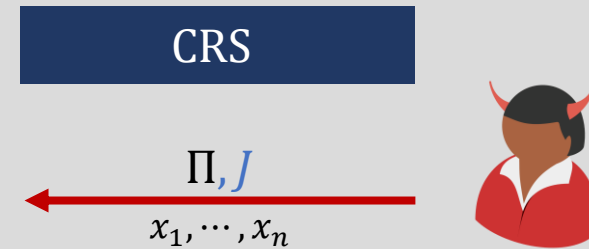
If  $\Pi$  verifies,  $R_L(x_j, w) = 1$  for some  $j \in J$

$J \subset [n]$  is **necessary** for  $f$  if  $f(b_1, \dots, b_n) = 0$  where  $b_i = 0$  iff  $i \in J$

# pBARG with Adaptive Subset Extraction

Monotone Policy  
 $f: \{0,1\}^n \rightarrow \{0,1\}$

CRS, td  $\leftarrow$  Setup()



$w \leftarrow$  Extract( $\Pi$ , td)

If  $\Pi$  verifies,  $R_L(x_j, w) = 1$  for some  $j \in J$  with probability  $\approx 1/n$

$J \subset [n]$  is necessary for  $f$  if  $f(b_1, \dots, b_n) = 0$  where  $b_i = 0$  iff  $i \in J$

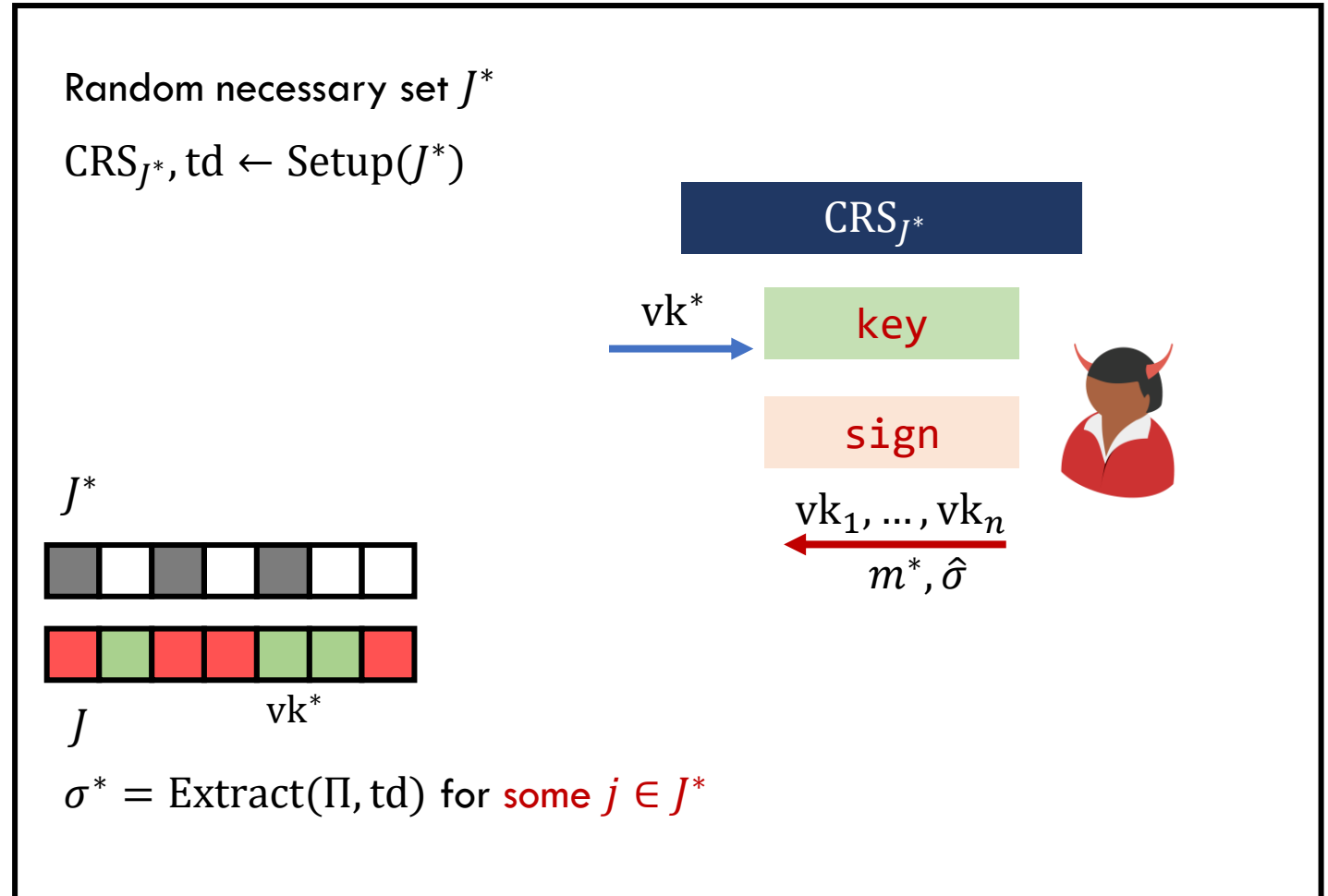

# Security Proof Monotone Policy Aggregate Signatures

Signature Scheme  
Challenger

$vk^*$



$m^*, \sigma^*$



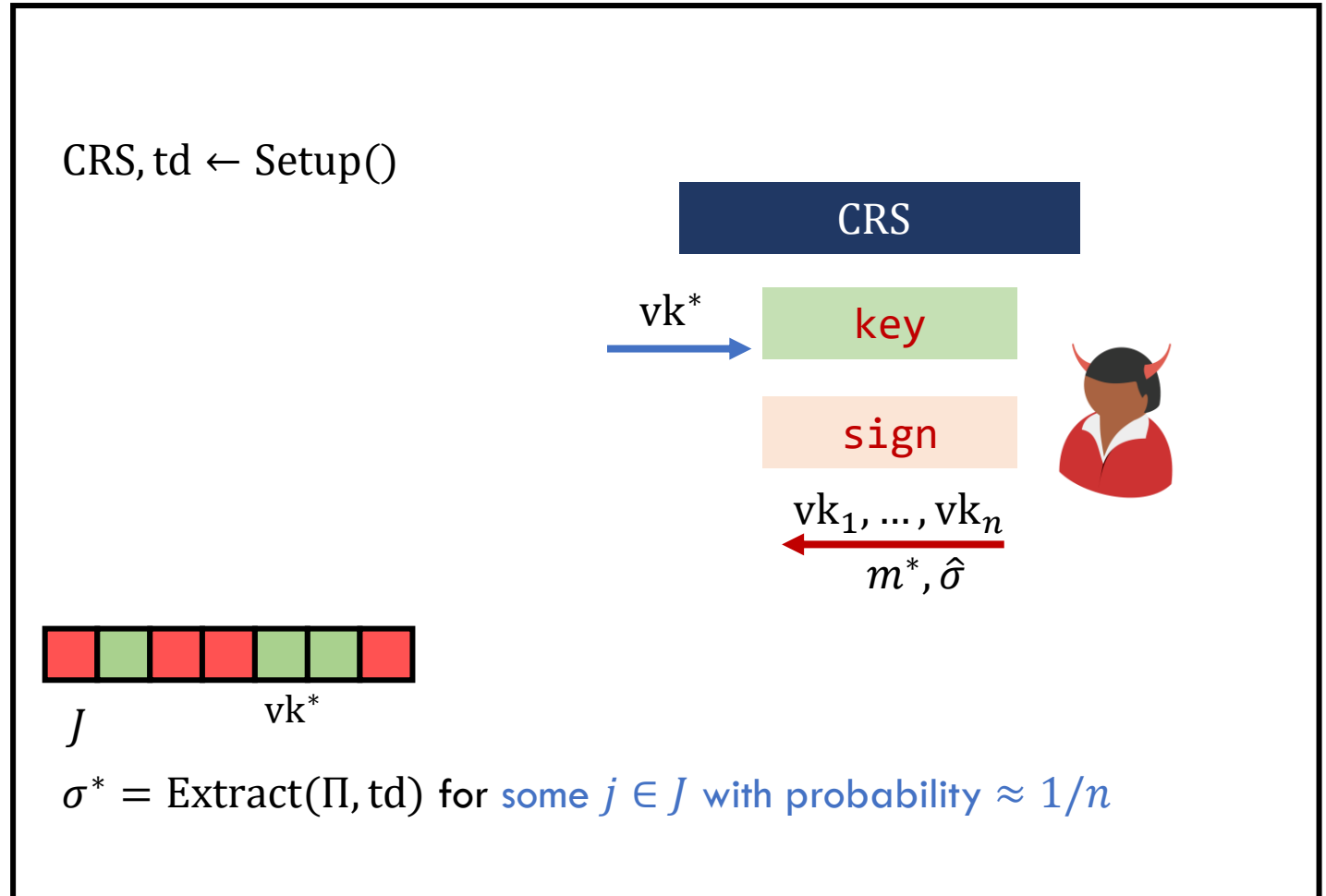

# Security Proof Monotone Policy Aggregate Signatures

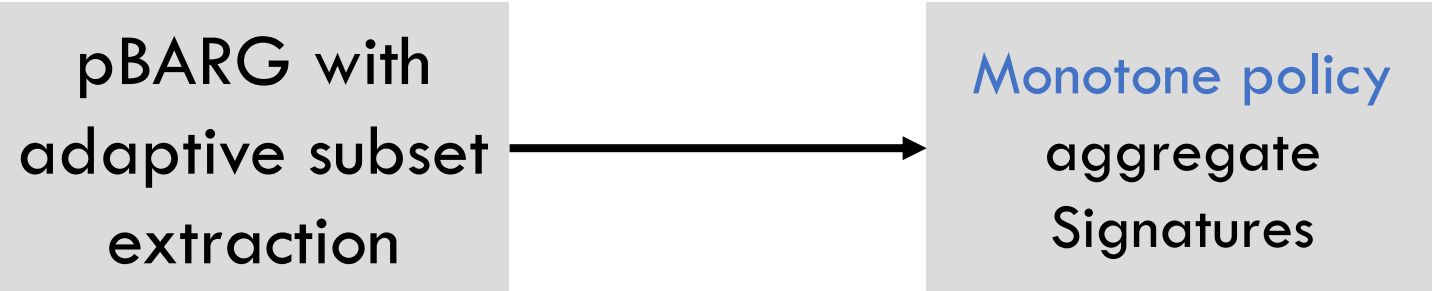
Signature Scheme  
Challenger

$vk^*$

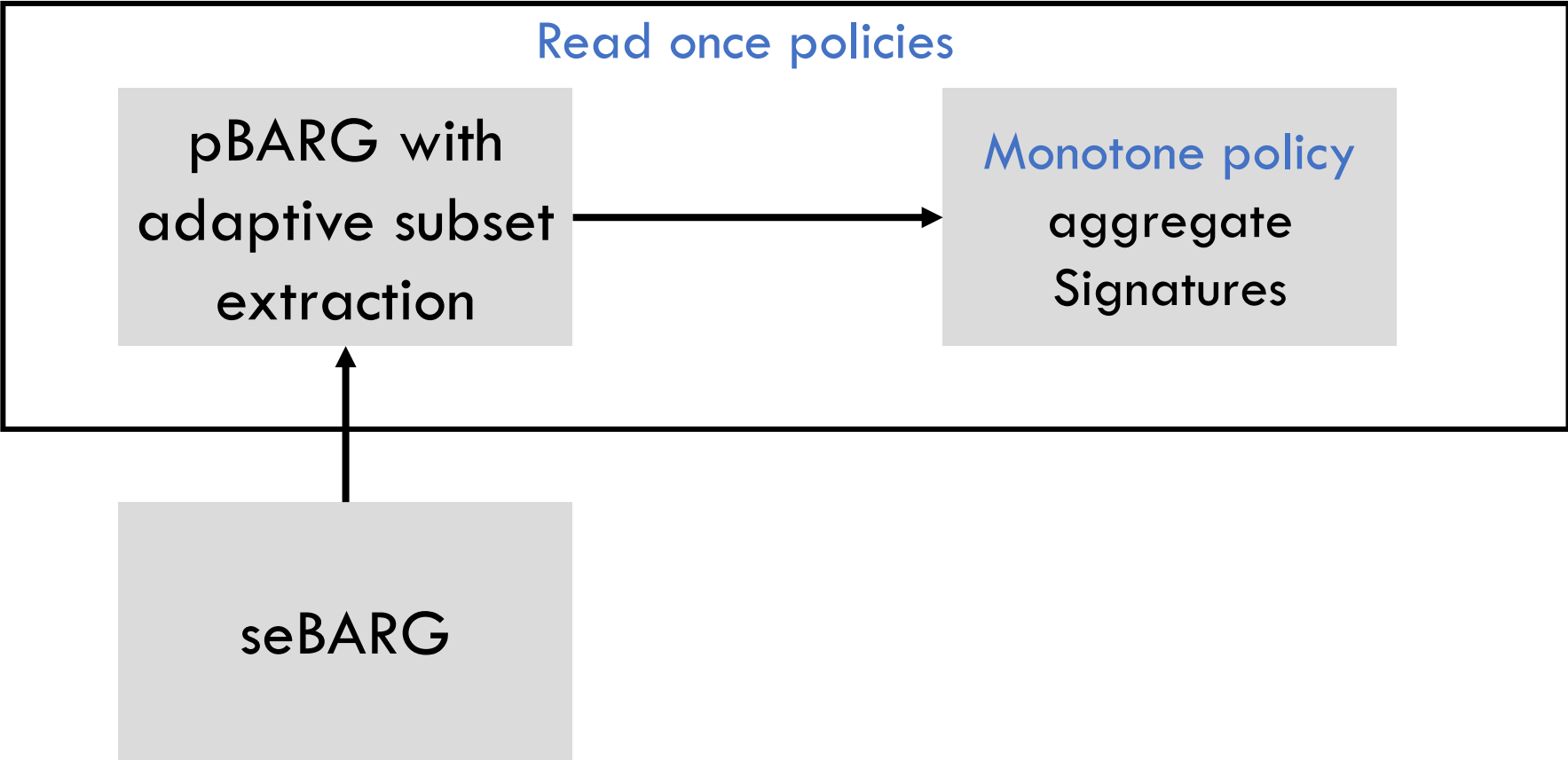


$m^*, \sigma^*$

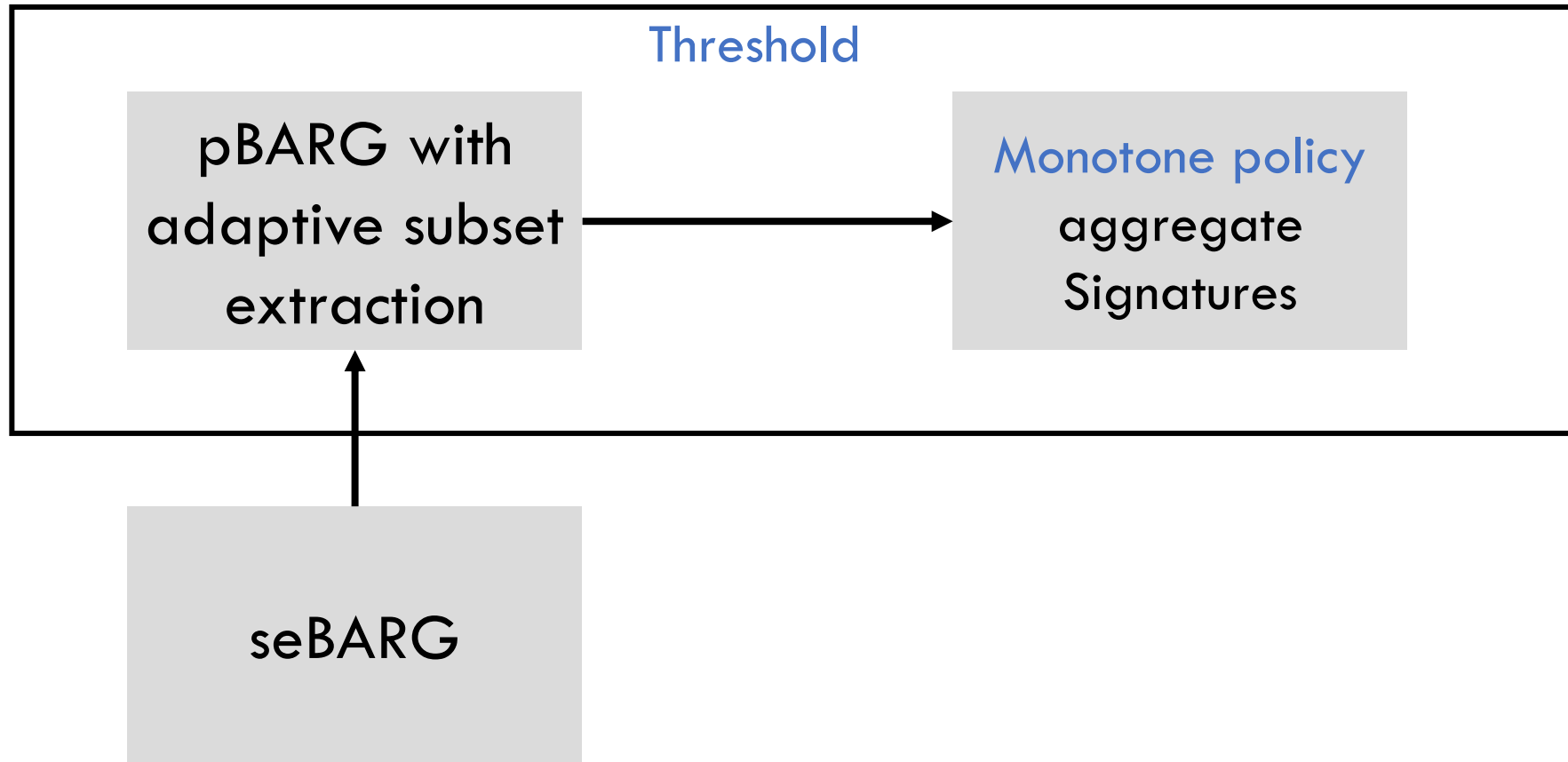








# Today



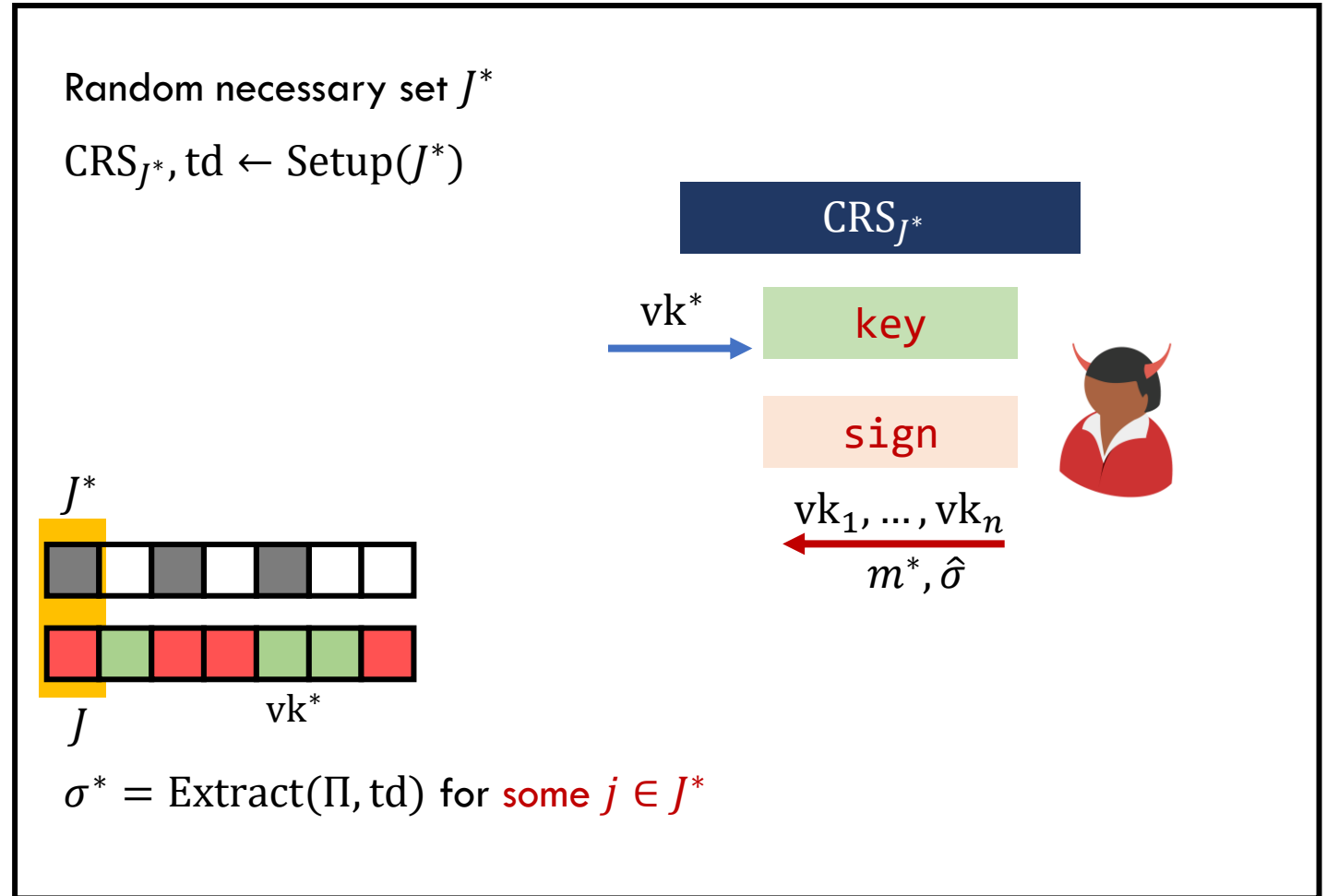
# Previous Attempt



Fails unless guessed  $J^*$  is the same as adaptively chosen  $J$ .

May be exponentially many  $J^*$ .

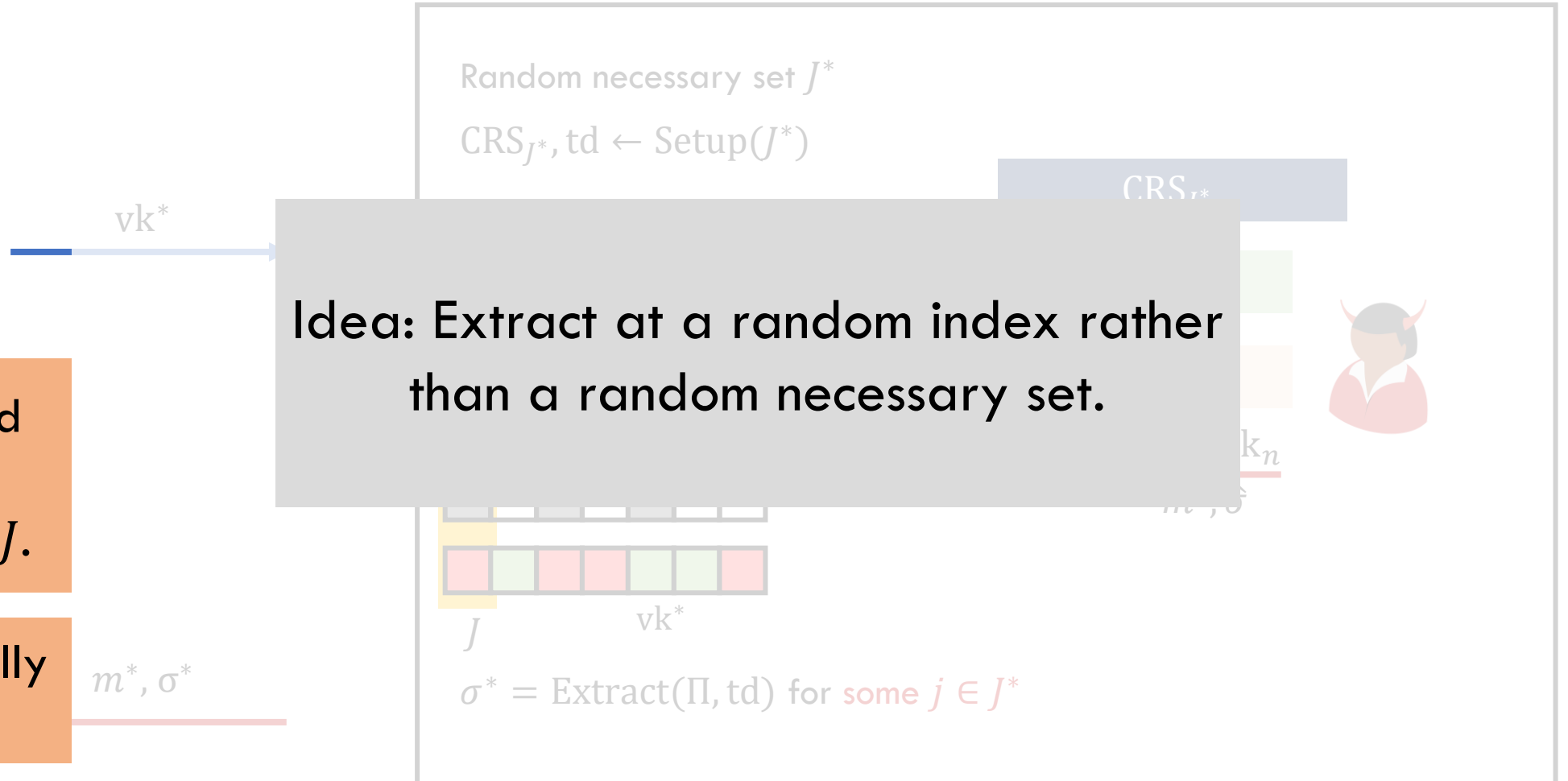
$m^*, \sigma^*$



# Previous Attempt

Fails unless guessed  $J^*$  is the same as adaptively chosen  $J$ .

May be exponentially many  $J^*$ .



# Threshold BARG with Adaptive Subset Extraction

$x_1 w_1$

$\vdots$

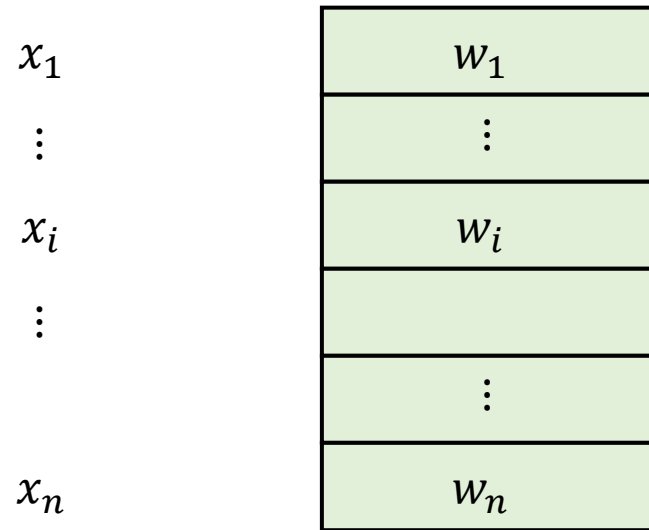
$x_i w_i$

$\vdots$

$x_n w_n$

At least  $t$  witnesses  $w_i$   
such that  $w_i \neq \perp$ .

# Threshold BARG with Adaptive Subset Extraction



# Threshold BARG with Adaptive Subset Extraction

$x_1$	$w_1$	$c_1$
$\vdots$		
$x_i$	$w_i$	$c_i$
$\vdots$		
$x_n$	$w_n$	$c_n$

$$c_i = c_{i-1} + R_L(x_i, w_i)$$

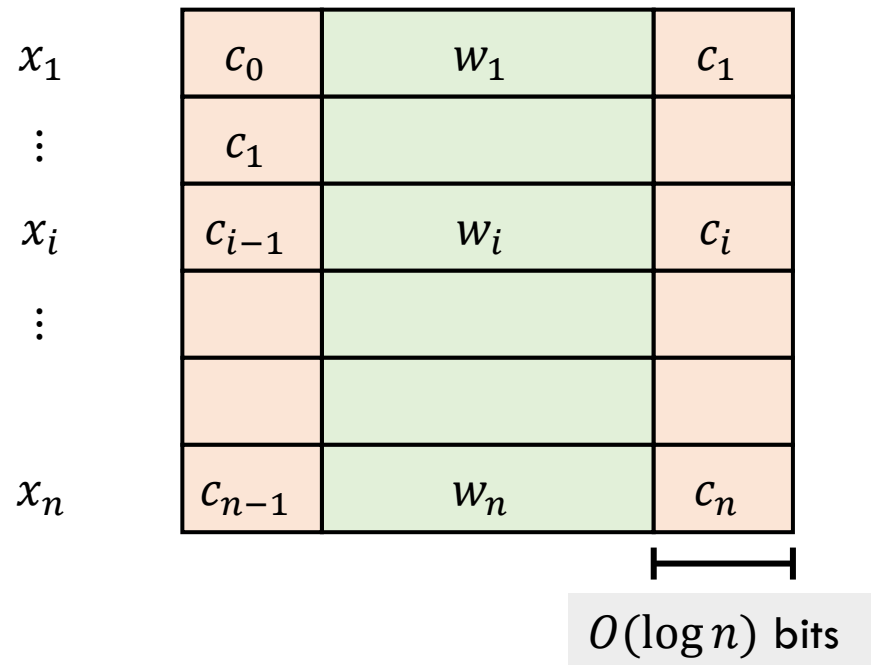
# Threshold BARG with Adaptive Subset Extraction

$x_1$	$c_0$	$w_1$	$c_1$
$\vdots$	$c_1$		
$x_i$	$c_{i-1}$	$w_i$	$c_i$
$\vdots$			
$x_n$	$c_{n-1}$	$w_n$	$c_n$

$$c_i = c_{i-1} + R_L(x_i, w_i)$$



# Threshold BARG with Adaptive Subset Extraction



$$c_i = c_{i-1} + R_L(x_i, w_i)$$

# Threshold BARG with Adaptive Subset Extraction

$x_1$	$c_0$	$w_1$	$c_1$
$\vdots$	$c_1$		
$x_i$	$c_{i-1}$	$w_i$	$c_i$
$\vdots$			
$x_n$	$c_{n-1}$	$w_n$	$c_n$

$\underbrace{\hspace{2cm}}_{O(\log n) \text{ bits}}$

Batch  $\forall i \in [n]$

Statement:  $x_i$

Witness:  $c_{i-1}, w_i, c_i$

$$\text{s.t. } c_i = c_{i-1} + R_L(x_i, w_i)$$

$$c_i = c_{i-1} + R_L(x_i, w_i)$$

# Threshold BARG with Adaptive Subset Extraction

Assumed to be same value.

$x_1$	$c_0$	$w_1$	$c_1$
$\vdots$	$c_1$		
$x_i$	$c_{i-1}$	$w_i$	$c_i$
$\vdots$			
$x_n$	$c_{n-1}$	$w_n$	$c_n$

$\underbrace{\hspace{10em}}_{O(\log n) \text{ bits}}$

Batch  $\forall i \in [n]$

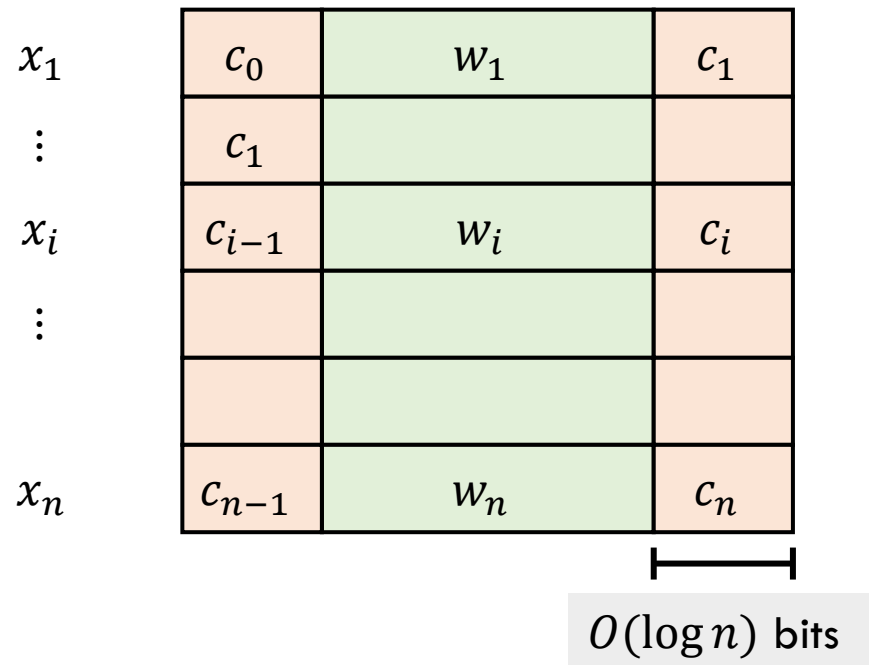
Statement:  $x_i$

Witness:  $c_{i-1}, w_i, c_i$

$$\text{s.t. } c_i = c_{i-1} + R_L(x_i, w_i)$$

$$c_i = c_{i-1} + R_L(x_i, w_i)$$

# Threshold BARG with Adaptive Subset Extraction



Batch  $\forall i \in [n]$

Statement:  $x_i, \text{rt}$

Witness:  $c_{i-1}, \pi_{i-1}, w_i, c_i, \pi_i$

s.t.  $c_i = c_{i-1} + R_L(x_i, w_i)$

$\pi_{i-1}$  valid opening to  $c_{i-1}$

$\pi_i$  valid opening to  $c_i$

$$\text{rt} = H\left(\boxed{c_0} \quad \boxed{c_1} \quad \dots \quad \boxed{c_n}\right)$$

# Threshold BARG with Adaptive Subset Extraction

$x_1$	$c_0$	$w_1$	$c_1$
$\vdots$	$c_1$		
$x_i$	$c_{i-1}$	$w_i$	$c_i$
$\vdots$			
$x_n$	$c_{n-1}$	$w_n$	$c_n$

$\underbrace{\hspace{2cm}}_{O(\log n) \text{ bits}}$

Batch  $\forall i \in [n]$

Statement:  $x_i$

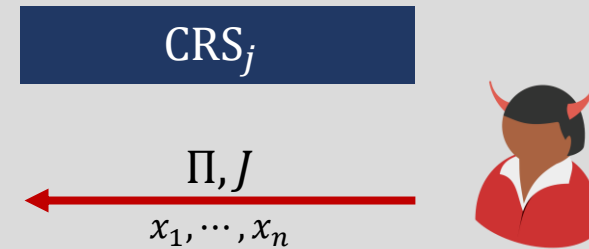
Witness:  $c_{i-1}, w_i, c_i$

$$\text{s.t. } c_i = c_{i-1} + R_L(x_i, w_i)$$



# Adaptive Subset Extraction

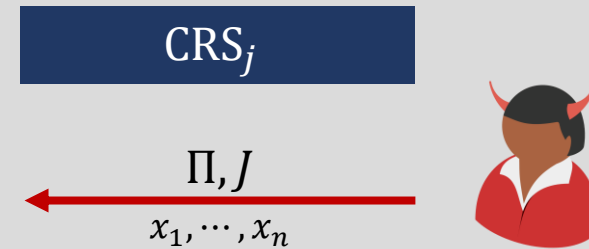
$j \leftarrow [n]$   
 $\text{CRS}_j, \text{td} \leftarrow \text{Setup}(j)$



$w \leftarrow \text{Extract}(\Pi, \text{td})$

# Adaptive Subset Extraction

$j \leftarrow [n]$   
 $\text{CRS}_j, \text{td} \leftarrow \text{Setup}(j)$




$w \leftarrow \text{Extract}(\Pi, \text{td})$

$$\Pr_{j \leftarrow [n]} [j \in J \wedge R_L(x_j, w_j) = 1] \stackrel{?}{\approx} 1/n$$




# Adaptive Subset Extraction

Can  keep avoiding  $j$  by setting  $w_j = \perp$ ?

$j \leftarrow [n]$   
 $\text{CRS}_j, \text{td} \leftarrow \text{Setup}(j)$

**CRS<sub>j</sub>**


$\Pi, J$   
 $x_1, \dots, x_n$



$w \leftarrow \text{Extract}(\Pi, \text{td})$

$$\Pr_{j \leftarrow [n]} [j \in J \wedge R_L(x_j, w_j) = 1] \stackrel{?}{\approx} 1/n$$

# Adaptive Subset Extraction

Can  keep avoiding  $j$  by setting  $w_j = \perp$ ?

Checking requires extracting using  $td \Rightarrow$  **cannot** rely on CRS hiding.

$j \leftarrow [n]$   
 $CRS_j, td \leftarrow \text{Setup}(j)$

$CRS_j$


$\Pi, J$   
 $x_1, \dots, x_n$



$w \leftarrow \text{Extract}(\Pi, td)$

$$\Pr_{j \leftarrow [n]} [j \in J \wedge R_L(x_j, w_j) = 1] \stackrel{?}{\approx} 1/n$$

# Adaptive Subset Extraction

Can  keep avoiding  $j$  by setting  $w_j = \perp$ ?

Sufficient to determine which statements were true to perform above check.

$j \leftarrow [n]$   
 $\text{CRS}_j, \text{td} \leftarrow \text{Setup}(j)$

$\text{CRS}_j$


$\Pi, J$   
 $x_1, \dots, x_n$



$w \leftarrow \text{Extract}(\Pi, \text{td})$

$$\Pr_{j \leftarrow [n]} [j \in J \wedge R_L(x_j, w_j) = 1] \stackrel{?}{\approx} 1/n$$

# Adaptive Subset Extraction

Can  keep avoiding  $j$  by setting  $w_j = \perp$ ?

Sufficient to determine which statements were true to perform above check.

$j \leftarrow [n]$   
 $\text{CRS}_j, \text{td} \leftarrow \text{Setup}(j)$

$\text{CRS}_j$


$\Pi, J$   
 $x_1, \dots, x_n$



$w \leftarrow \text{Extract}(\Pi, \text{td})$

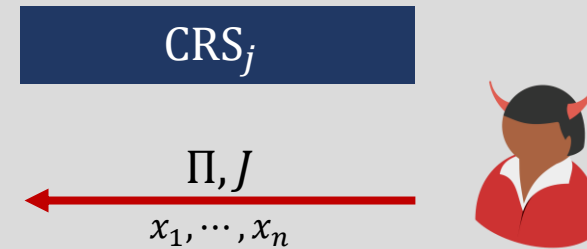
Goal: Determine which  $t$  statements proved were true.

# Adaptive Subset Extraction

Can  keep avoiding  $j$  by setting  $w_j = \perp$ ?

Sufficient to determine which statements were true to perform above check.

$j \leftarrow [n]$   
 $\text{CRS}_j, \text{td} \leftarrow \text{Setup}(j)$



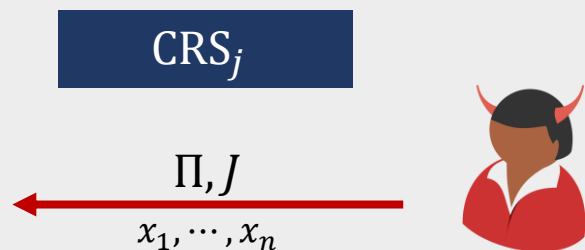
$w \leftarrow \text{Extract}(\Pi, \text{td})$

Approximately Simulate  
Goal: Determine which  $t$  statements proved were true.

There exists a simulator  $\text{Sim}$  such that for all  $j \in [n]$

**Real( $j$ )**

$\text{CRS}_j, \text{td} \leftarrow \text{Setup}(j)$



$c_{j-1}, w, c_j \leftarrow \text{Extract}(\text{td})$

Output  $(x_j, w)$

$\approx$

**Ideal ( $j$ )**

$x_1, \dots, x_n, w_1, \dots, w_n, J \leftarrow \text{Sim}()$

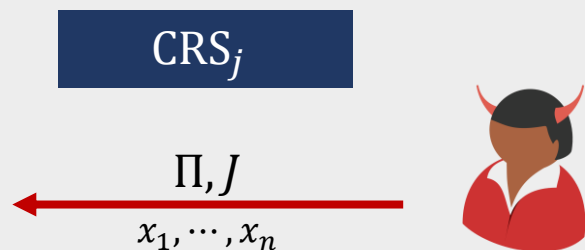
If  $|J| \leq k - t$  or  $|\{w_i \neq \perp\}| < t$   
abort

Output  $(x_j, w_j)$

There exists a simulator Sim such that for all  $j \in [n]$

Real( $j$ )

$\text{CRS}_j, \text{td} \leftarrow \text{Setup}(j)$



$c_{j-1}, w, c_j \leftarrow \text{Extract}(\text{td})$

Output  $(x_j, w)$

$\approx$

Ideal ( $j$ )

$x_1, \dots, x_n, w_1, \dots, w_n, J \leftarrow \text{Sim}()$

If  $|J| \leq k - t$  or  $|\{w_i \neq \perp\}| < t$   
abort

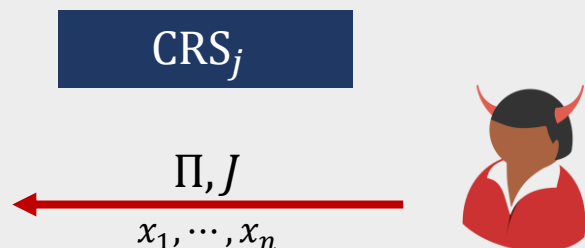
Output  $(x_j, w_j)$

$$\Pr_{j \leftarrow [n]} [R_L(x_j, w_j) = 1] \approx t/n$$

There exists a simulator  $\text{Sim}$  such that for all  $j \in [n]$

**Real( $j$ )**

$\text{CRS}_j, \text{td} \leftarrow \text{Setup}(j)$



$c_{j-1}, w, c_j \leftarrow \text{Extract}(\text{td})$

Output  $(x_j, w)$

$$\Pr_{j \leftarrow [n]} [R_L(x_j, w_j) = 1] \approx t/n$$

**Ideal ( $j$ )**

$x_1, \dots, x_n, w_1, \dots, w_n, J \leftarrow \text{Sim}()$

If  $|J| \leq k - t$  or  $|\{w_i \neq \perp\}| < t$   
abort

Output  $(x_j, w_j)$

$\approx$

$\Leftarrow$

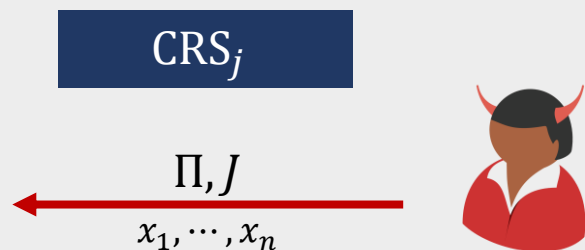
$$\Pr_{j \leftarrow [n]} [R_L(x_j, w_j) = 1] \approx t/n$$



There exists a simulator Sim such that for all  $j \in [n]$

Real( $j$ )

$\text{CRS}_j, \text{td} \leftarrow \text{Setup}(j)$



$c_{j-1}, w, c_j \leftarrow \text{Extract}(\text{td})$

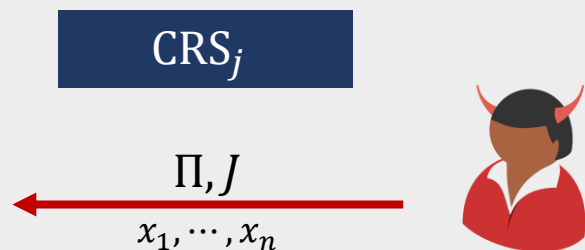
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$$\Pr_{j \leftarrow [n]} [R_L(x_j, w_j) = 1] \approx t/n$$

There exists a simulator Sim such that for all  $j \in [n]$

**Real( $j$ )**

$\text{CRS}_j, \text{td} \leftarrow \text{Setup}(j)$



$c_{j-1}, w, c_j \leftarrow \text{Extract}(\text{td})$

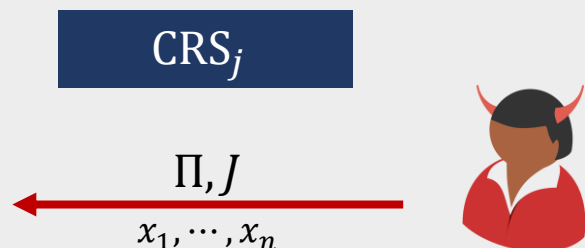
Output  $(x_j, w)$

$$\Pr_{j \leftarrow [n]} [R_L(x_j, w_j) = 1] \approx t/n$$

There exists a simulator Sim such that for all  $j \in [n]$

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$c_{j-1}, w, c_j \leftarrow \text{Extract}(\text{td})$

Output  $(x_j, w)$

$$\Pr_{j \leftarrow [n]} [R_L(x_j, w_j) = 1] \approx t/n$$

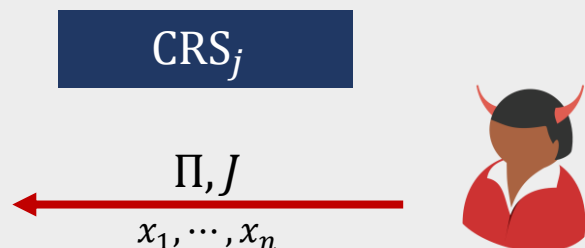
$$\Pr_{j \leftarrow [n]} [j \in J] \approx (n - t + 1)/n$$

By CRS indistinguishability and that the size of any necessary set is  $(n - t + 1)$

There exists a simulator Sim such that for all  $j \in [n]$

**Real( $j$ )**

$\text{CRS}_j, \text{td} \leftarrow \text{Setup}(j)$



$c_{j-1}, w, c_j \leftarrow \text{Extract}(\text{td})$

Output  $(x_j, w)$

$$\Pr_{j \leftarrow [n]} [R_L(x_j, w_j) = 1] \approx t/n$$

$$\Pr_{j \leftarrow [n]} [j \in J] \approx (n - t + 1)/n$$

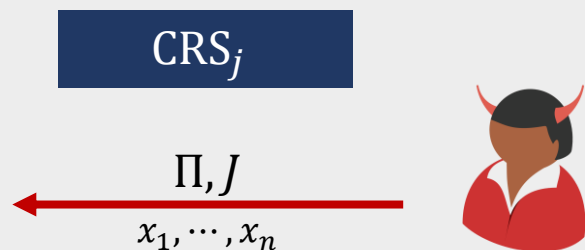
By CRS indistinguishability and that the size of any necessary set is  $(n - t + 1)$

$$\Pr_{j \leftarrow [n]} [j \in J \wedge R_L(x_j, w_j) = 1] \approx 1/n$$

There exists a simulator  $\text{Sim}$  such that for all  $j \in [n]$

**Real( $j$ )**

$\text{CRS}_j, \text{td} \leftarrow \text{Setup}(j)$



$c_{j-1}, w, c_j \leftarrow \text{Extract}(\text{td})$

Output  $(x_j, w)$

$\approx$

**Ideal ( $j$ )**

$x_1, \dots, x_n, w_1, \dots, w_n, J \leftarrow \text{Sim}()$

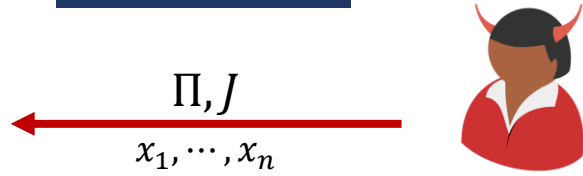
If  $|J| \leq k - t$  or  $|\{w_i \neq \perp\}| < t$   
abort

Output  $(x_j, w_j)$

# Simulator Sketch

$\text{CRS}_1, \text{td} \leftarrow \text{Setup}(1)$

$\text{CRS}_1$



$c_0, w, c_1 \leftarrow \text{Extract}(\text{td})$

Output  $(x_1, c_0, w, c_1)$

# Simulator Sketch

$\text{CRS}_{1, \text{td}} \leftarrow \text{Setup}(1)$

$x_1$  

# Simulator Sketch

$\text{CRS}_1, \text{td} \leftarrow \text{Setup}(1)$

$x_1$  

$\text{CRS}_1, \text{td} \leftarrow \text{Setup}(1)$

$x'_1$  

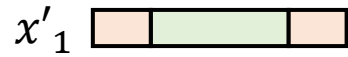


# Simulator Sketch

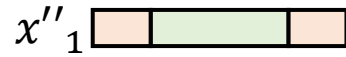
$\text{CRS}_1, \text{td} \leftarrow \text{Setup}(1)$



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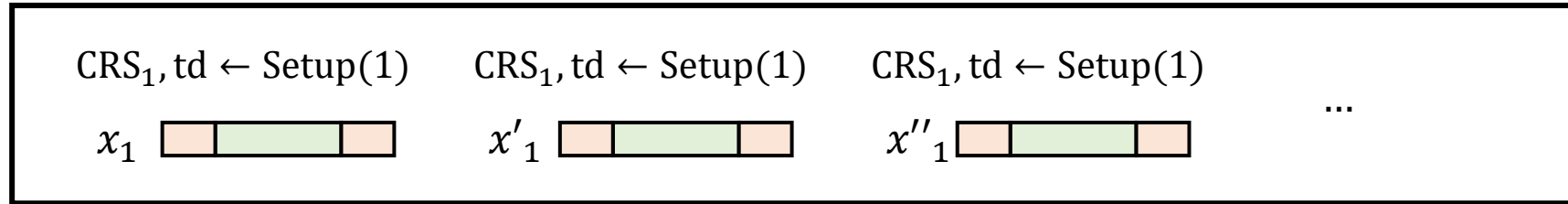


$\text{CRS}_1, \text{td} \leftarrow \text{Setup}(1)$

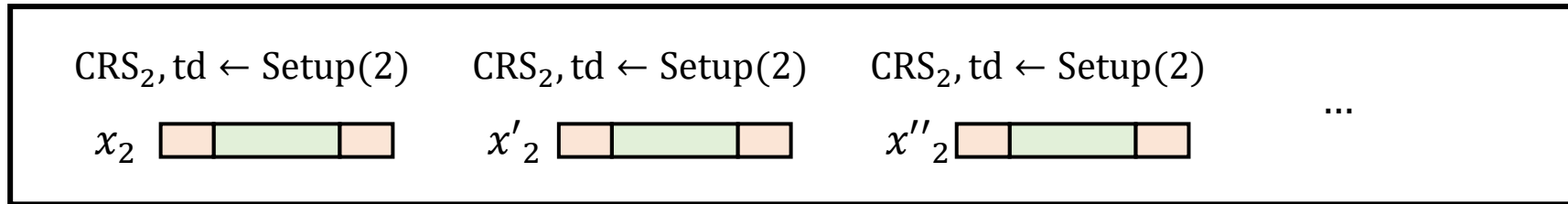
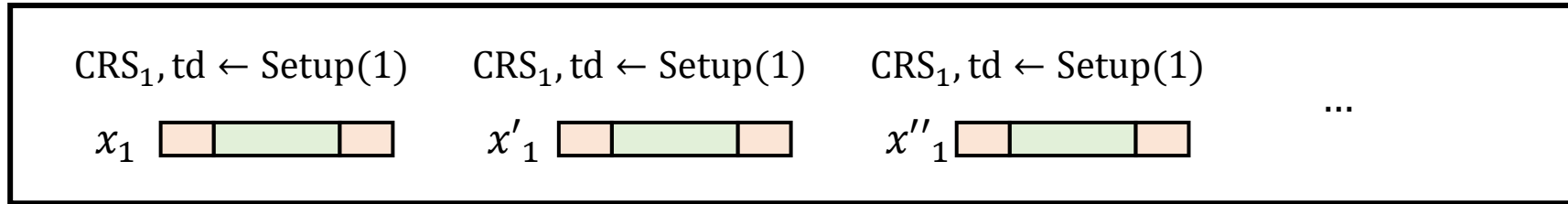


...

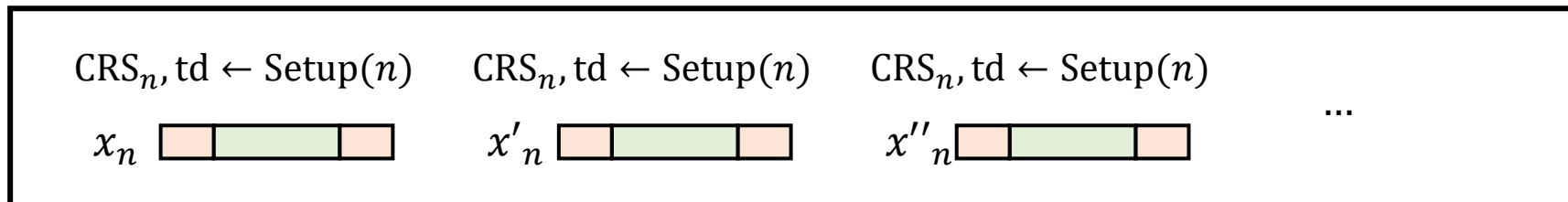
# Simulator Sketch



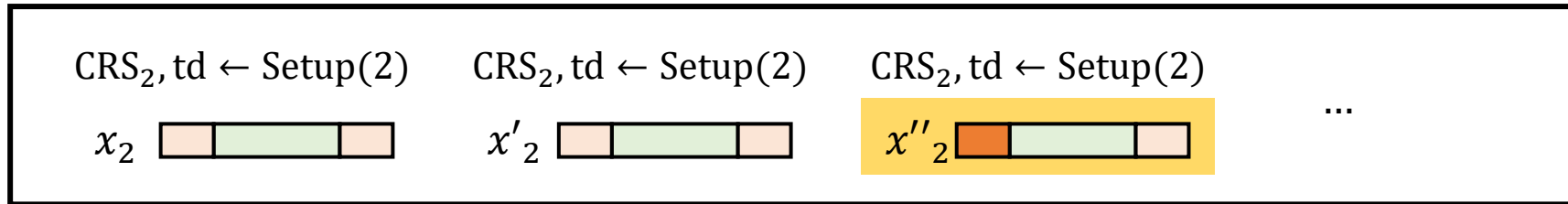
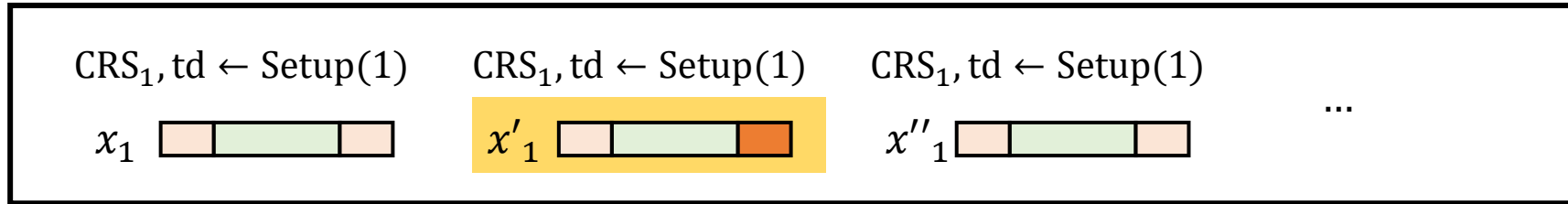
# Simulator Sketch



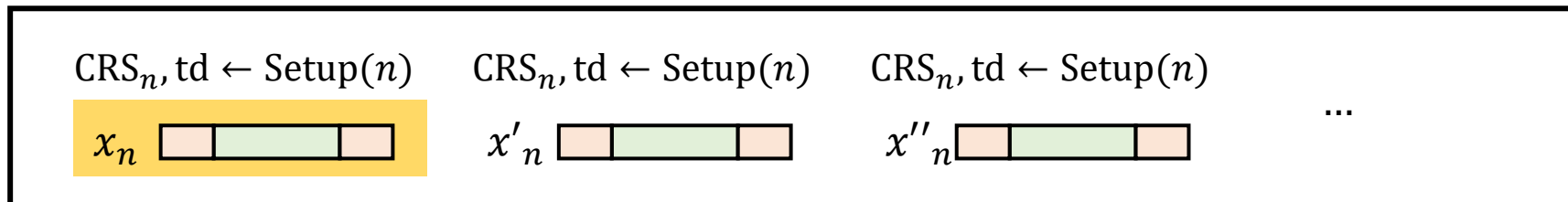
⋮



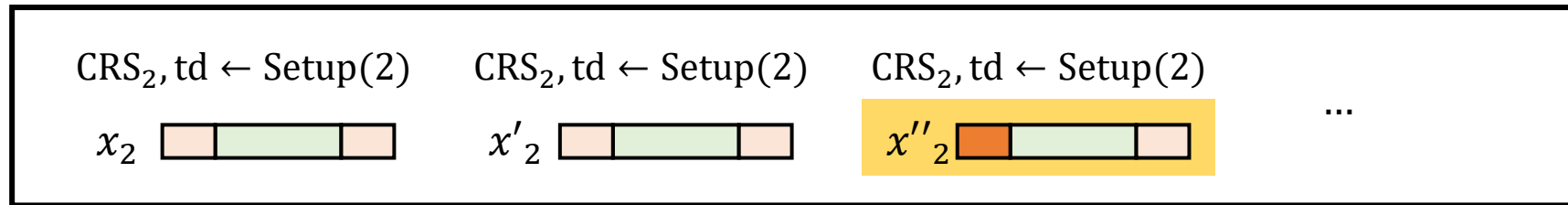
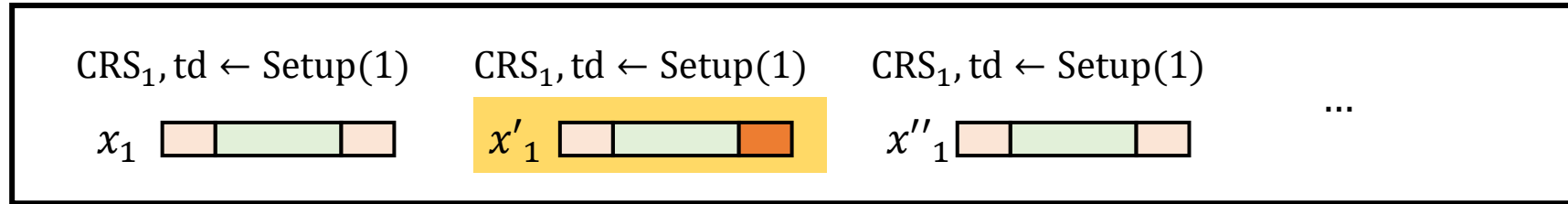
# Simulator Sketch



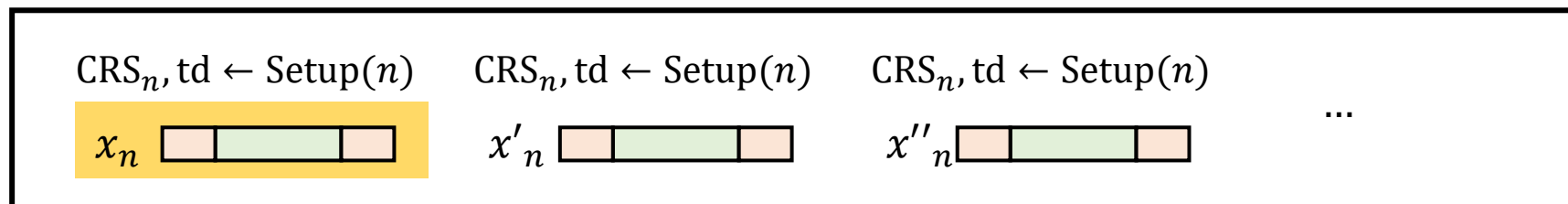
⋮



# Simulator Sketch



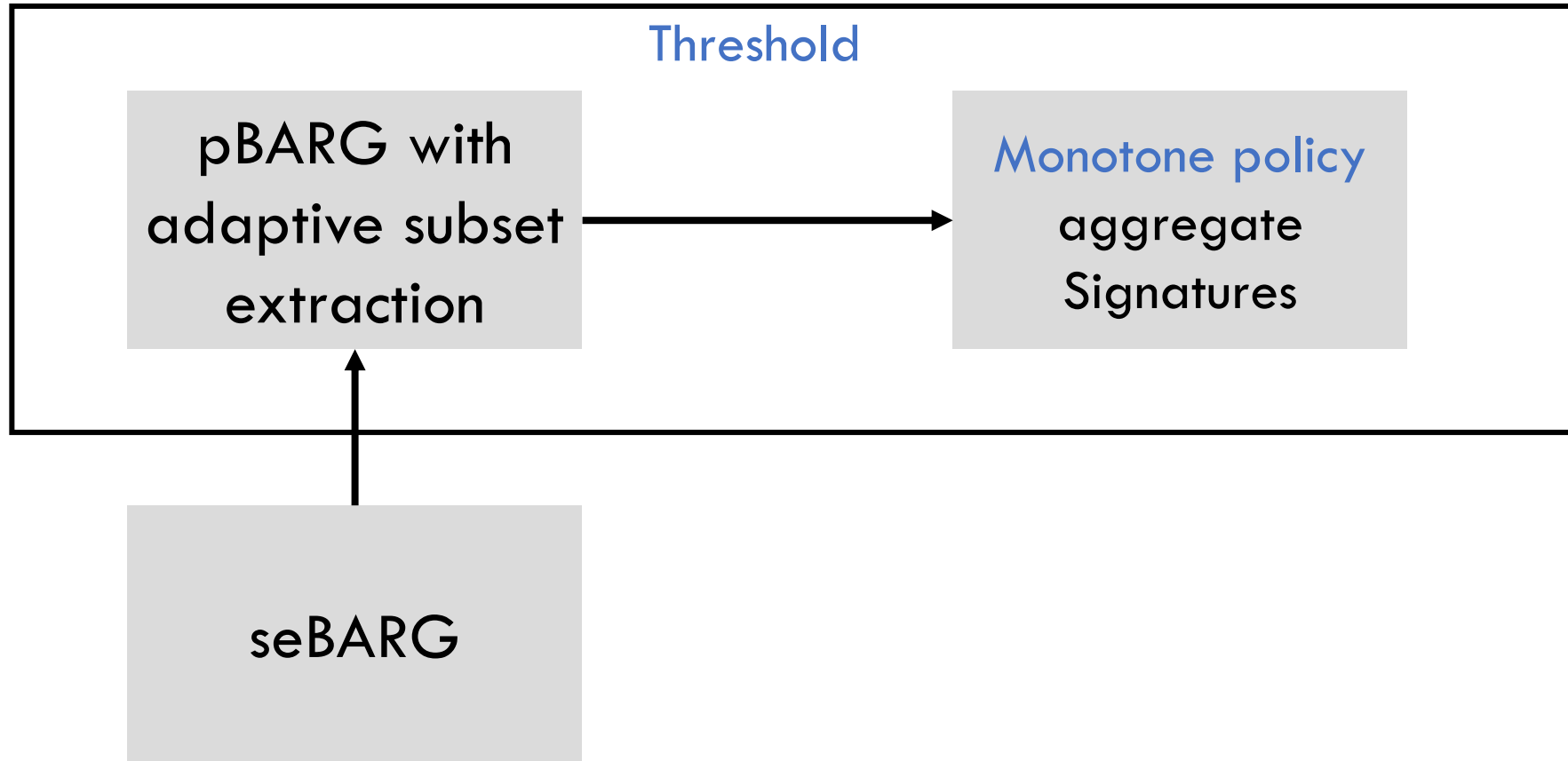
⋮



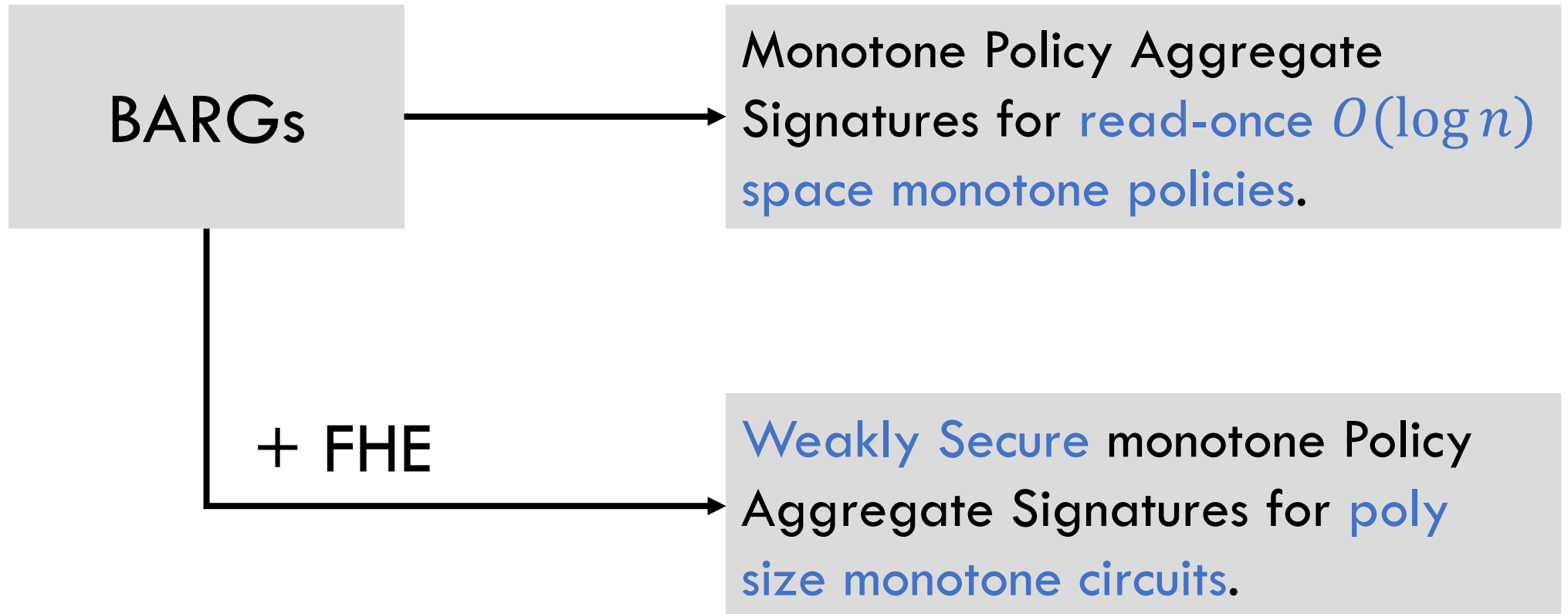
At least  $t$  valid witnesses.

If  $|c_i| = O(\log n)$

# Today



# Our Results



# Thank you. Questions?

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Paper on ePrint soon!