

SNARGs for P from LWE



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Succinct Non-Interactive Arguments (SNARGs)

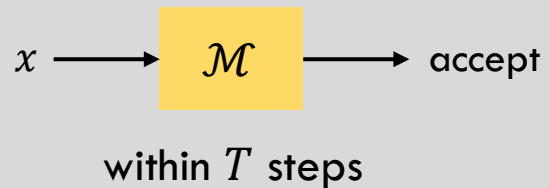
Common Reference String (CRS)



\mathcal{M}, x



\mathcal{M}, x



Succinct Non-Interactive Arguments (SNARGs)

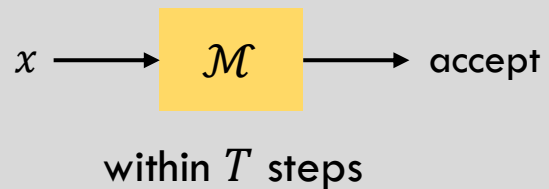
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wants to delegate computation to



Succinct Non-Interactive Arguments (SNARGs)

Common Reference String (CRS)



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Π



\mathcal{M}, x

$x \longrightarrow \mathcal{M} \longrightarrow \text{accept}$

within T steps

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Π is publicly verifiable

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Succinct Non-Interactive Arguments (SNARGs)

Common Reference String (CRS)



\mathcal{M}, x

$\leftarrow \text{polylog}(T) \rightarrow$

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\mathcal{M}, x

Verifier **running time:**
 $\text{polylog}(T)$

Π is **publicly verifiable**

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No PPT  can produce accepting Π if

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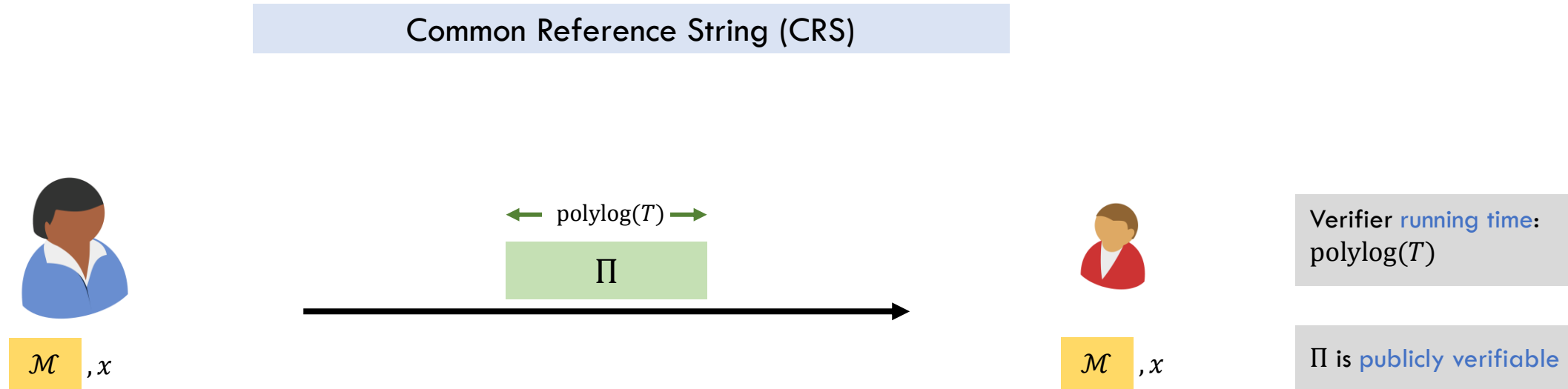
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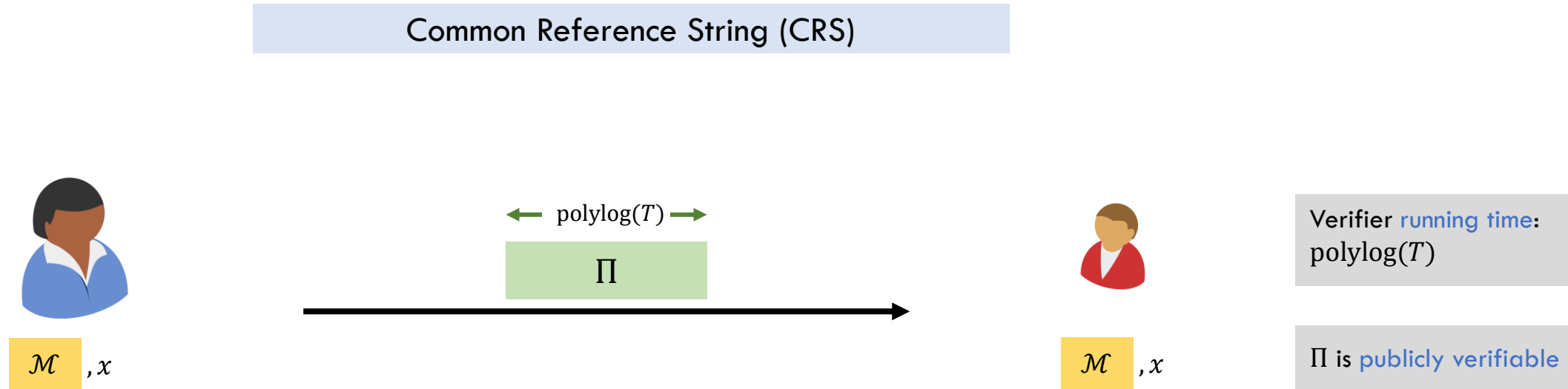
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Succinct Non-Interactive Arguments (SNARGs)



What kind of computation can we hope to **delegate** based on **standard assumptions**?

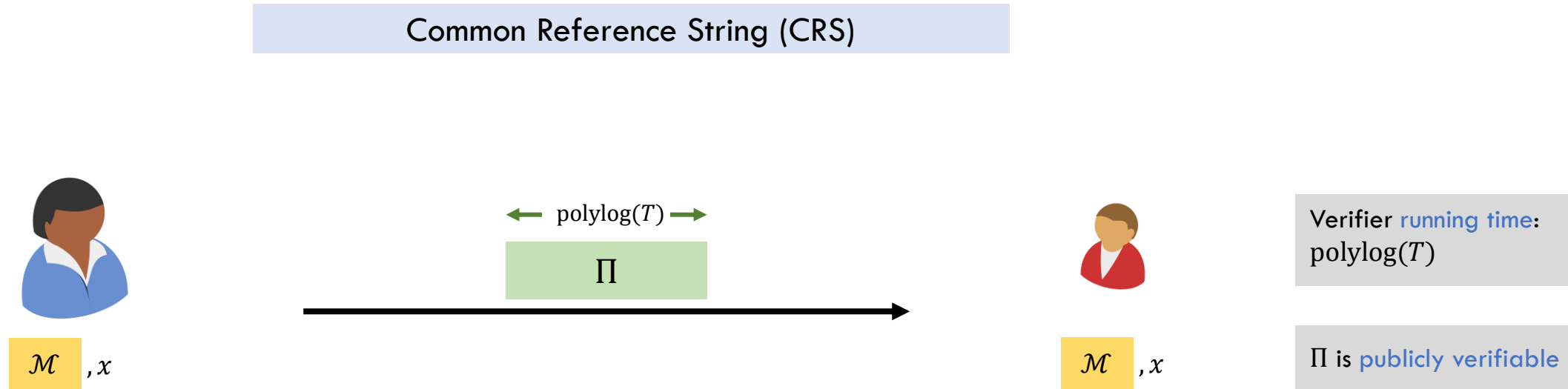
Succinct Non-Interactive Arguments (SNARGs)



What kind of computation can we hope to **delegate** based on **standard assumptions**?

- Nondeterministic polynomial-time computation (NP)? **Unlikely!** [Gentry-Wichs'11]

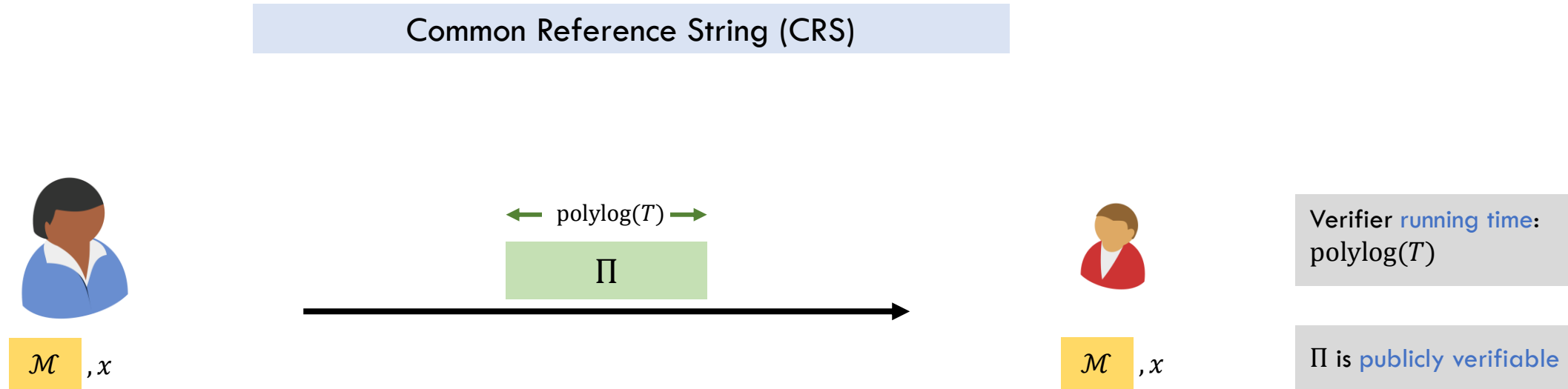
Succinct Non-Interactive Arguments (SNARGs)



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- Deterministic computation?

Succinct Non-Interactive Arguments (SNARGs)



What kind of computation can we hope to **delegate** based on **standard assumptions**?

- Nondeterministic polynomial-time computation (NP)? **Unlikely!** [Gentry-Wichs'11]
- Deterministic computation?
- Sub-classes of NP?

SNARGs for Batch NP

CRS



C, x_1, \dots, x_k

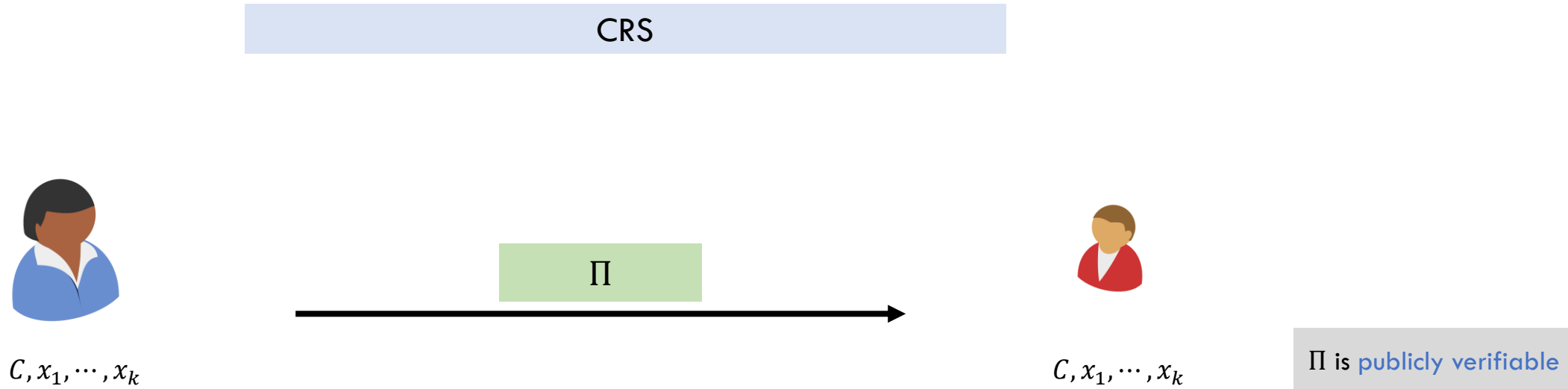


C, x_1, \dots, x_k

$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$

$\forall i \in [k], (C, x_i) \in \text{SAT}$

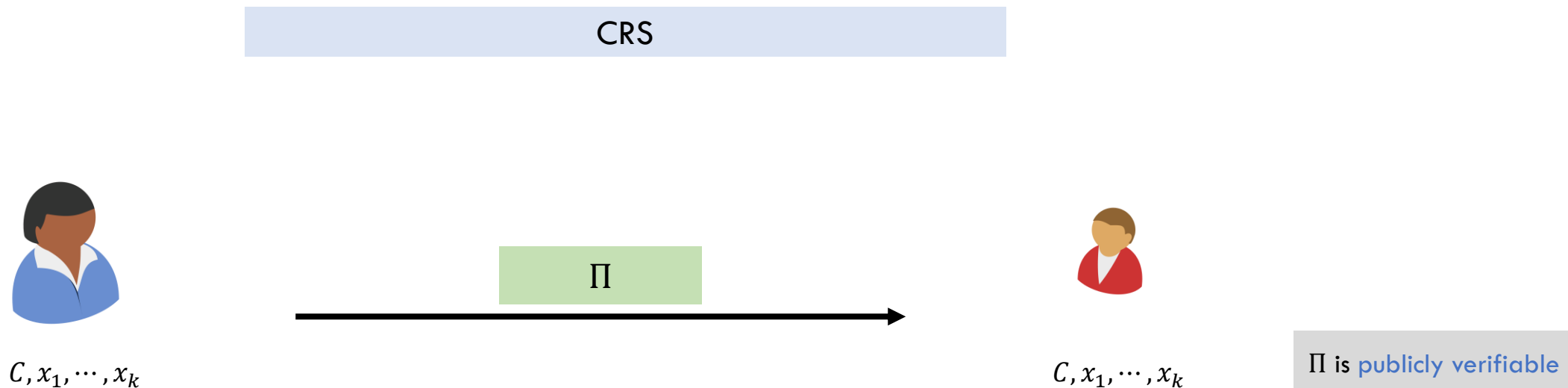
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SNARGs for Batch NP



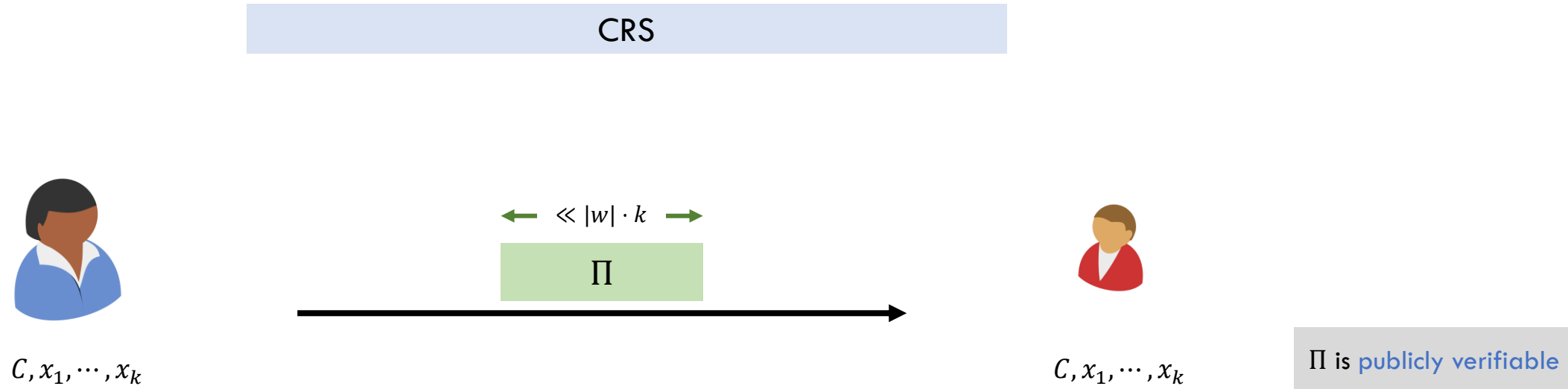
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No PPT  can produce **accepting** Π if

$$\exists i^* \in [k], (C, x_{i^*}) \notin \text{SAT}$$

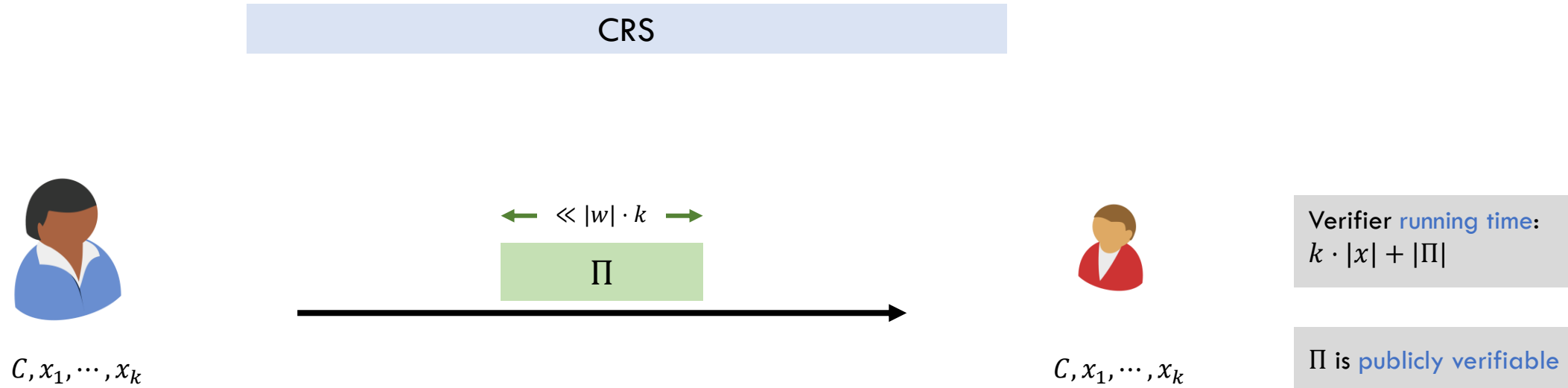
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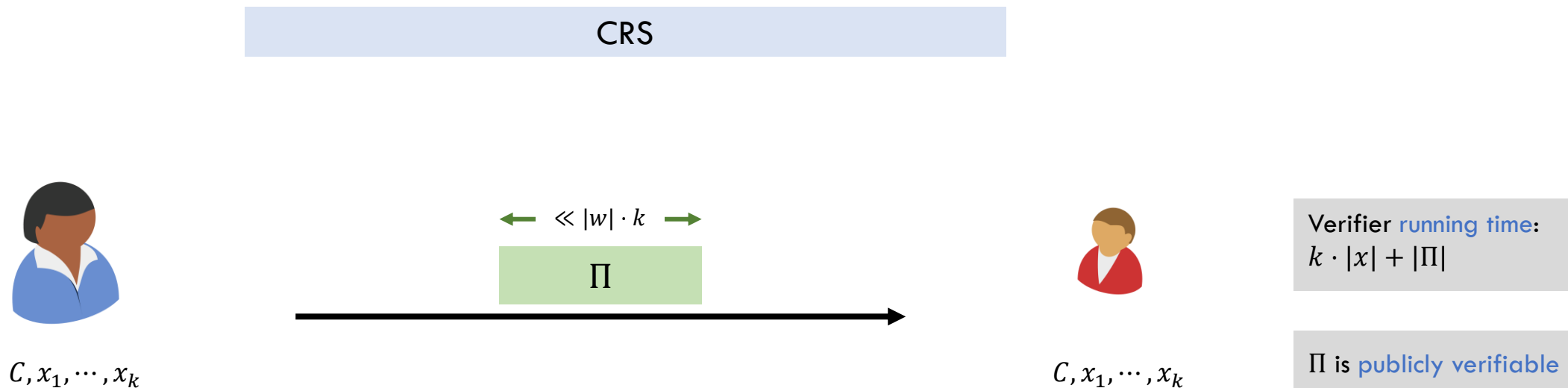
SNARGs for Batch NP



$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

SNARGs for Batch NP



$$\text{SAT}^{\otimes k} = \{(C, x_1, \dots, x_k) \mid \forall i \in [k], (C, x_i) \in \text{SAT}\}$$

$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

Prior Works

Prior Works

Non-falsifiable assumptions/ Random oracle model

[Micali'94, Groth'10, Lipmaa'12, Damgård-Faust-Hazay'12, Gennaro-Gentry-Parno-Raykova'13, Bitansky-Chiesa-Ishai-Ostrovsky-Paneth'13, Bitansky-Canetti-Chiesa-Tromer'13, Bitansky-Canetti-Chiesa-Goldwasser-Lin-Rubinfeld-Tromer'17]

Some works can
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Some works can delegate NP

“Less standard” assumptions

[Canetti-Holmgren-Jain-Vaikuntanathan'15, Koppula-Lewko-Waters'15, Bitansky-Garg-Lin-Pass-Telang'15, Canetti-Holmgren'16, Ananth-Chen-Chung-Lin-Lin'16, Chen-Chow-Chung-Lai-Lin-Zhou'16, Paneth-Rothblum'17, Canetti-Chen-Holmgren-Lombardi-Rothblum-Rothblum-Wichs'19, Kalai-Paneth-Yang'19]

Delegation for P and some for batch NP

Do there exists **SNARGs** for **P** and **batch NP**
based on **standard assumptions**?

Do there exists **SNARGs** for **P**
based on **standard assumptions**?

Previously best known: [Jawale-Kalai-Khurana-Zhang'21] for **depth bounded computation** based on **sub-exponential hardness of LWE**.

Do there exist SNARGs for batch NP based on standard assumptions?

Previously best known: [C-Jain-Jin'21 a] assuming QR + (LWE/sub-exp DDH)

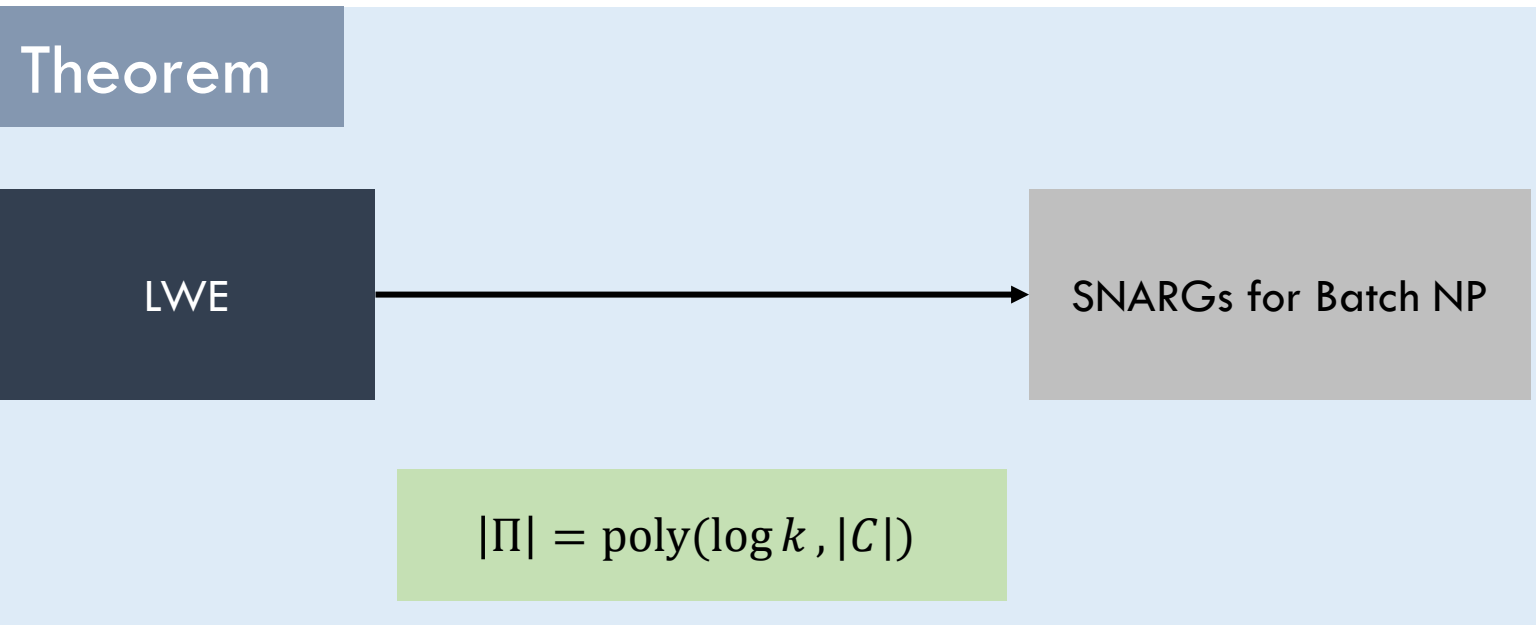
$$|\Pi| = \tilde{O}(|C| + \sqrt{k|C|})$$

QR – Quadratic residuosity, LWE – Learning with Error, DDH – Decisional Diffie-Hellman

$SAT = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$

$\forall i \in [k], (C, x_i) \in SAT$

Results

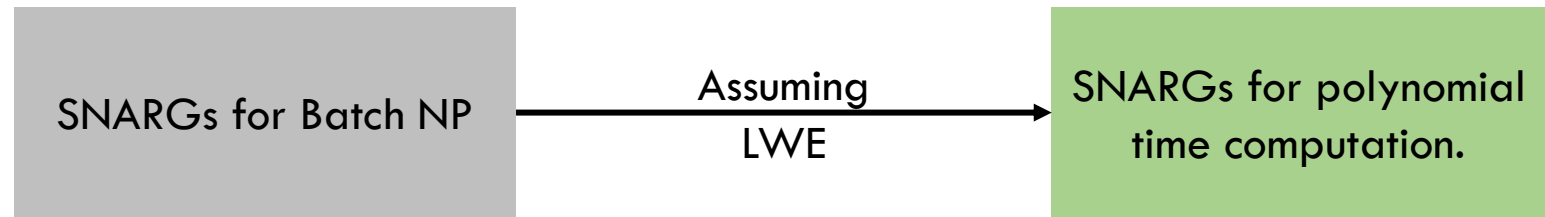


LWE – Learning with Error

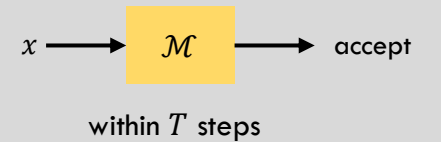
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LWE – Learning with Error



Results

Theorem

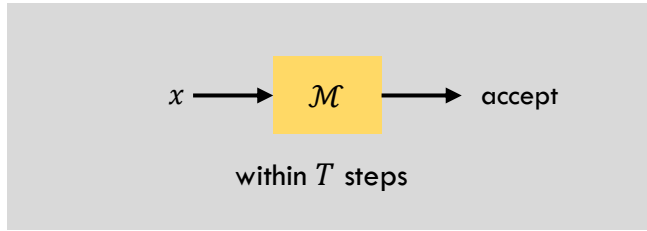
LWE

SNARGs for Batch NP

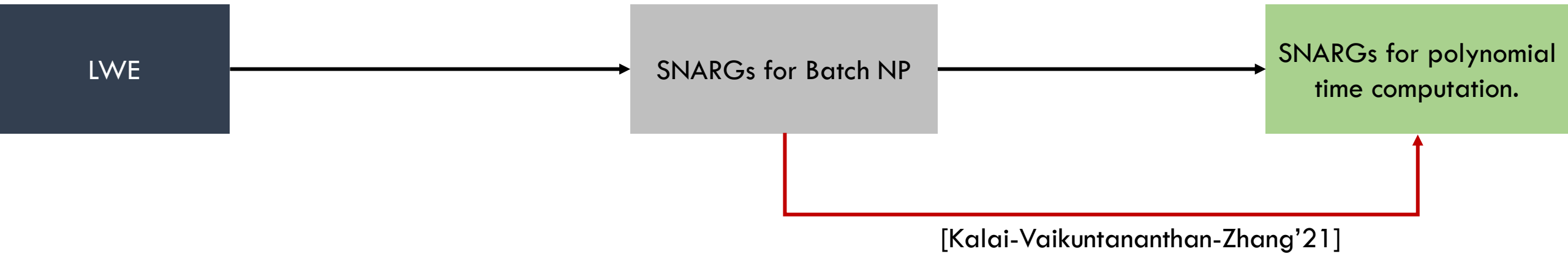
SNARGs for polynomial time computation.

$$|\text{CRS}|, |\Pi|, |\text{👤}| = \text{polylog}(T)$$

LWE – Learning with Error

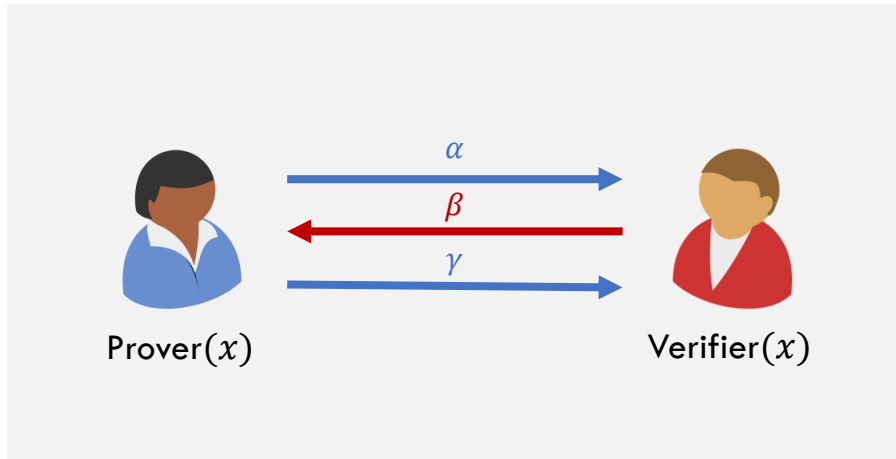


Results



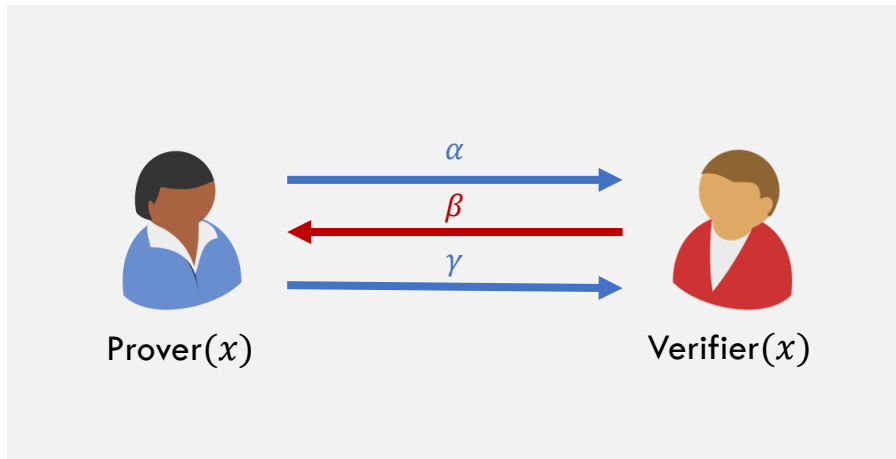
Tool: Fiat-Shamir (FS) Methodology

Fiat-Shamir (FS) Methodology



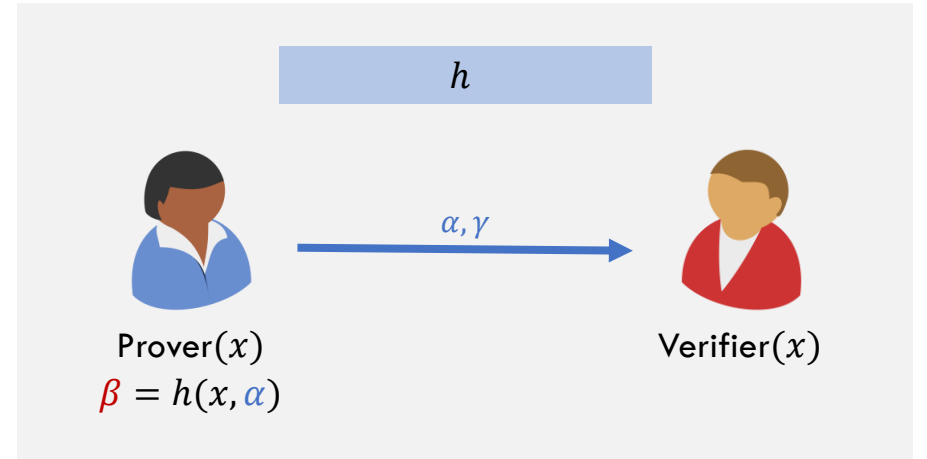
β is a random string

Fiat-Shamir (FS) Methodology

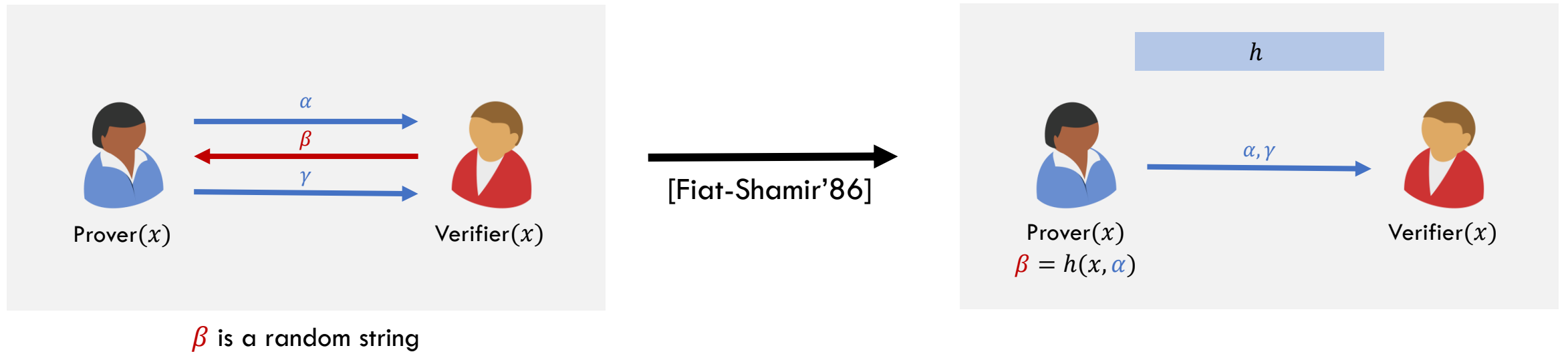


β is a random string

→
[Fiat-Shamir'86]



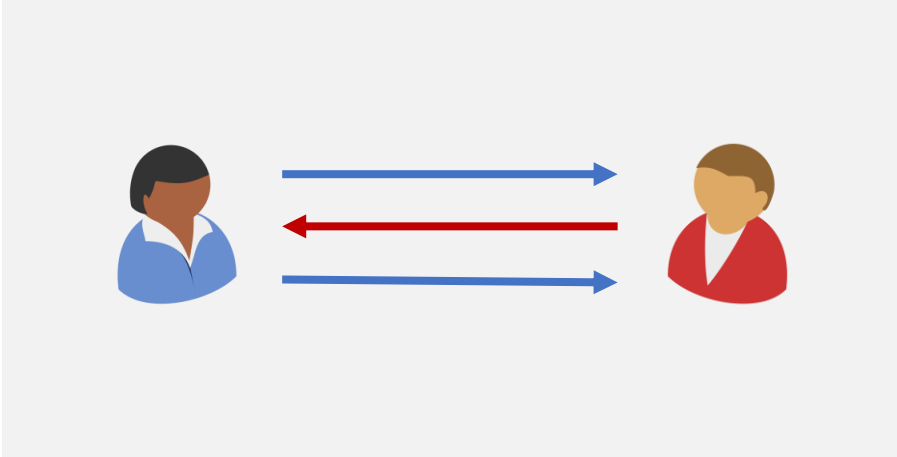
Fiat-Shamir (FS) Methodology



FS methodology is secure for certain protocols under a variety of assumptions (via [correlation intractable hash functions](#))

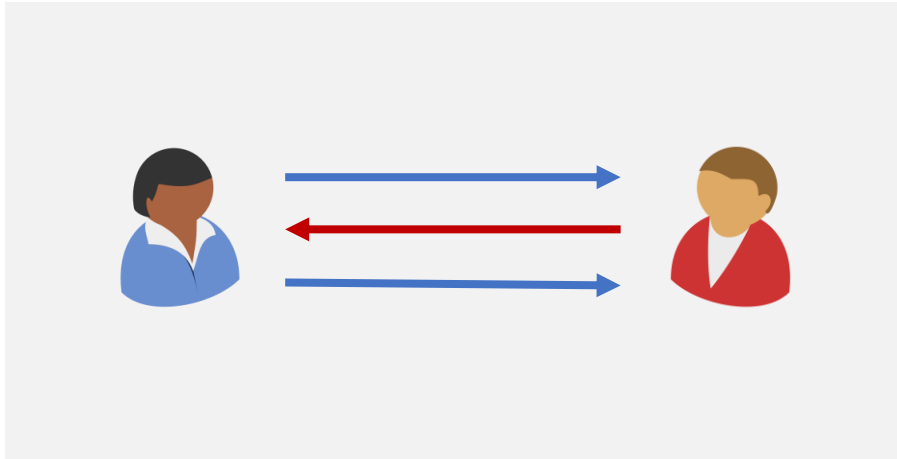
[Kalai-Rothblum-Rothblum'17, Canetti-Chen-Reyzin-Rothblum'18, Holmgren-Lombardi'18, Canetti-Chen-Holmgren-Lombardi-Rothblum-Rothblum-Wichs'19, Peikert-Sheihian'19, Brakerski-Koppula-Mour'20, Couteau-Katsumata-Ursu'20, Jain-Jin'21, Jawale-Kalai-Khurana-Zhang'21, Holmgren-Lombardi-Rothblum'21]

Fiat-Shamir (FS) Methodology



[Kilian'92]

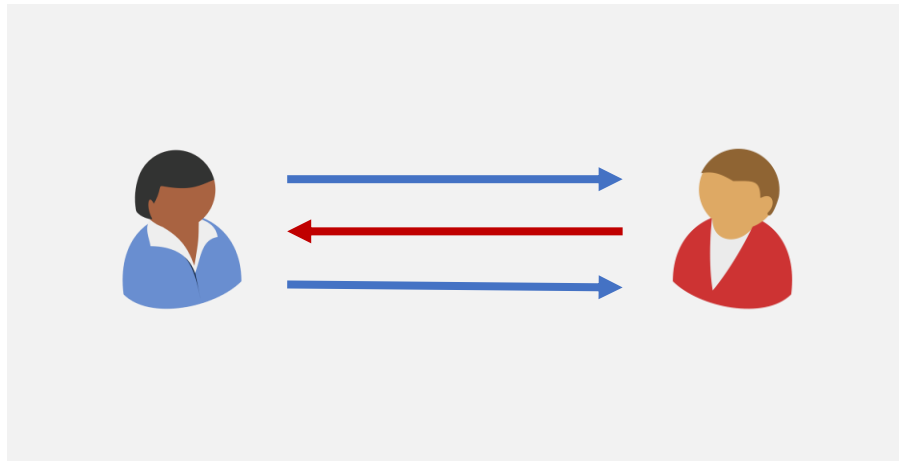
Fiat-Shamir (FS) Methodology



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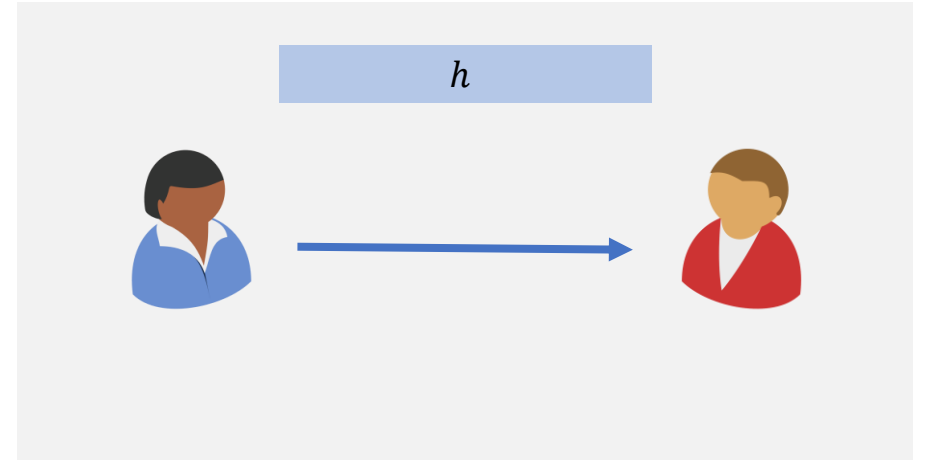
Succinct interactive arguments for NP.

Fiat-Shamir (FS) Methodology



[Kilian'92]

Instantiate with
CIH

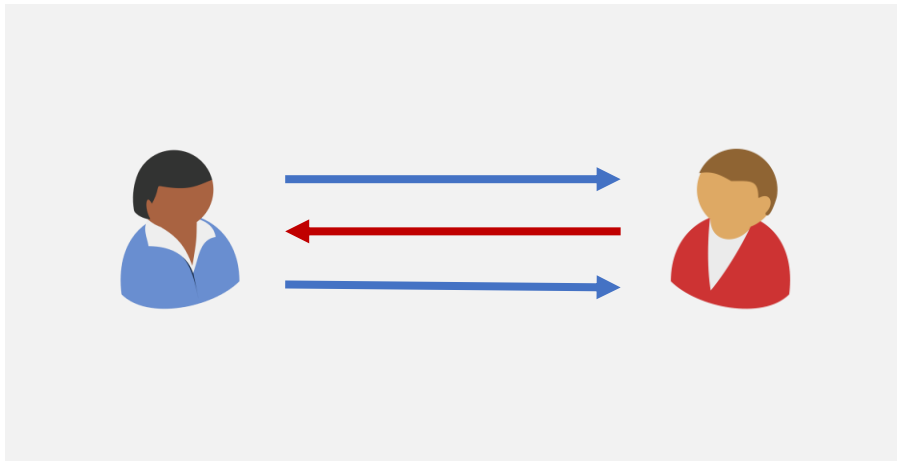


Succinct interactive arguments for NP.

[Bartusek-Bronfman-Holmgren-Ma-Rothblum'19]

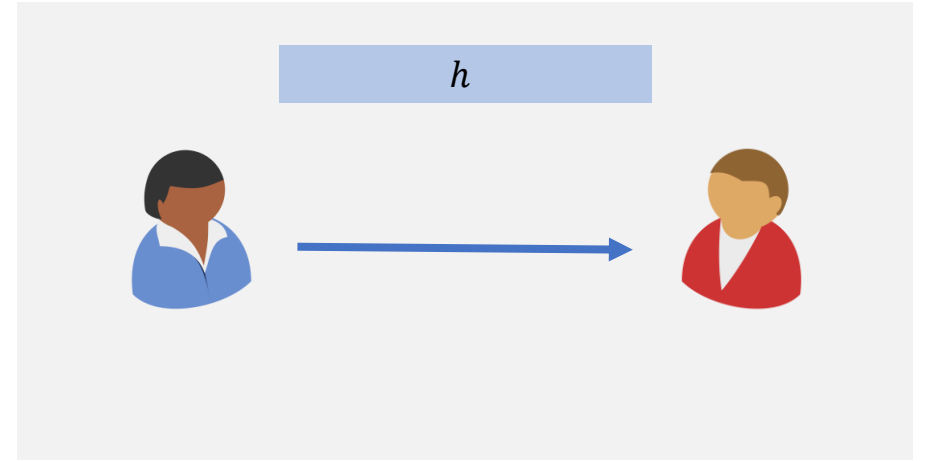
Instantiating hash function for Fiat-Shamir transformation of Kilian's protocol is hard.

Fiat-Shamir (FS) Methodology



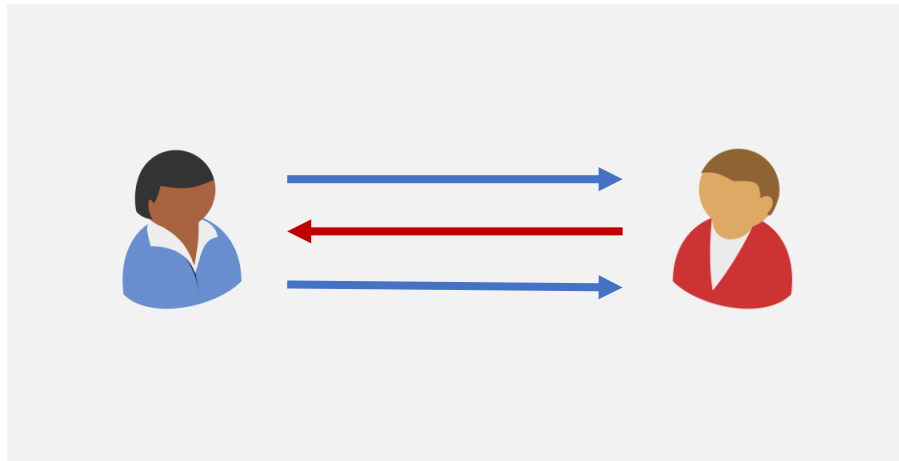
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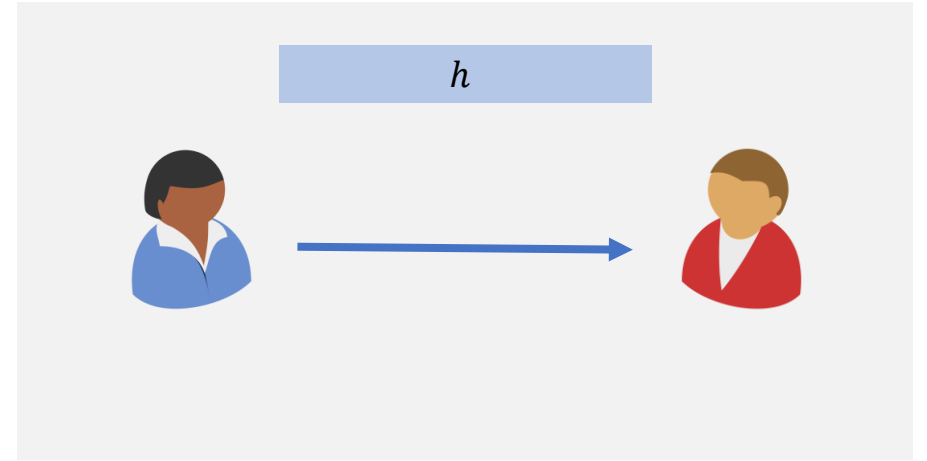
Succinct interactive **arguments** for NP.

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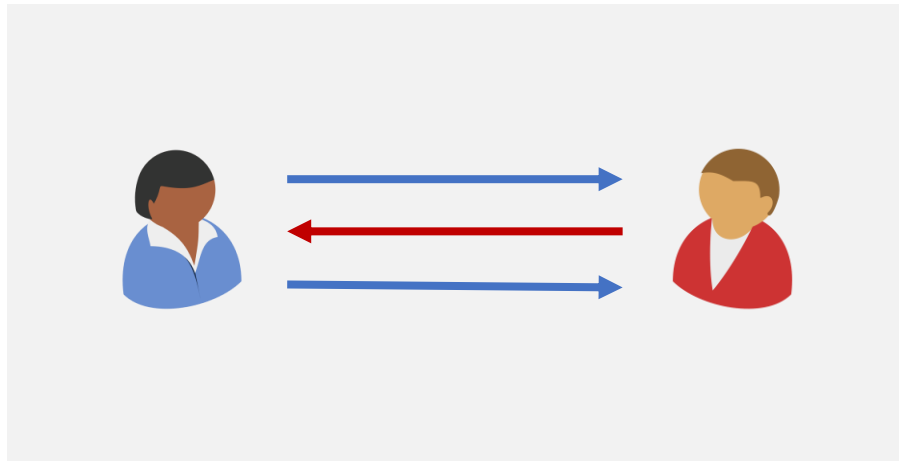
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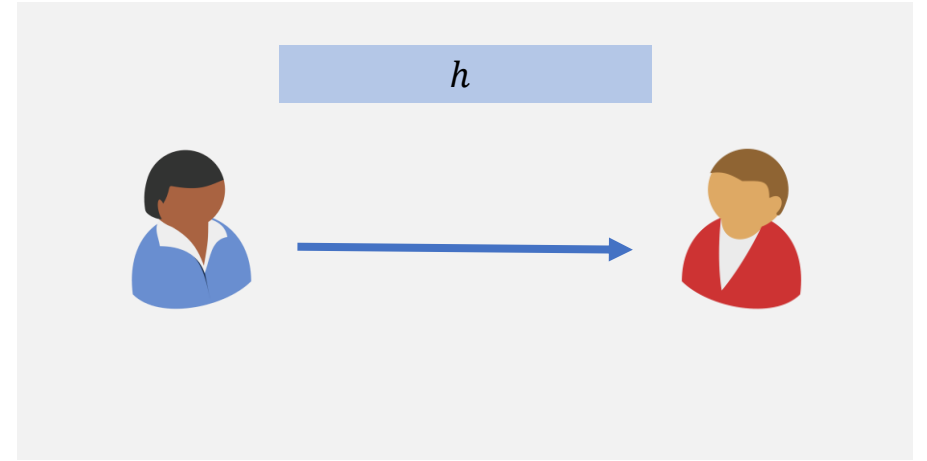
Known **instantiations** of CI Hash for Fiat-Shamir transform are for **proofs**.

Fiat-Shamir (FS) Methodology



[Goldwasser-Kalai-Rothblum'08]

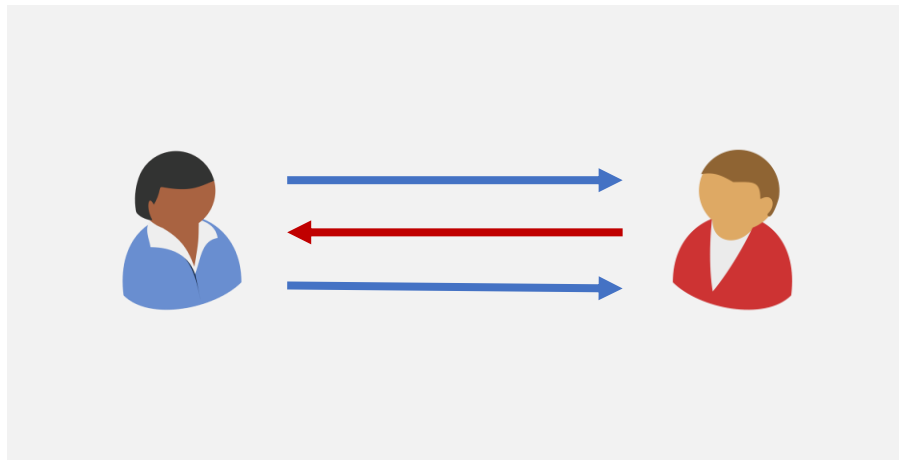
Instantiate with
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Succinct interactive proof for depth
bounded computation.

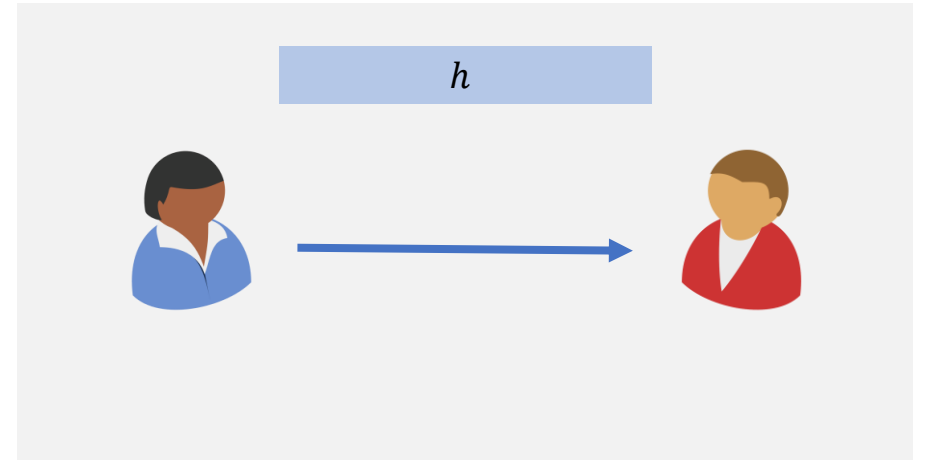
[Canetti-Chen-Holmgren-Lombardi-Rothblum-Rothblum-Wichs'19,
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[Goldwasser-Kalai-Rothblum'08]

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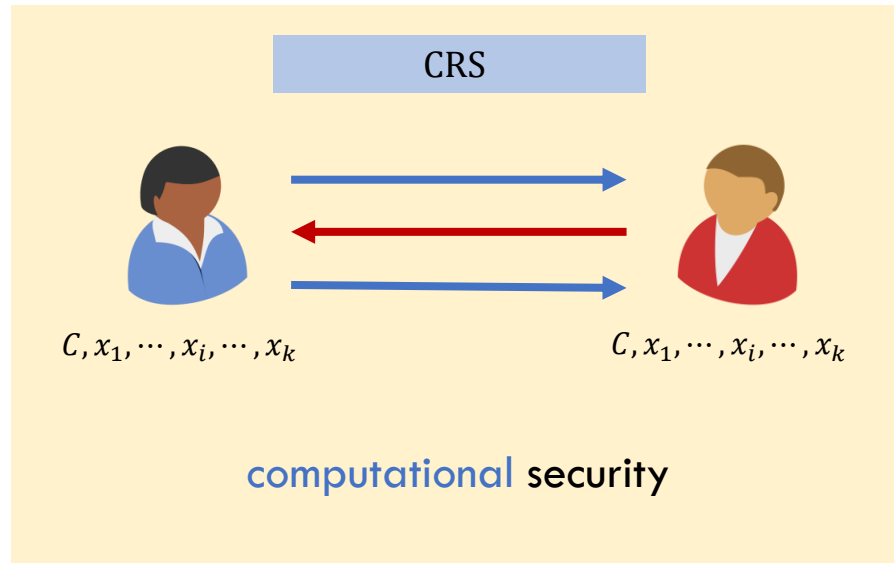
[Canetti-Chen-Holmgren-Lombardi-Rothblum-Rothblum-Wichs'19, Jawale-Kalai-Khurana-Zhang'21]

- Interactive proofs for all polynomial time computation **unlikely to exist**.
- **No known** interactive proof for batch NP.

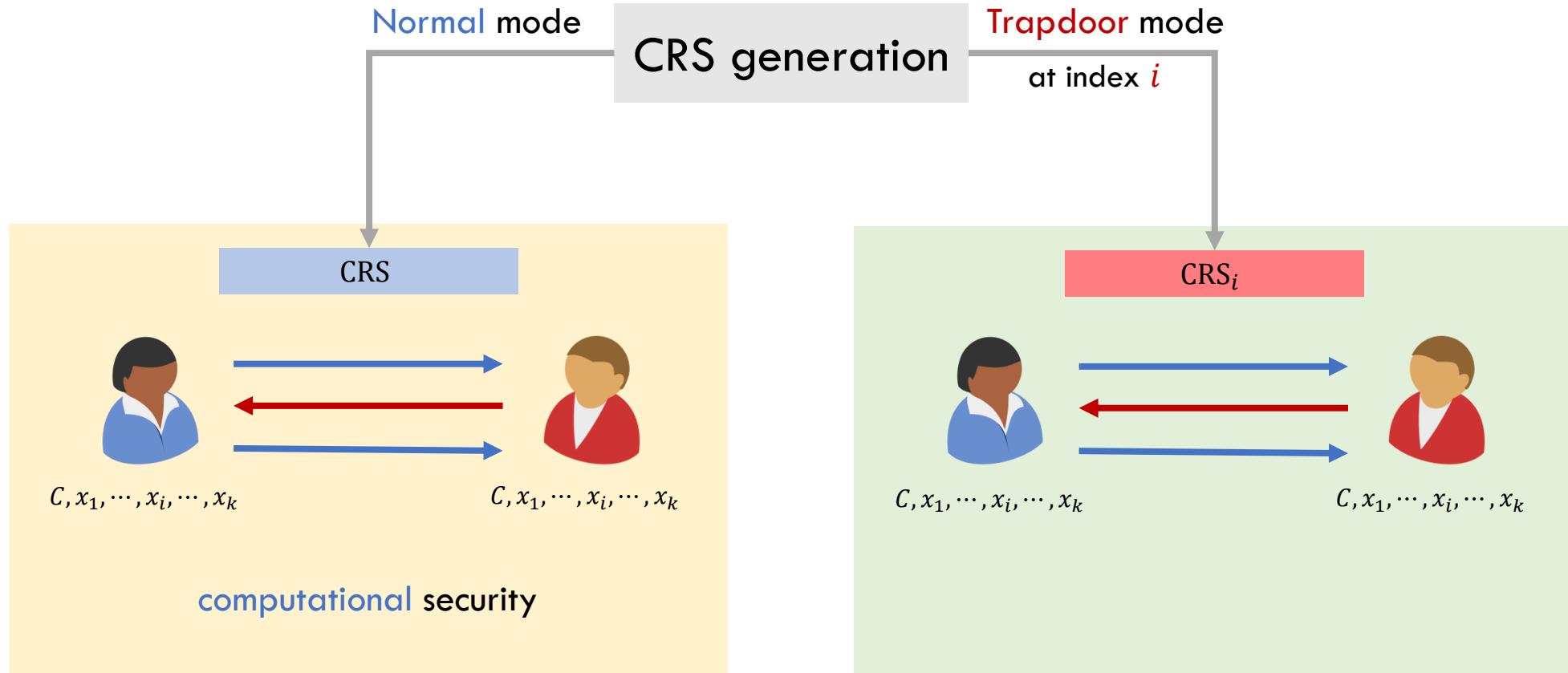
A Different Starting Point

Dual-Mode Interactive Batch Arguments [C-Jain-Jin'21 a]

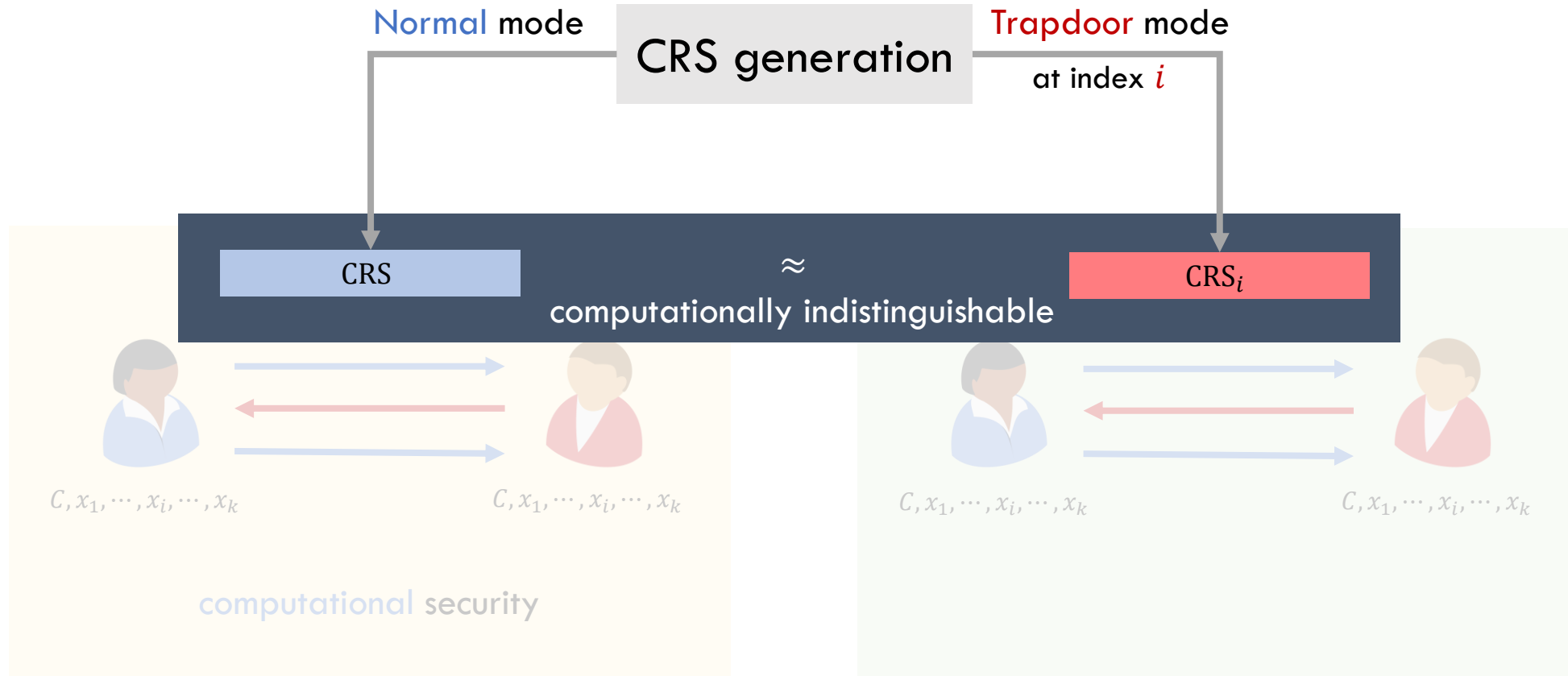
Dual-Mode Interactive Batch Arguments



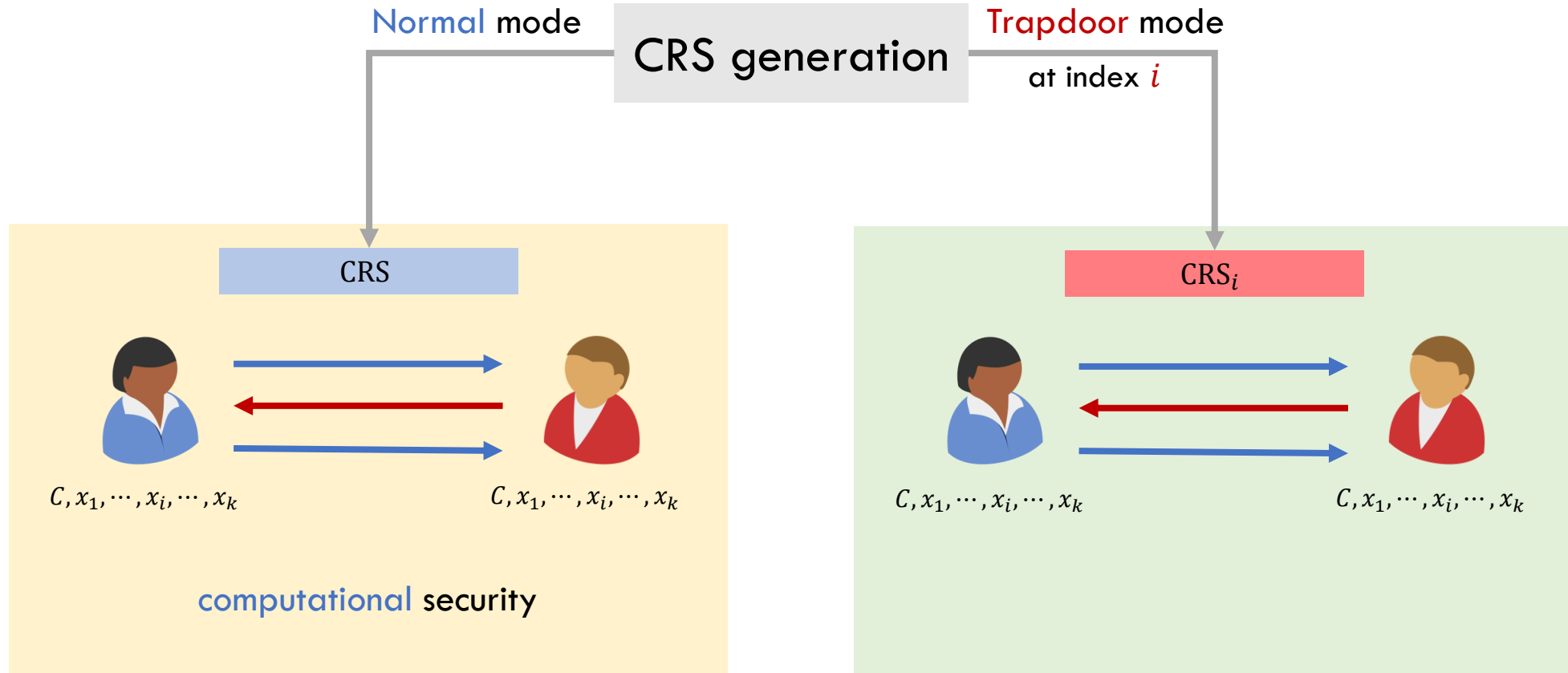
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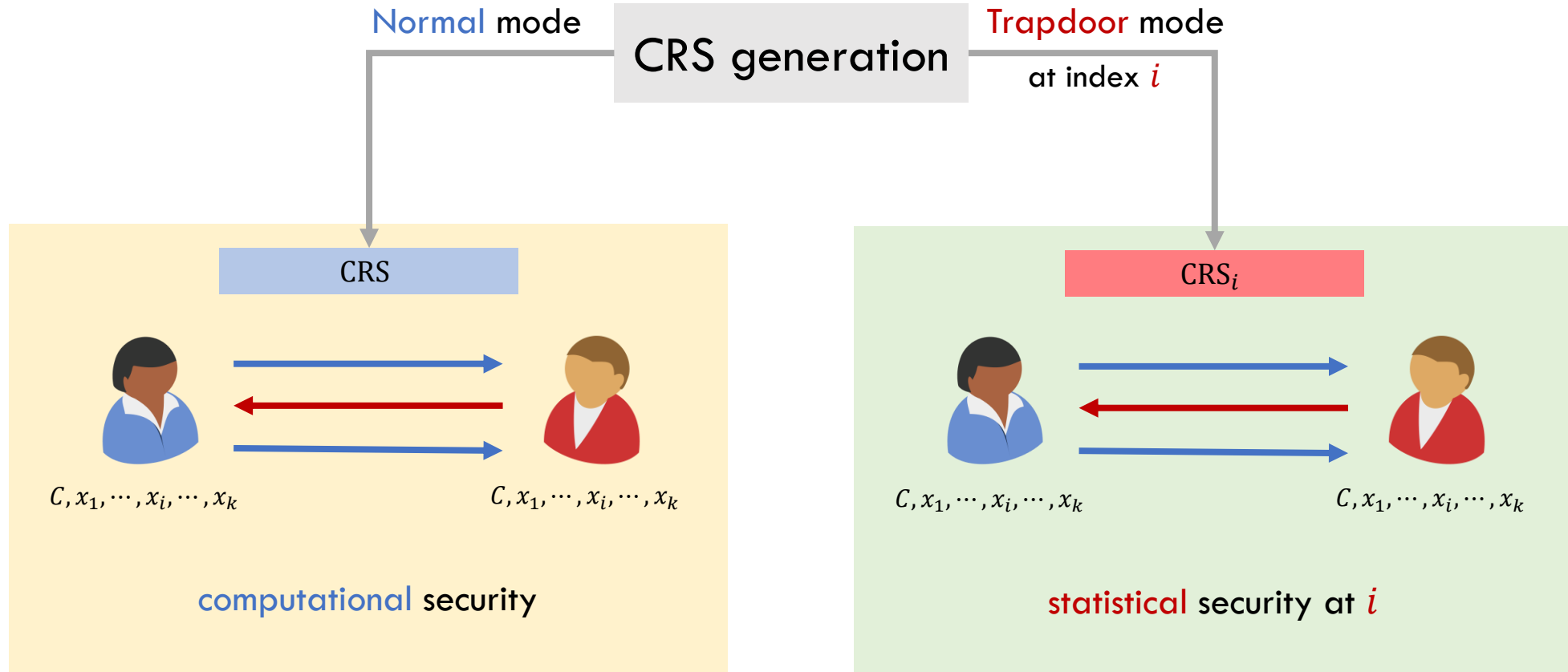
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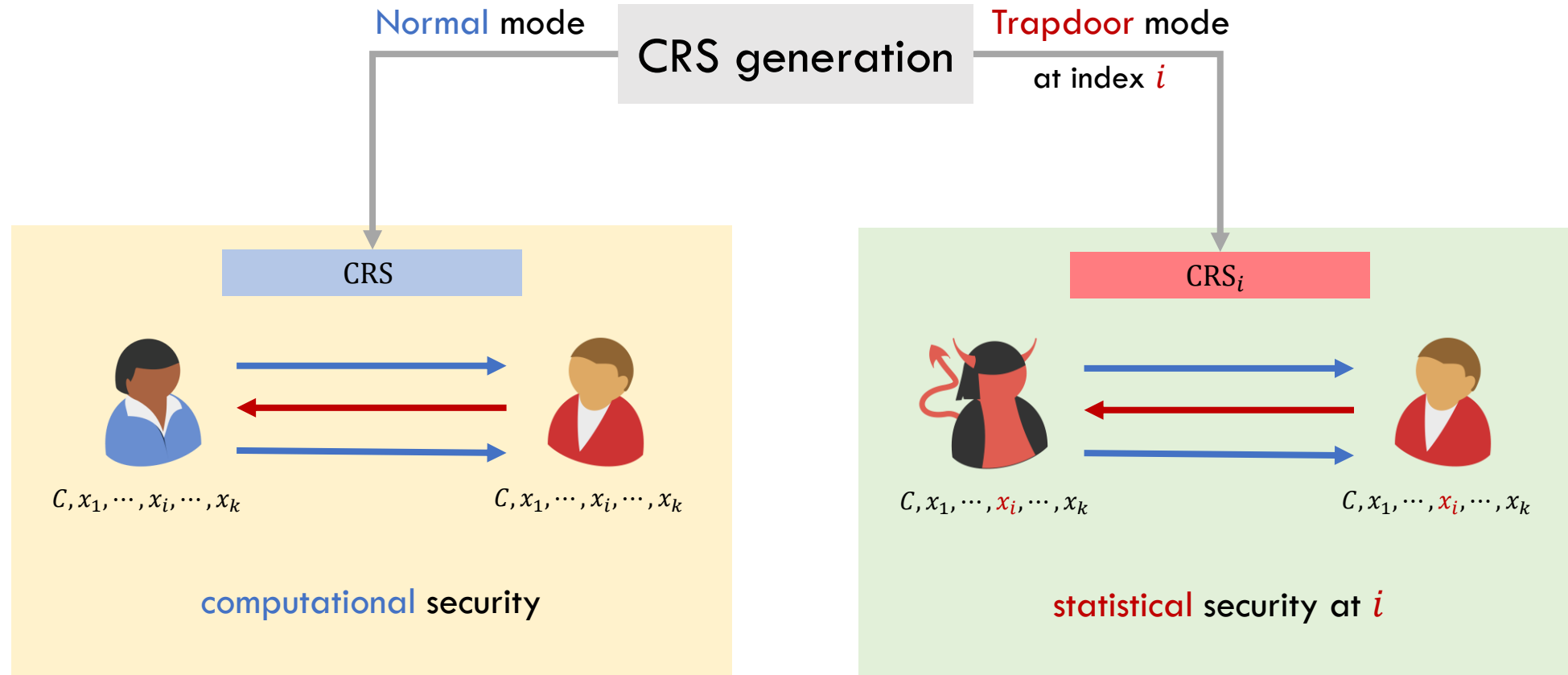
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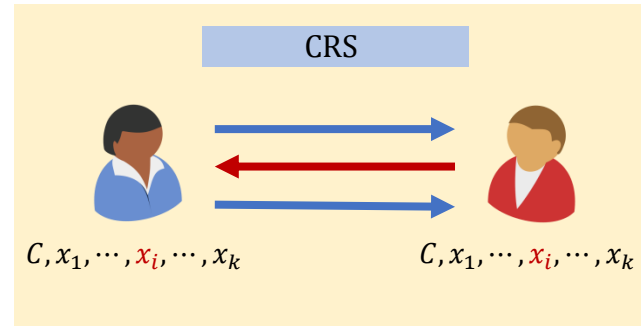


Dual-Mode Interactive Batch Arguments

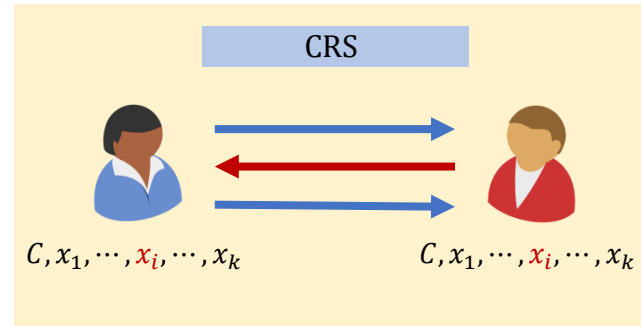


Even **unbounded**  cannot make  accept if $(C, x_i) \notin \text{SAT}$

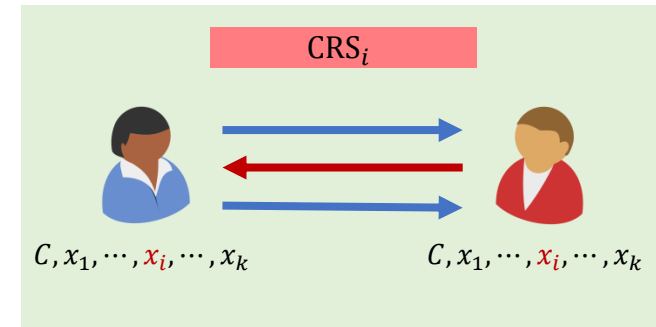
Security Intuition



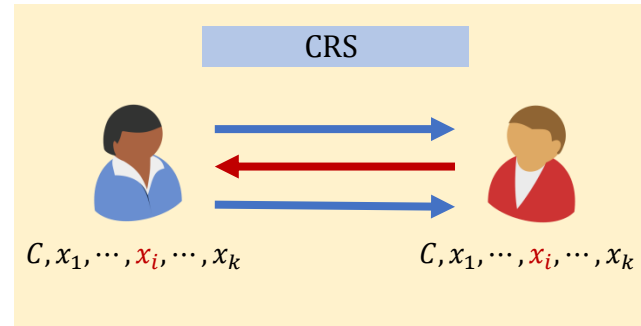
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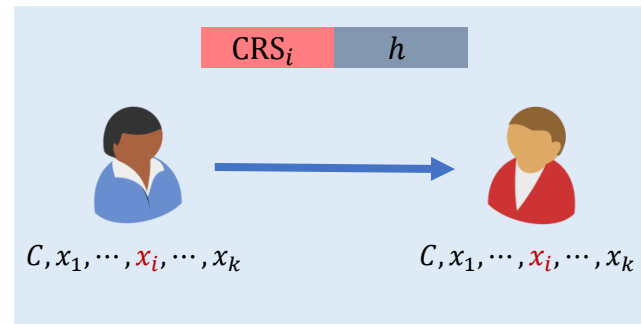
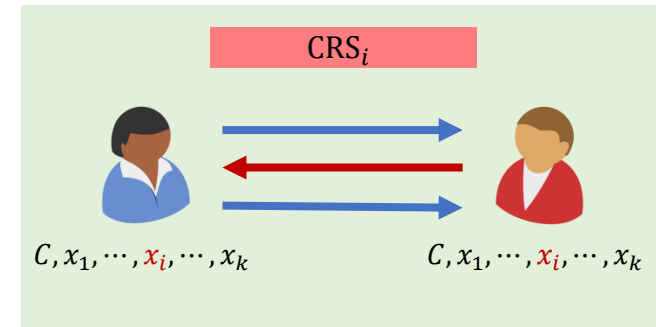
Switch to trapdoor mode at i



Security Intuition



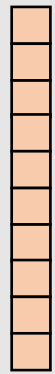
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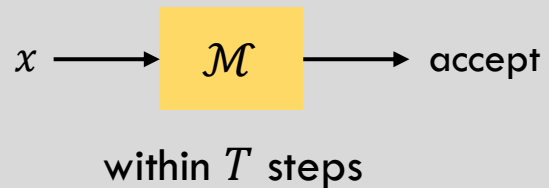
Rely on FS transformation

SNARGs for Batch NP \rightarrow SNARGs for P

SNARGs for Polynomial-time Computation

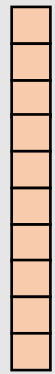


st_0

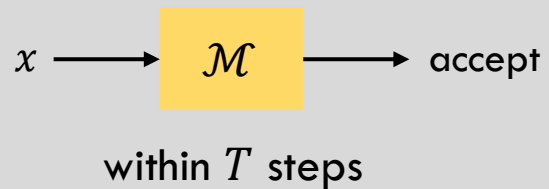


This talk: Bounded space computation

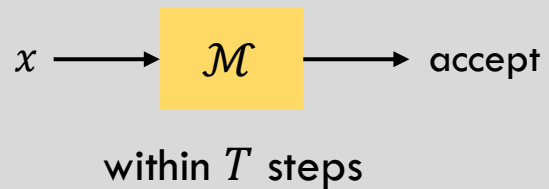
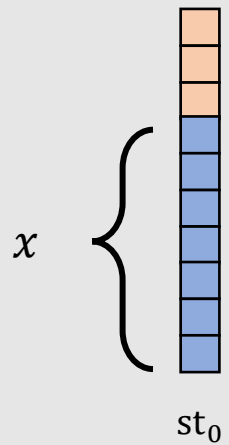
SNARGs for Polynomial-time Computation



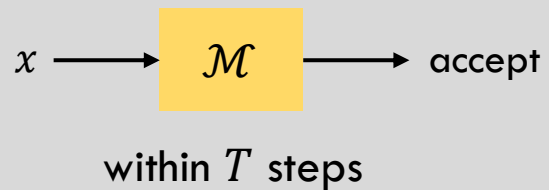
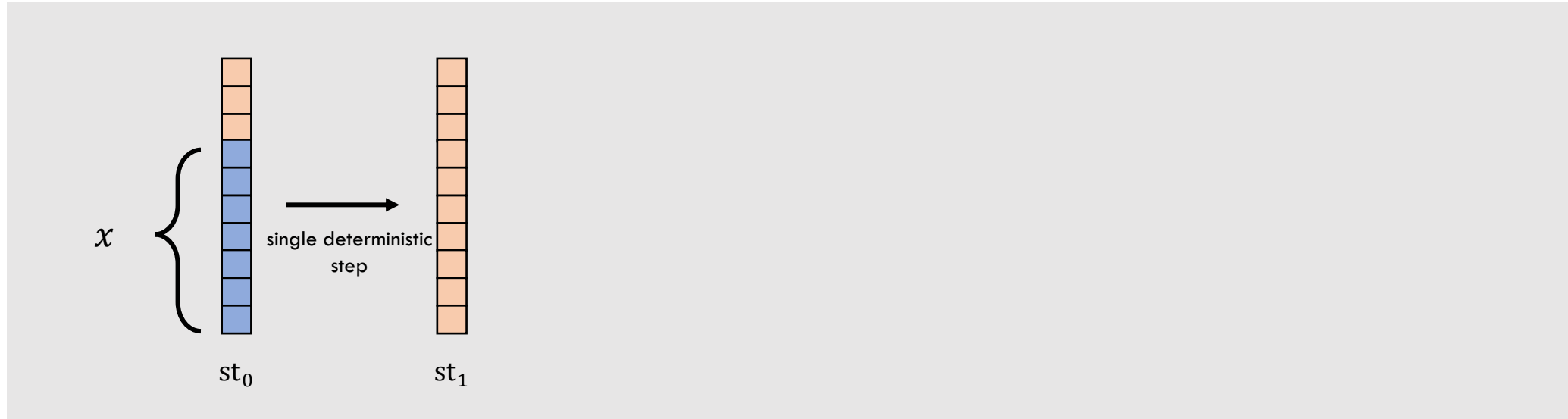
st_0



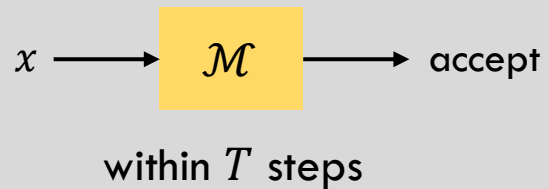
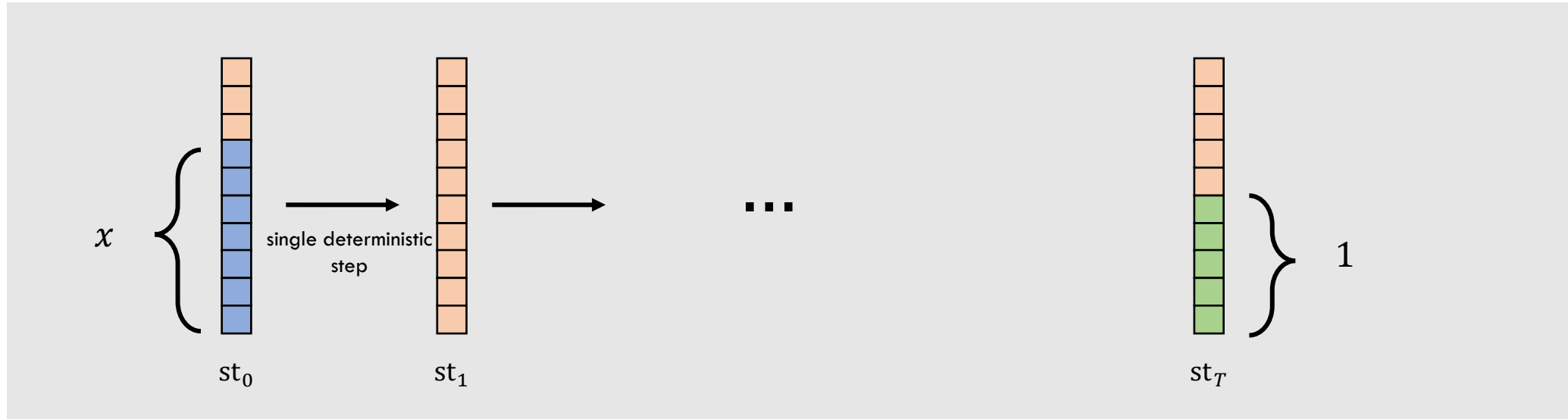
SNARGs for Polynomial-time Computation



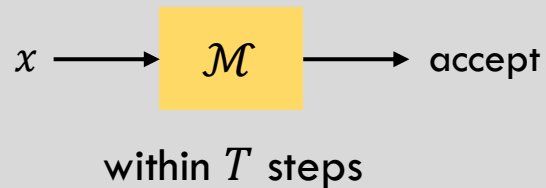
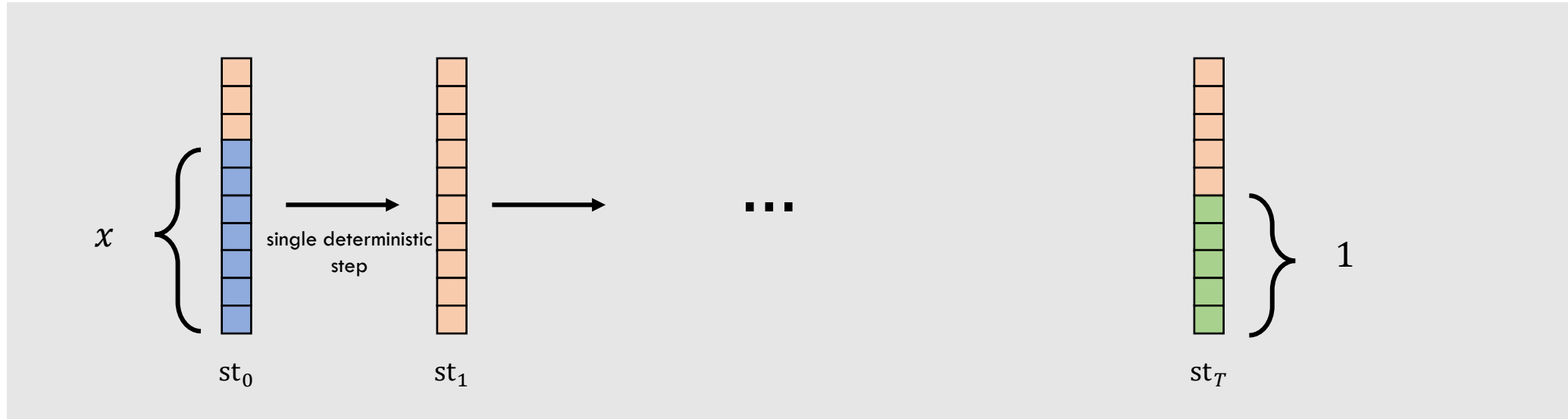
SNARGs for Polynomial-time Computation



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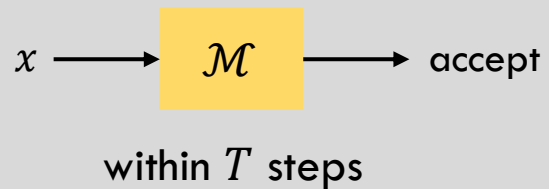
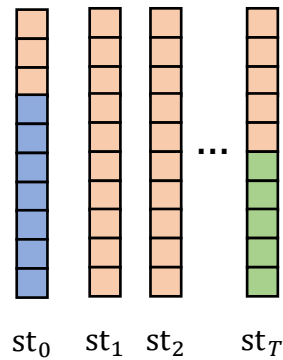


SNARGs for Polynomial-time Computation

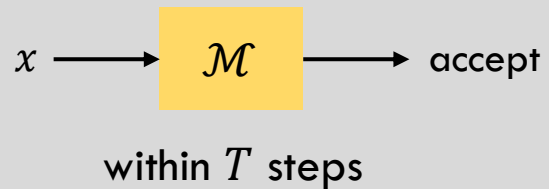
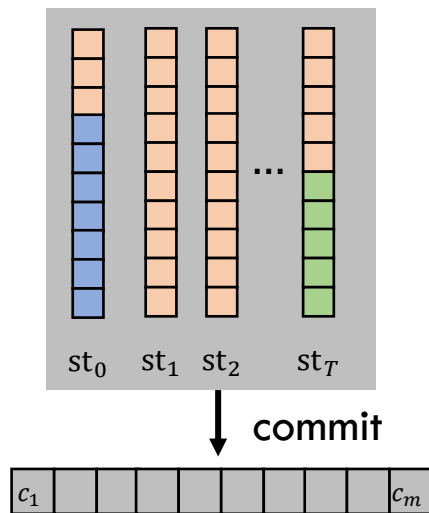


Prove for every $i \in [0, \dots, T - 1]$
 $st_i \rightarrow st_{i+1}$
is the correct transition.

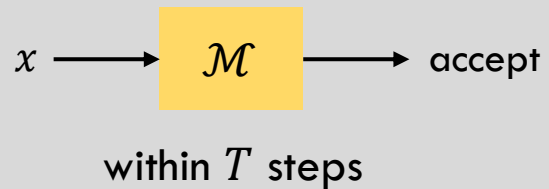
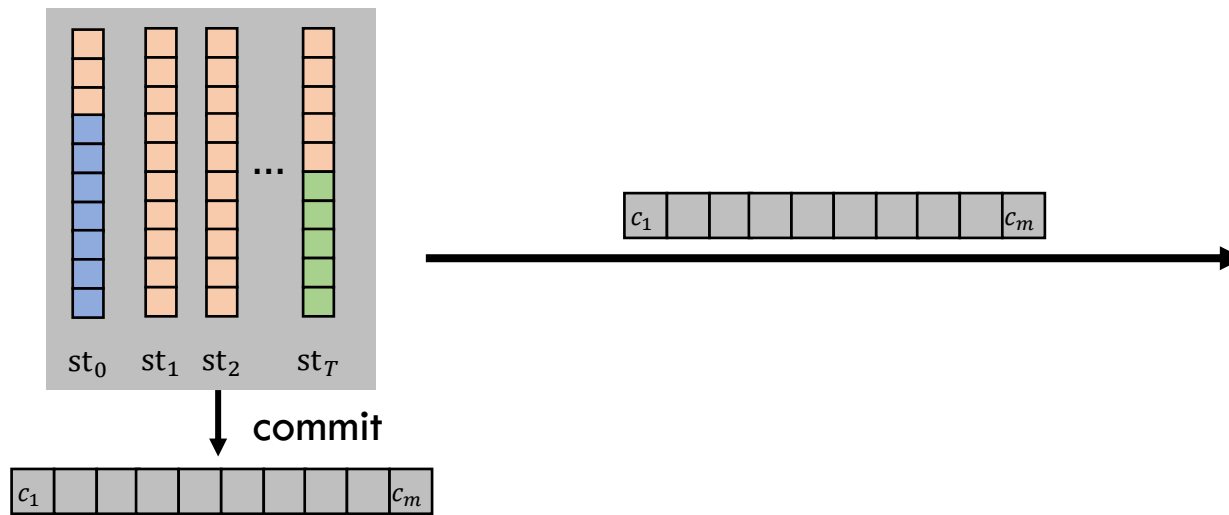
SNARGs for Polynomial-time Computation



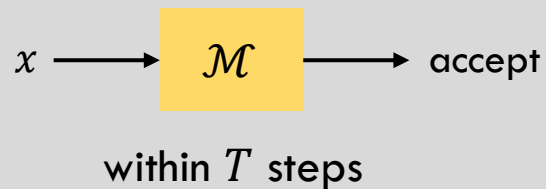
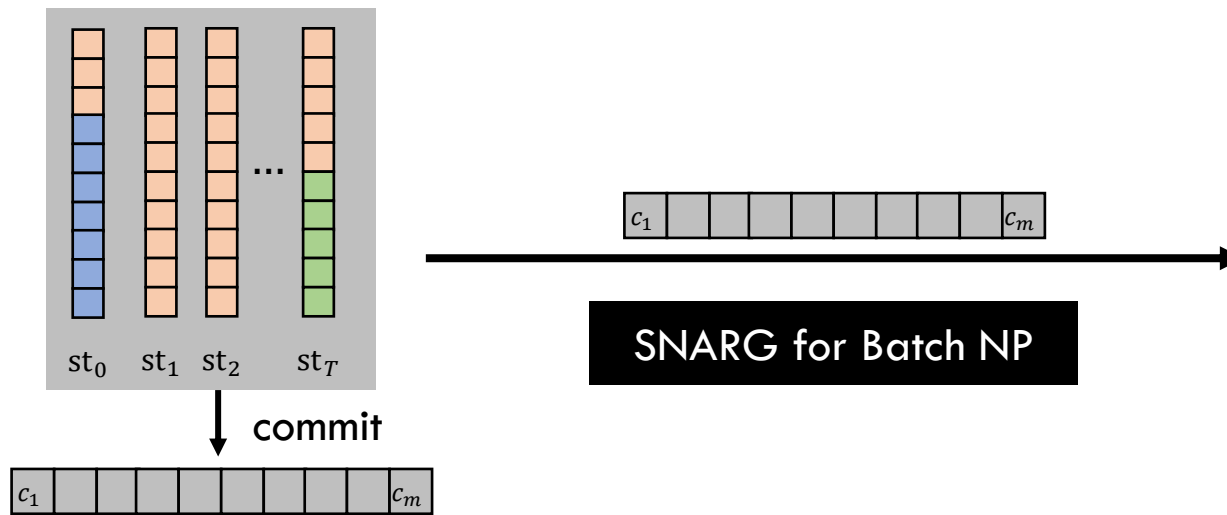
SNARGs for Polynomial-time Computation



SNARGs for Polynomial-time Computation



SNARGs for Polynomial-time Computation

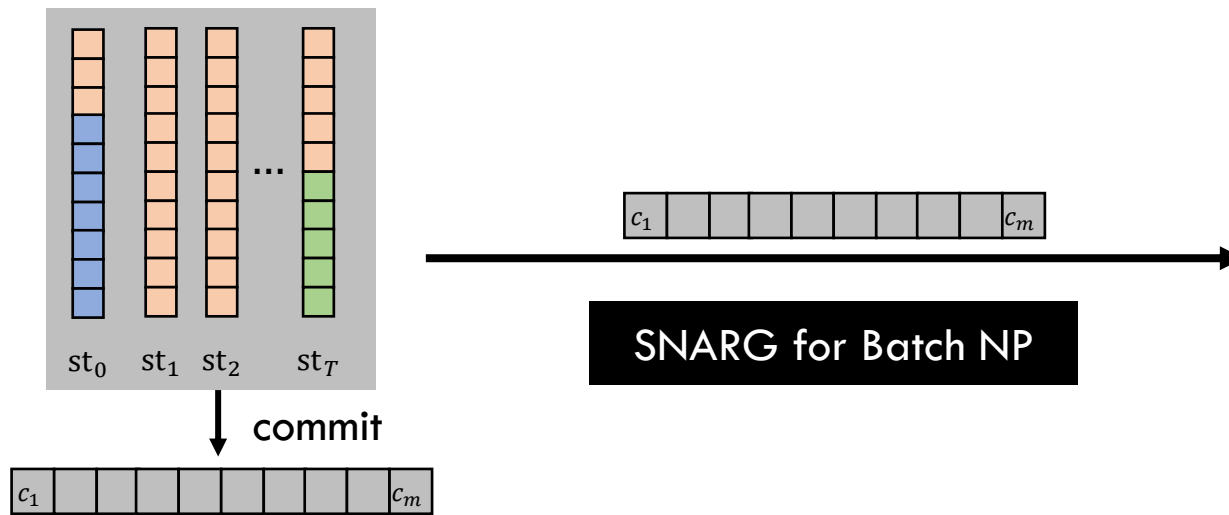


SNARG for Batch NP

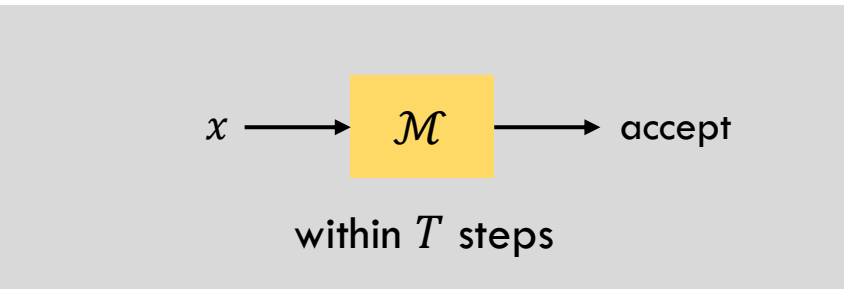
For every $i \in [0, \dots, T - 1]$

1. Commitment contains st_i and st_{i+1}
2. Valid transition $st_i \rightarrow st_{i+1}$

SNARGs for Polynomial-time Computation



Verifier needs to read all the instances $\Omega(T)$



SNARG for Batch NP
For every $i \in [0, \dots, T - 1]$

1. Commitment contains st_i and st_{i+1}
2. Valid transition $st_i \rightarrow st_{i+1}$

SNARGs for Batch Index

$$L_C = \{i \mid \exists w \text{ s.t. } C(i, w) = 1\}$$

$$\forall i \in [k], i \in L_C$$



C, k

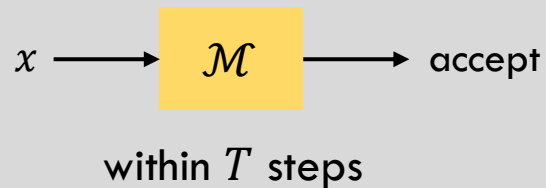
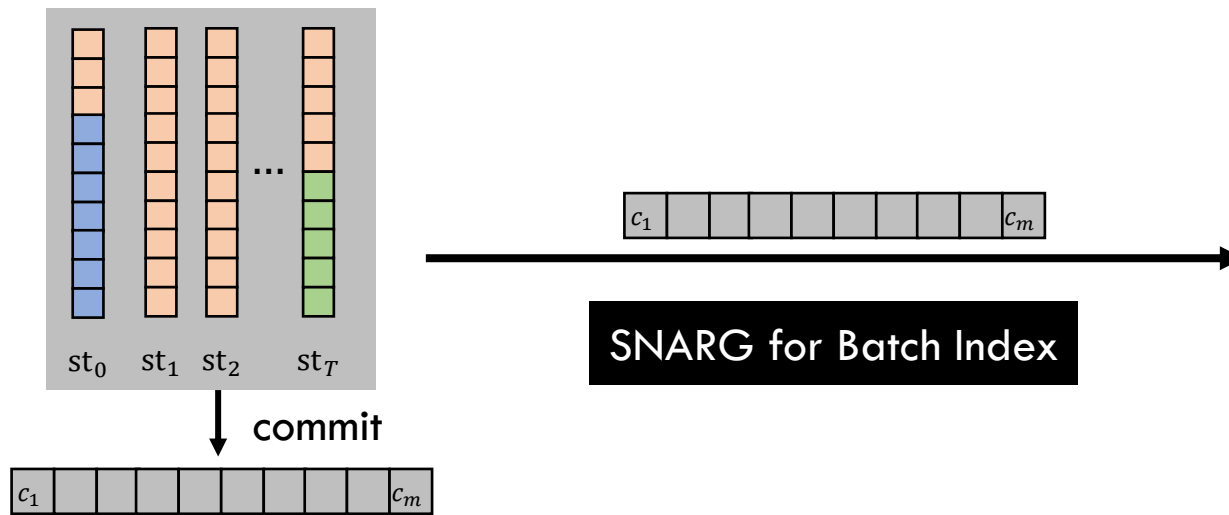
Π



C, k

Verifier **running time**:
 $\text{poly}(\log k, |C|)$

SNARGs for Polynomial-time Computation

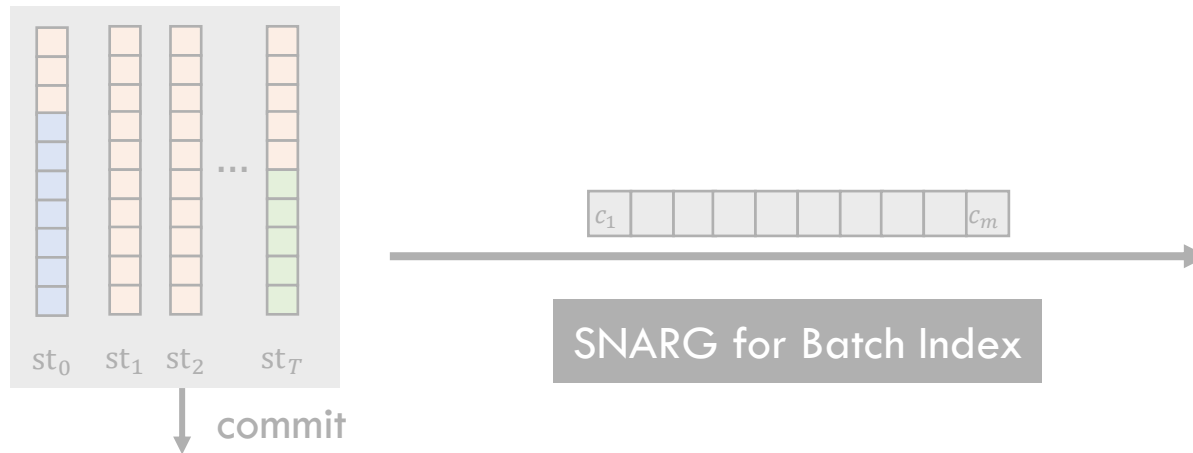


SNARG for Batch Index

For every $i \in [0, \dots, T - 1]$

1. Commitment contains st_i and st_{i+1}
2. Valid transition $st_i \rightarrow st_{i+1}$

SNARGs for Polynomial-time Computation



Further Challenges: Ensuring **Global consistency across all states**

Use **Somewhere Statistical Binding (SSB)** Commitments [Hubáček -Wichs'15] and **No-Signaling SSB Commitments** [González-Zacharakis'21]

x —

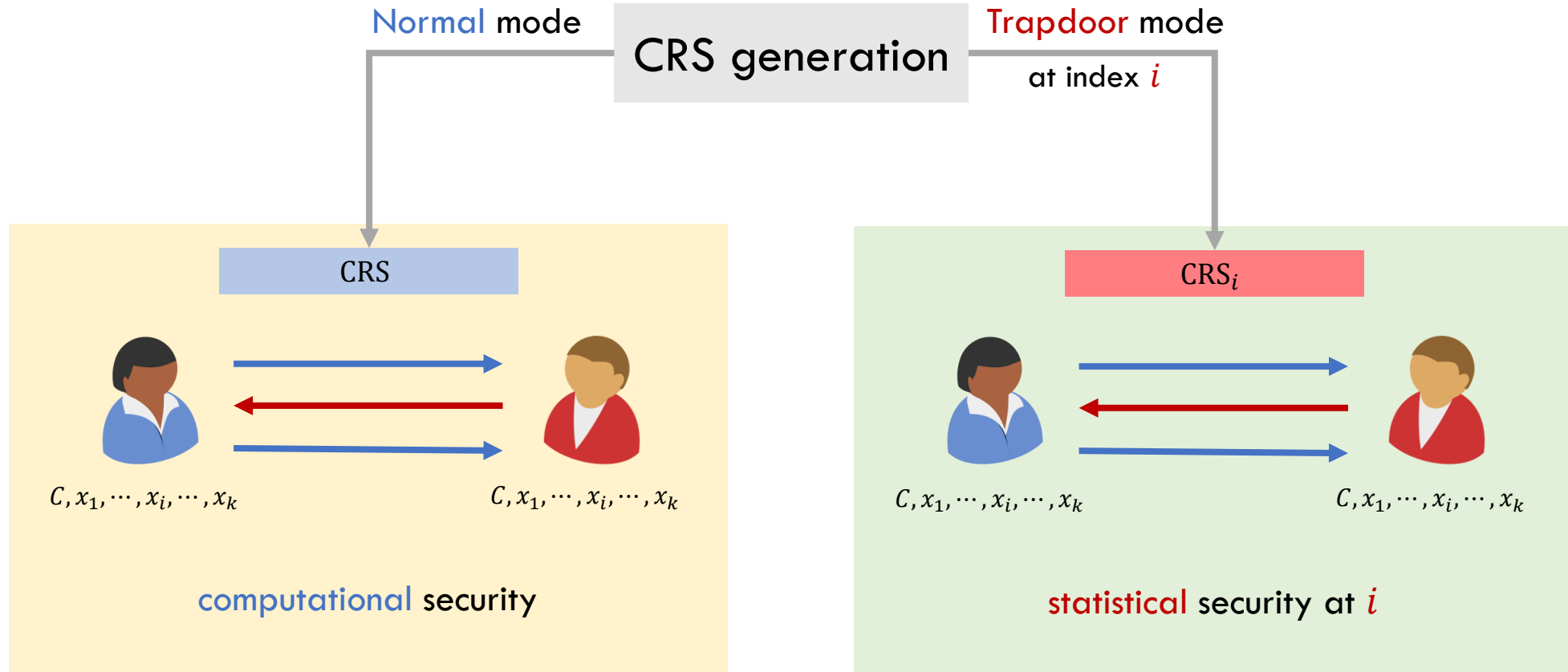
ex

and st_{i+1}

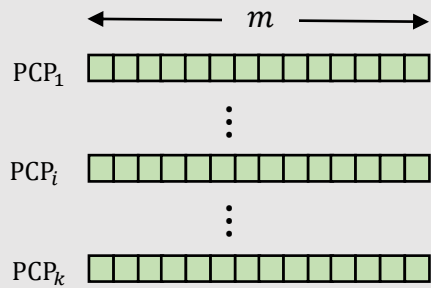
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SNARGs for Batch Index

Dual-Mode Interactive Batch Arguments



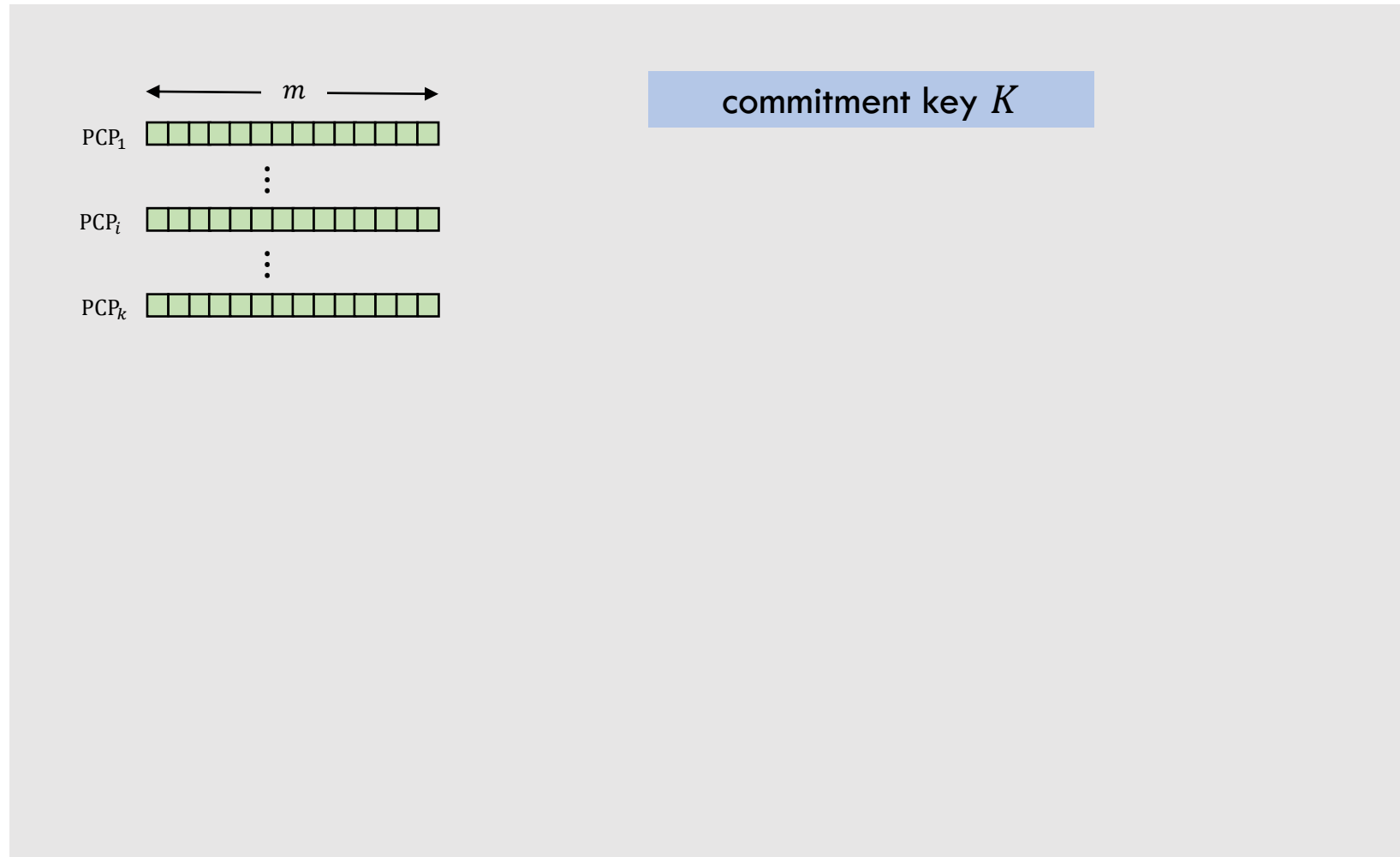
Dual Mode Argument for Batch Index



$$L_C = \{i \mid \exists w \text{ s.t. } C(i, w) = 1\}$$

$$\forall i \in [k], i \in L_C$$

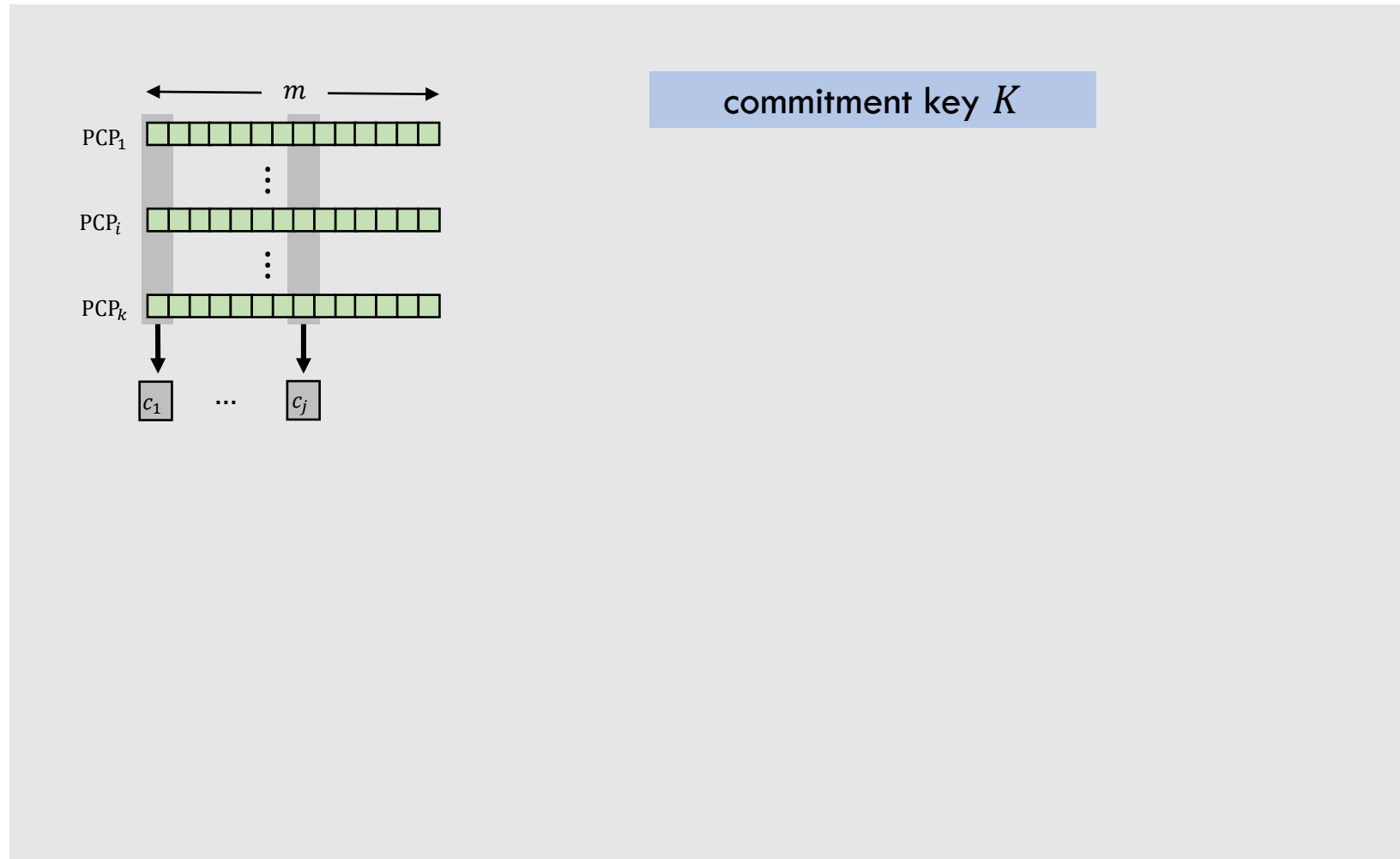
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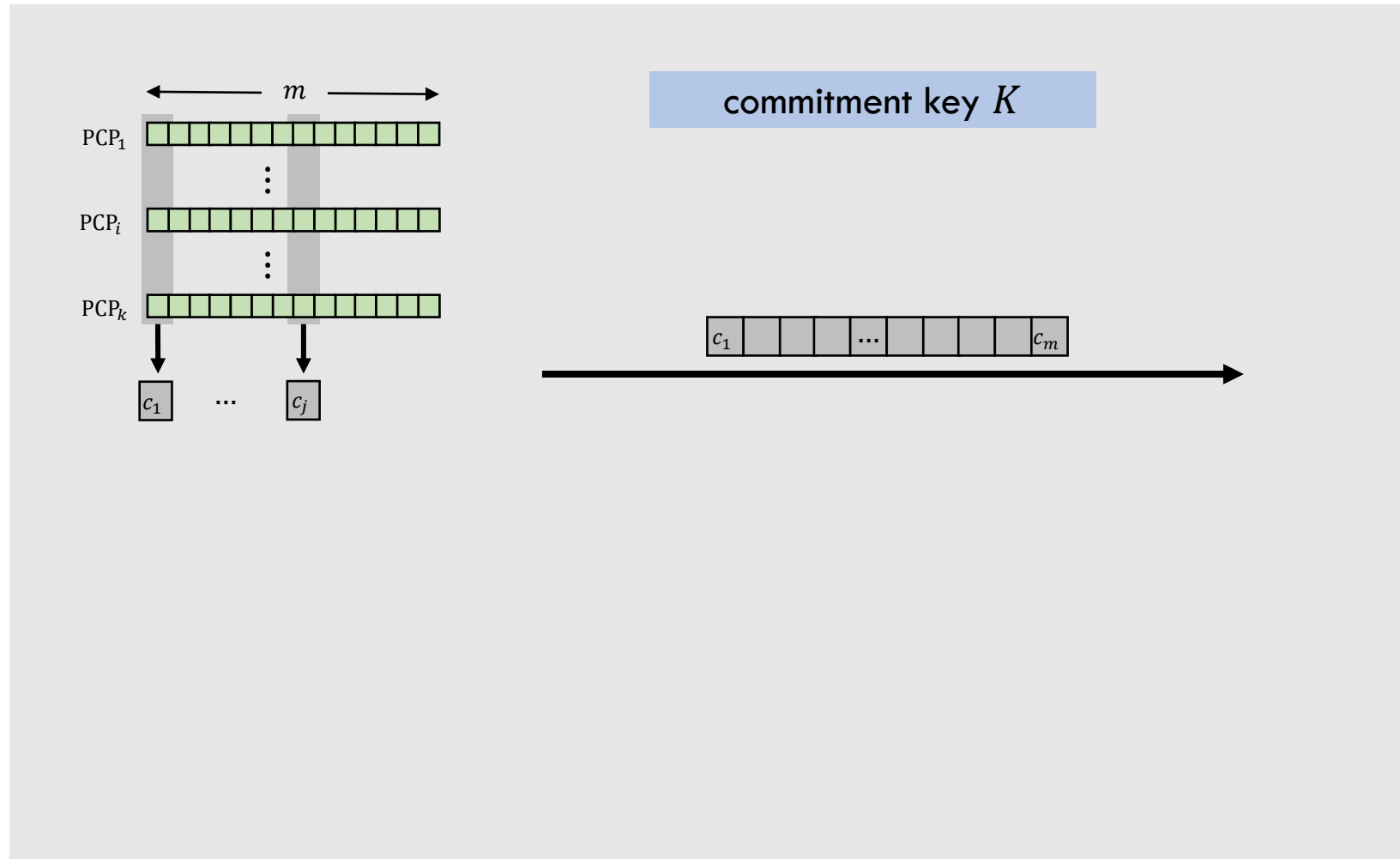
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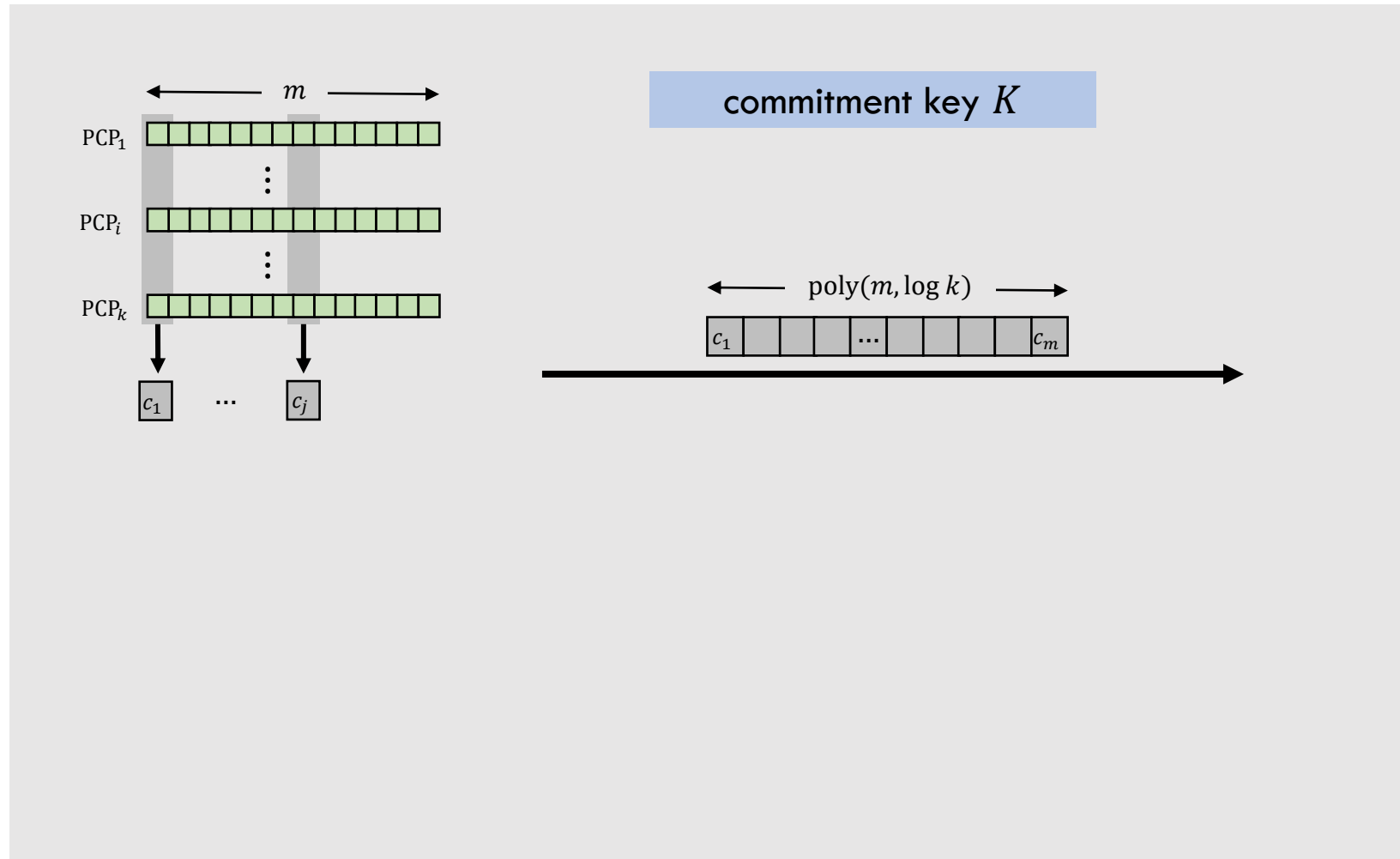
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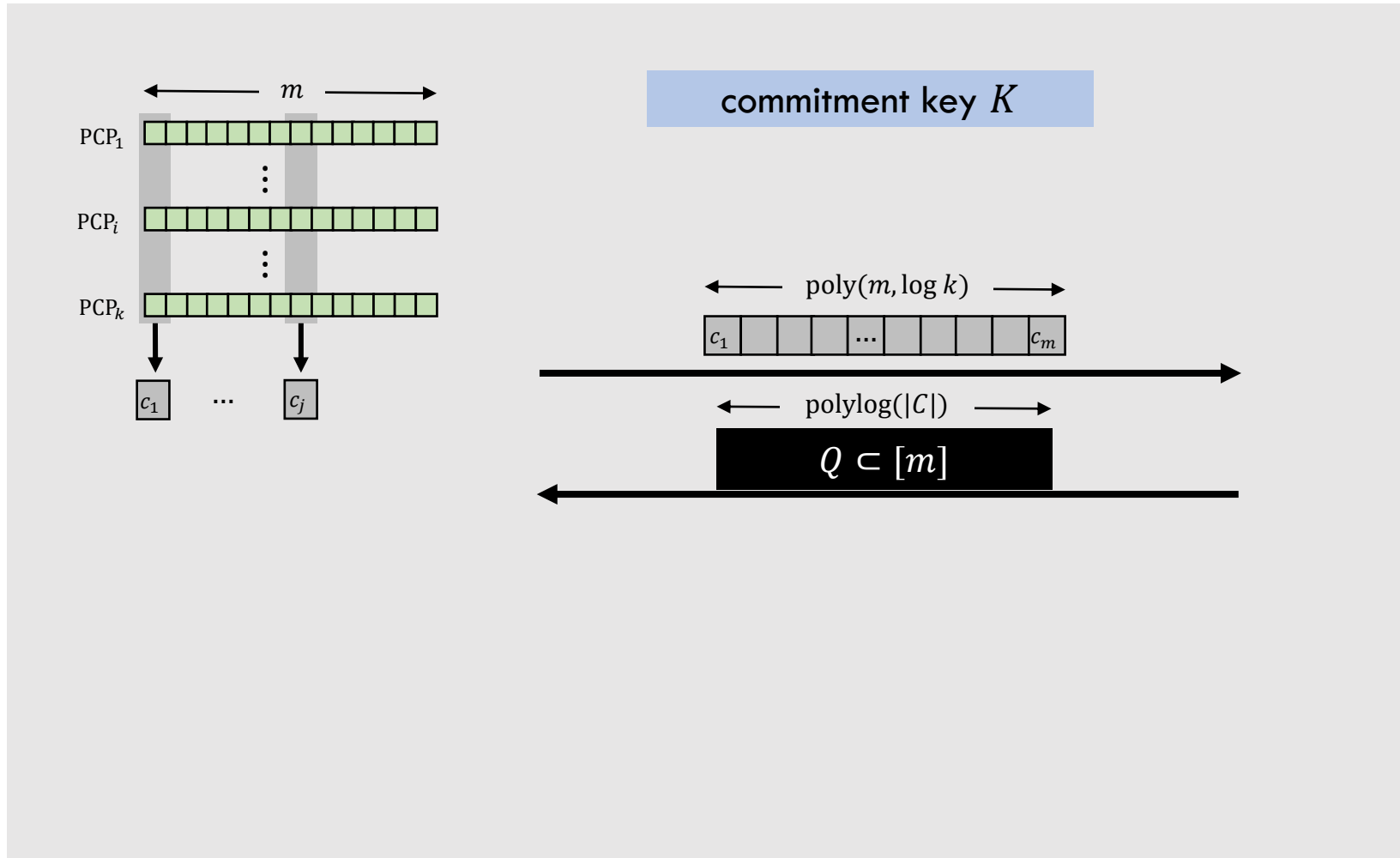
Dual Mode Argument for Batch Index



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Dual Mode Argument for Batch Index



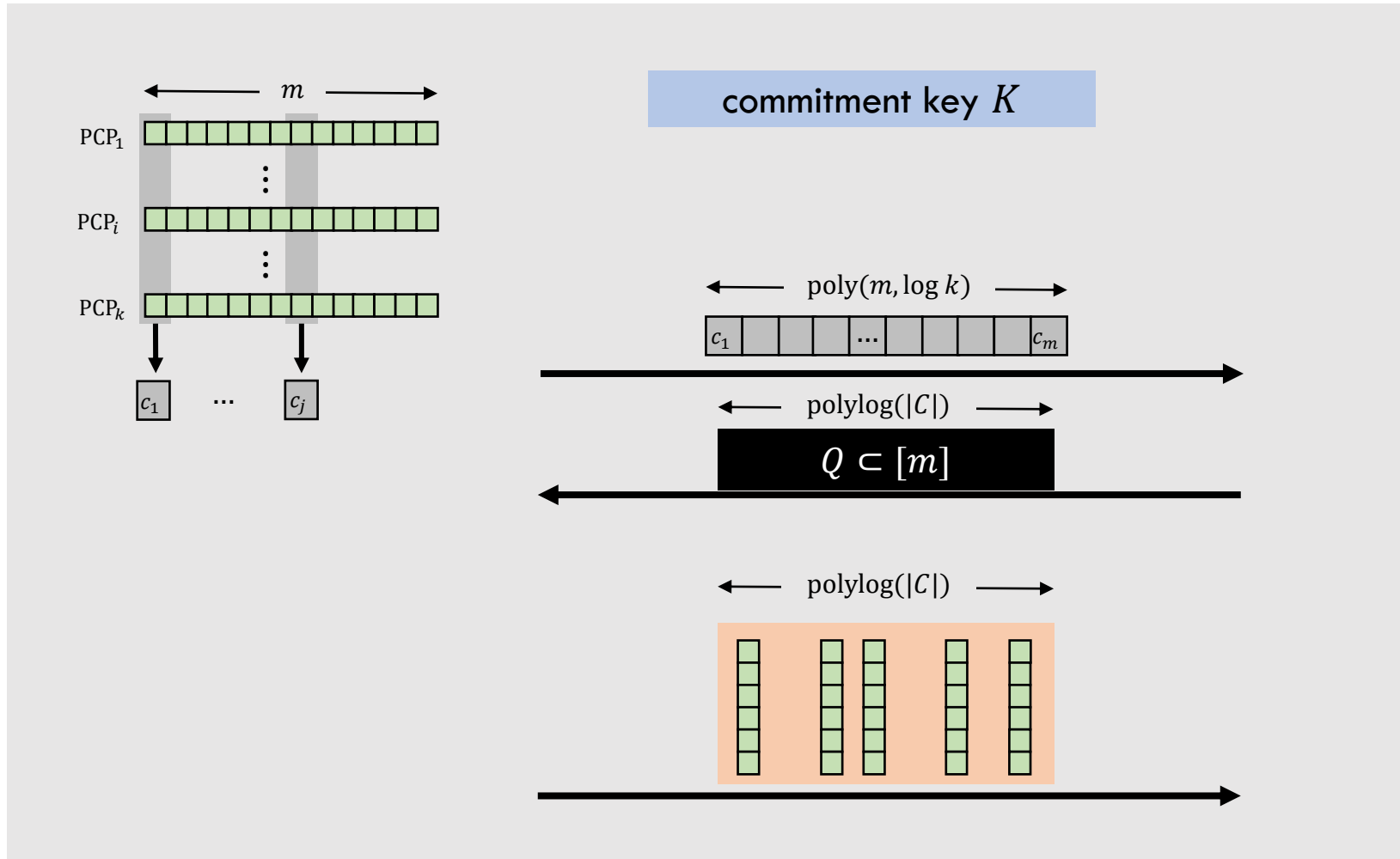
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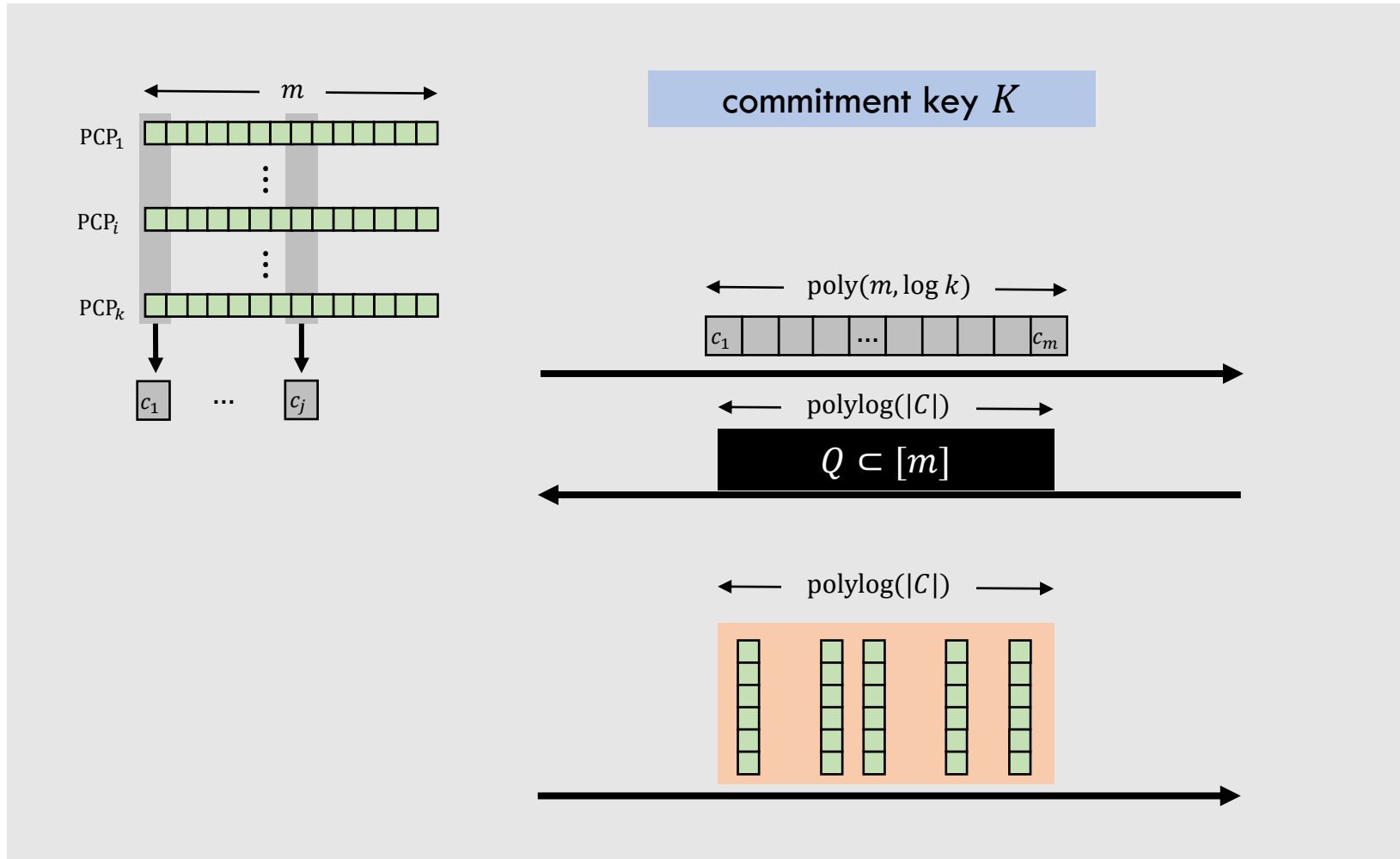
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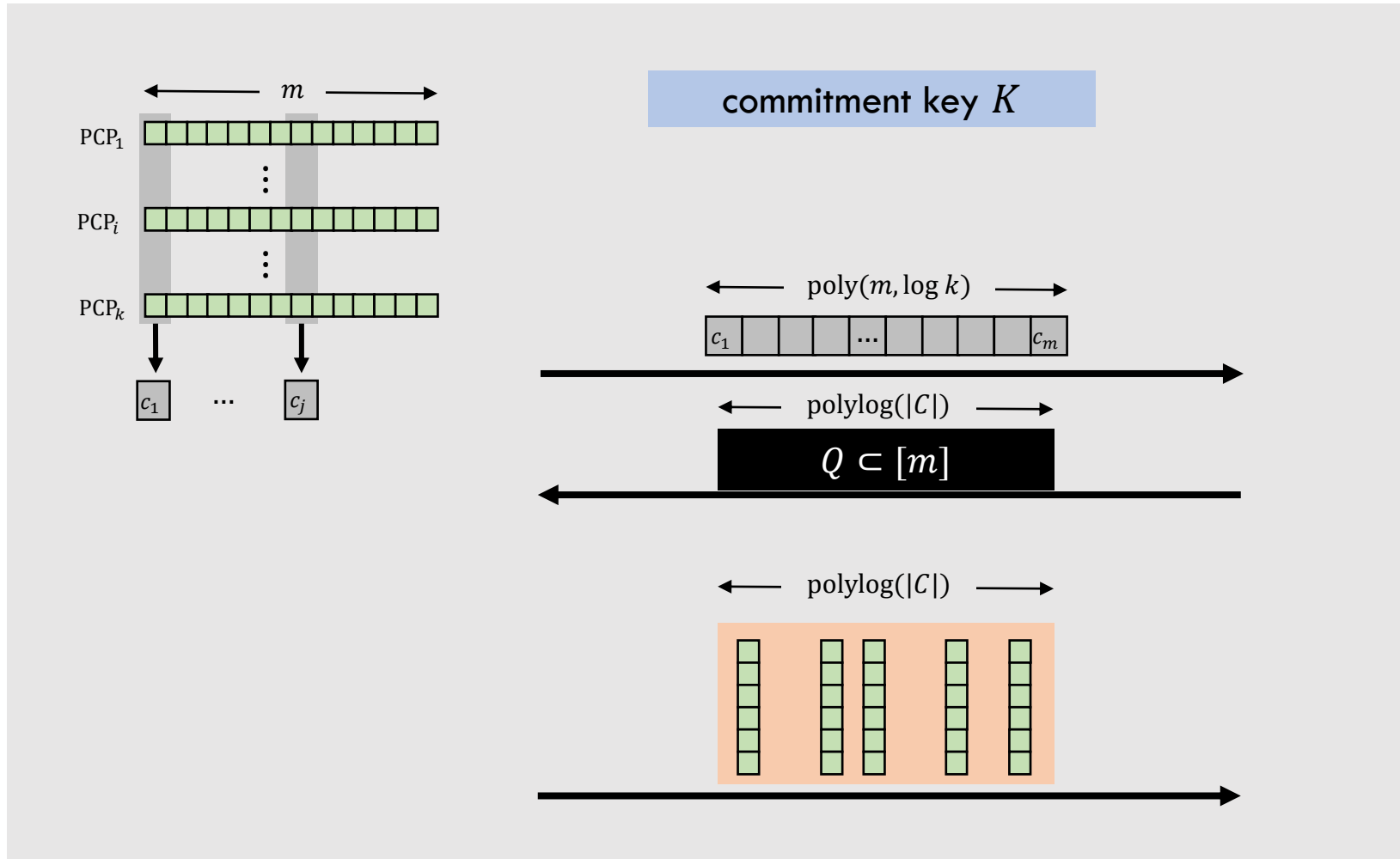
$$L_C = \{i \mid \exists w \text{ s.t. } C(i, w) = 1\}$$

$$\forall i \in [k], i \in L_C$$

Verify:

1. Commitment openings are valid.
2. PCP responses verify on Q .

Dual Mode Argument for Batch Index



$$L_C = \{i \mid \exists w \text{ s.t. } C(i, w) = 1\}$$

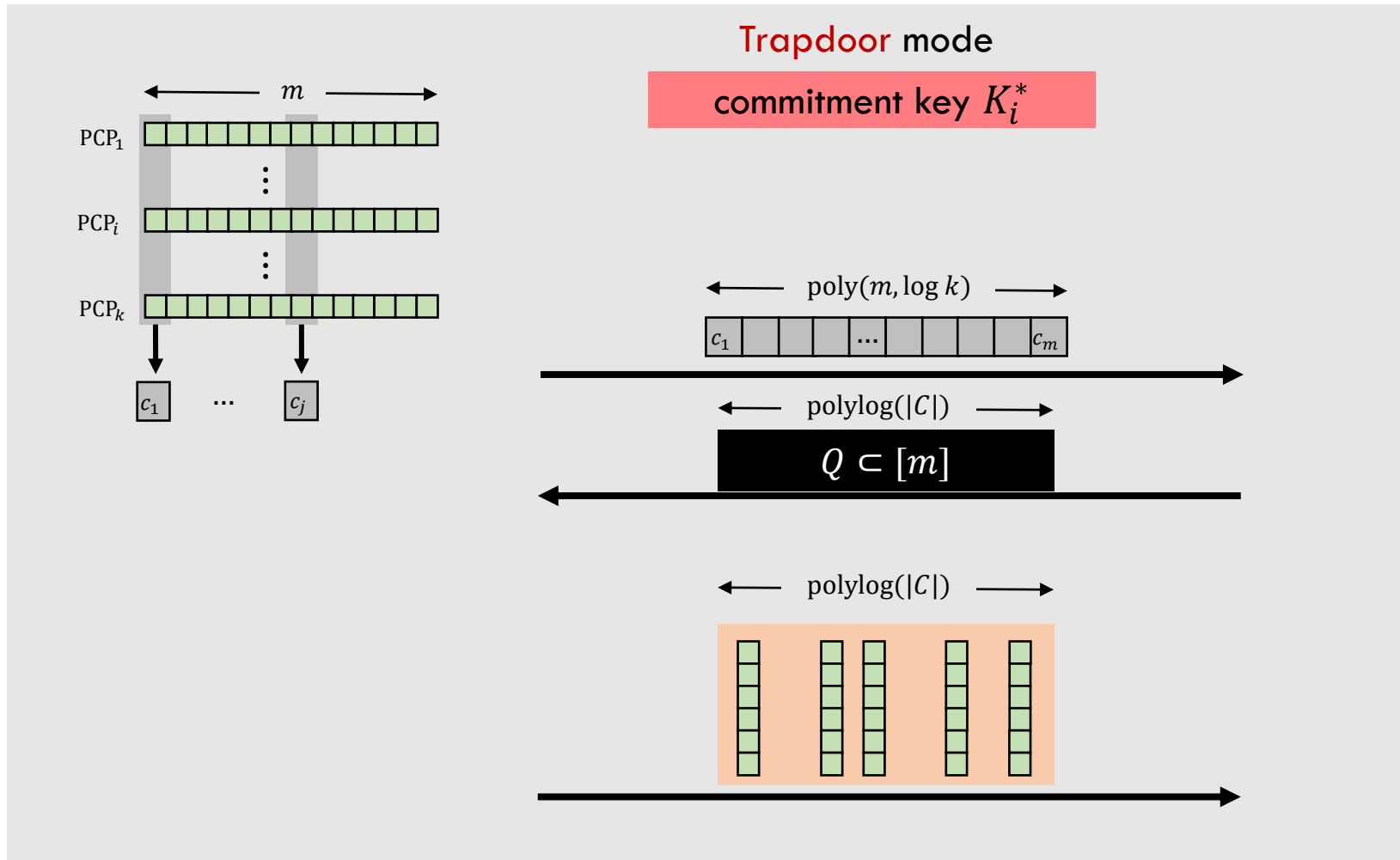
$$\forall i \in [k], i \in L_C$$

Somewhere Statistically Binding (SSB) Commitment Scheme

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Dual Mode Argument for Batch Index



$$L_C = \{i \mid \exists w \text{ s.t. } C(i, w) = 1\}$$

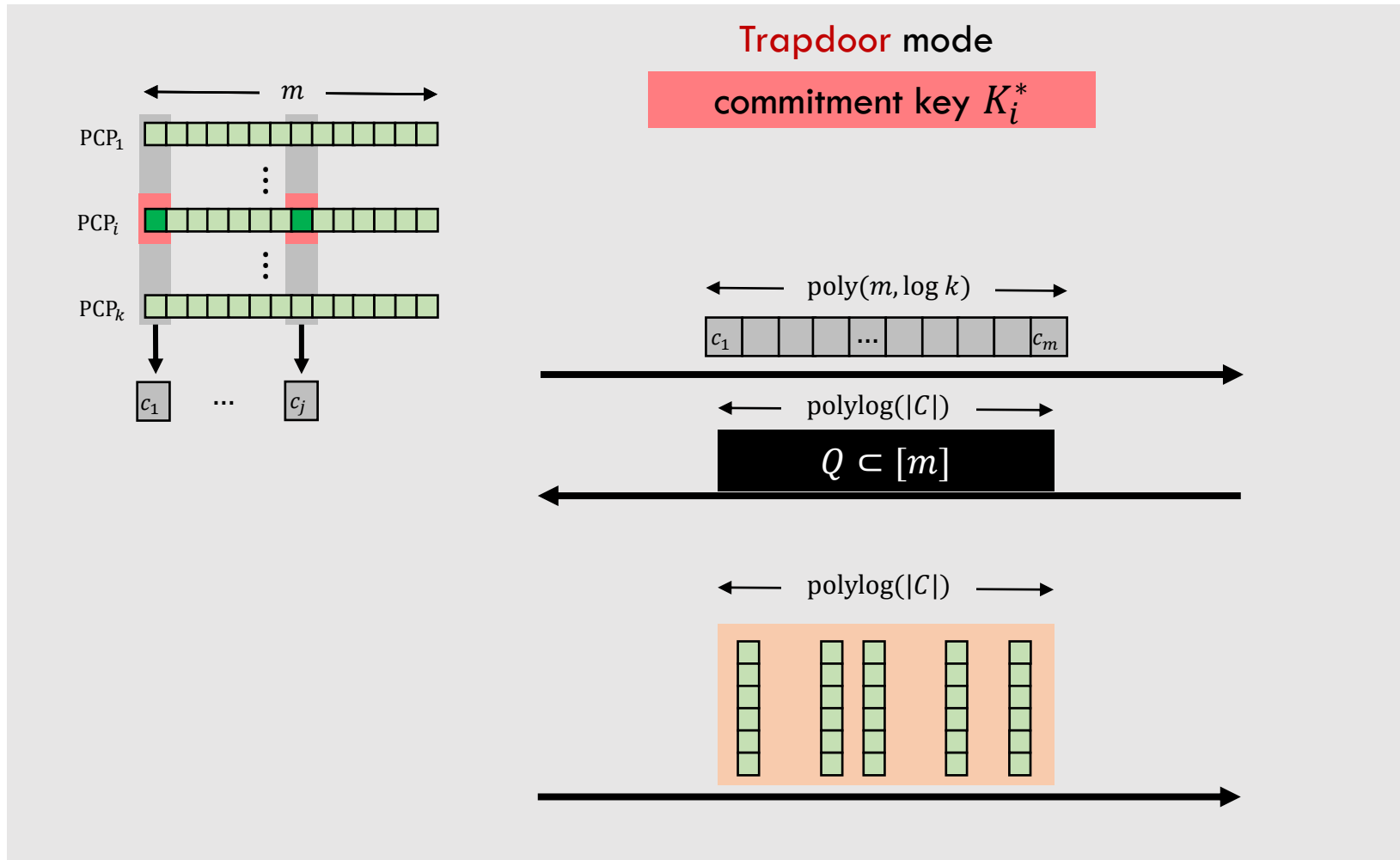
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Somewhere Statistically Binding (SSB) Commitment Scheme

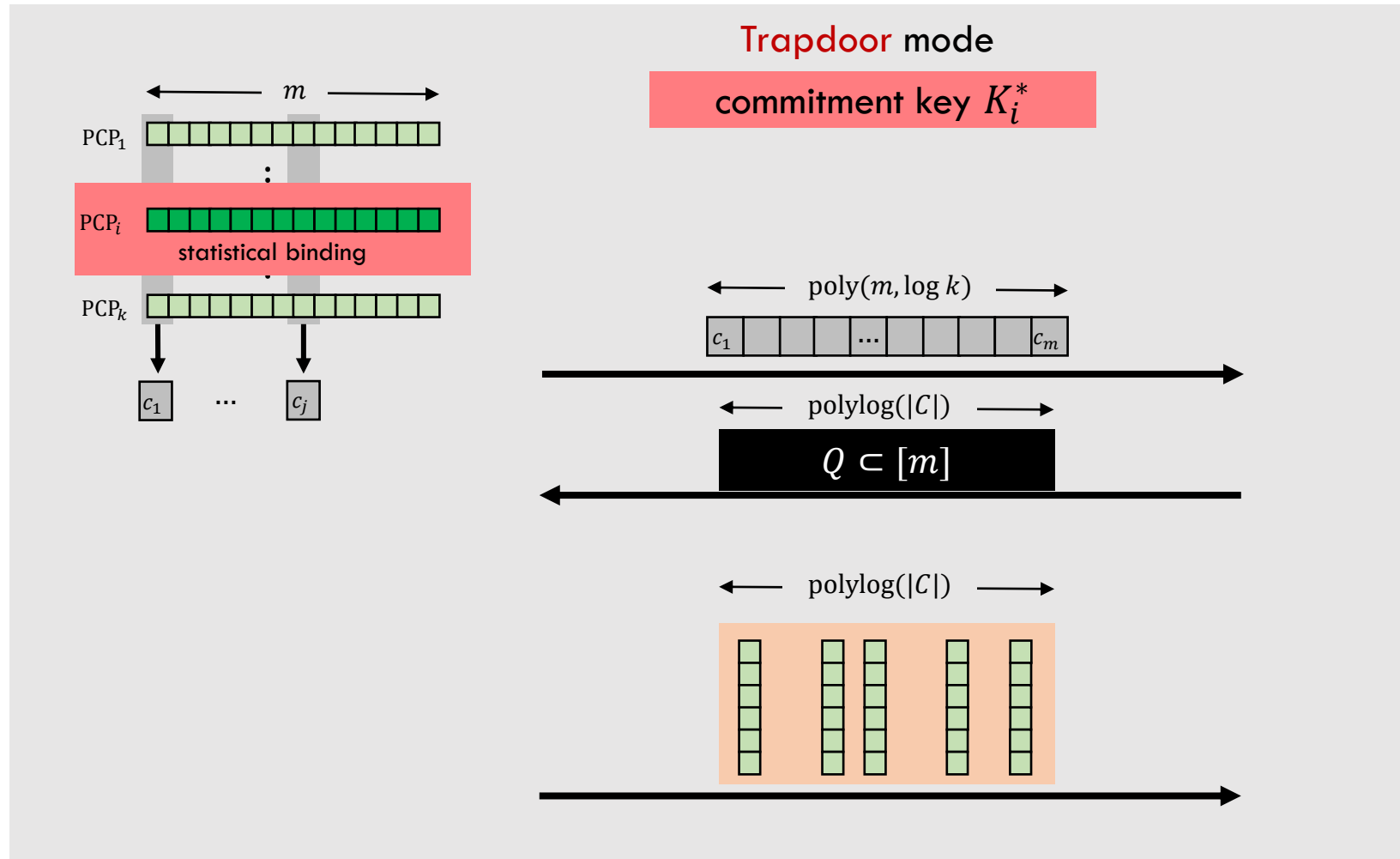
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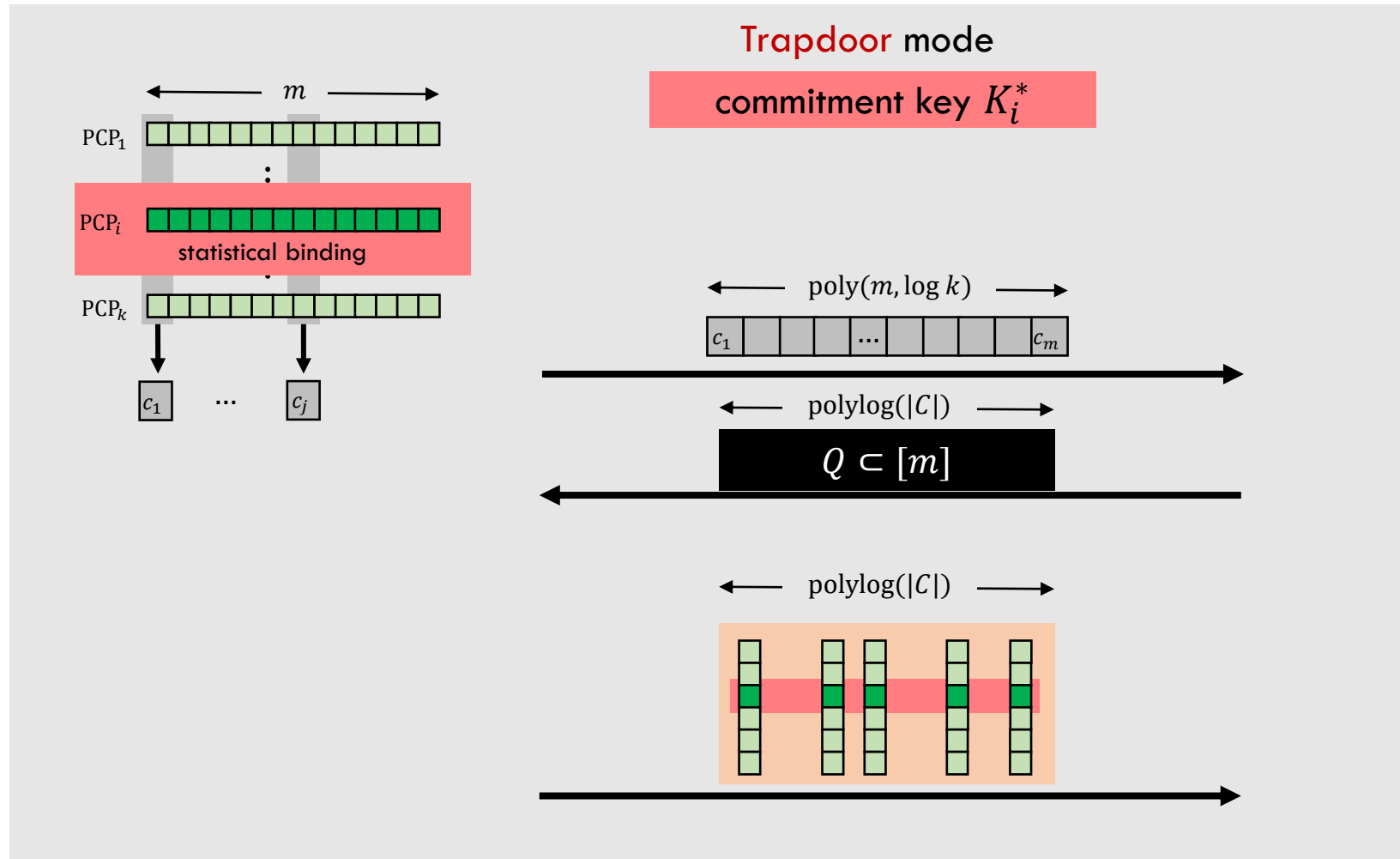
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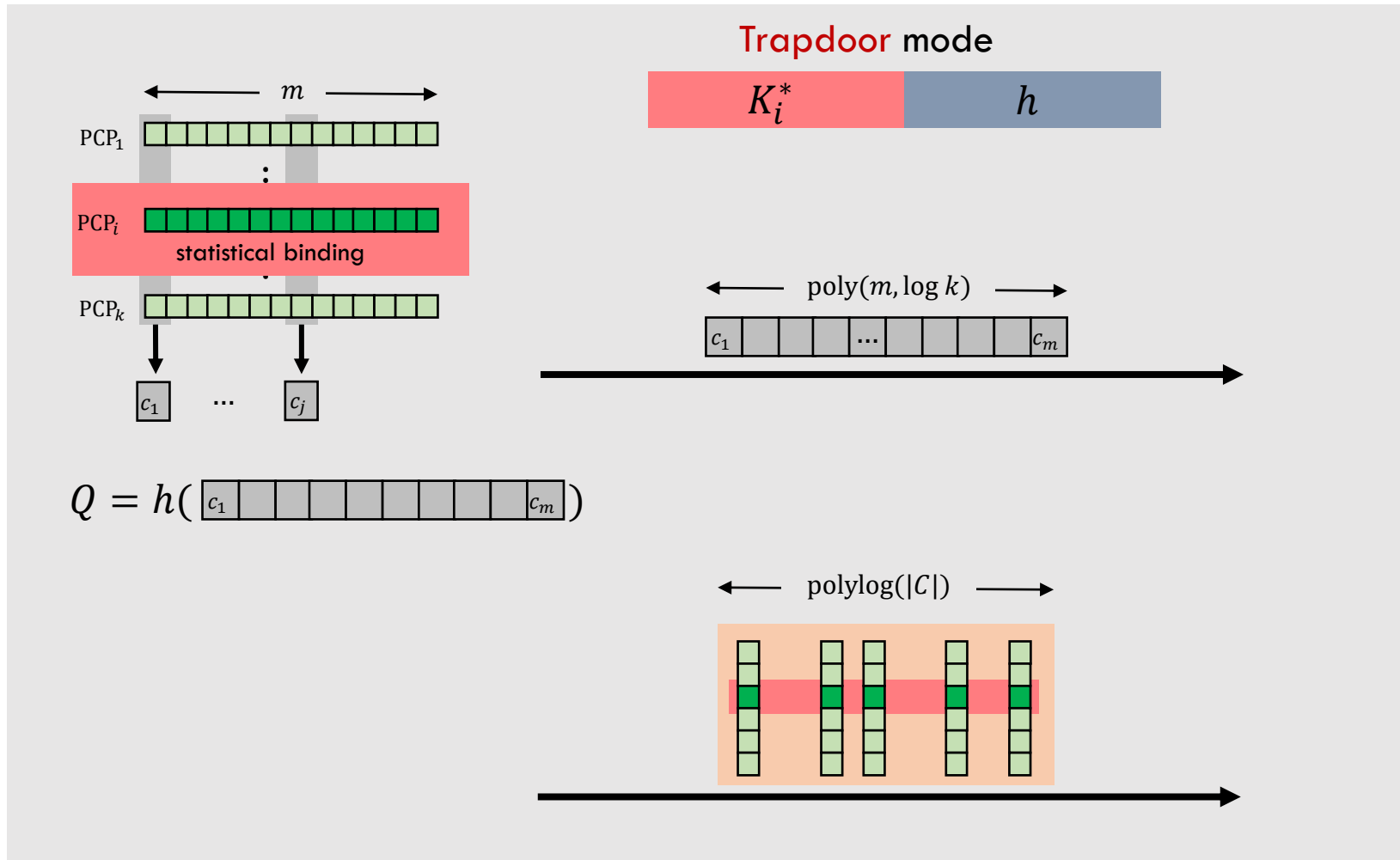
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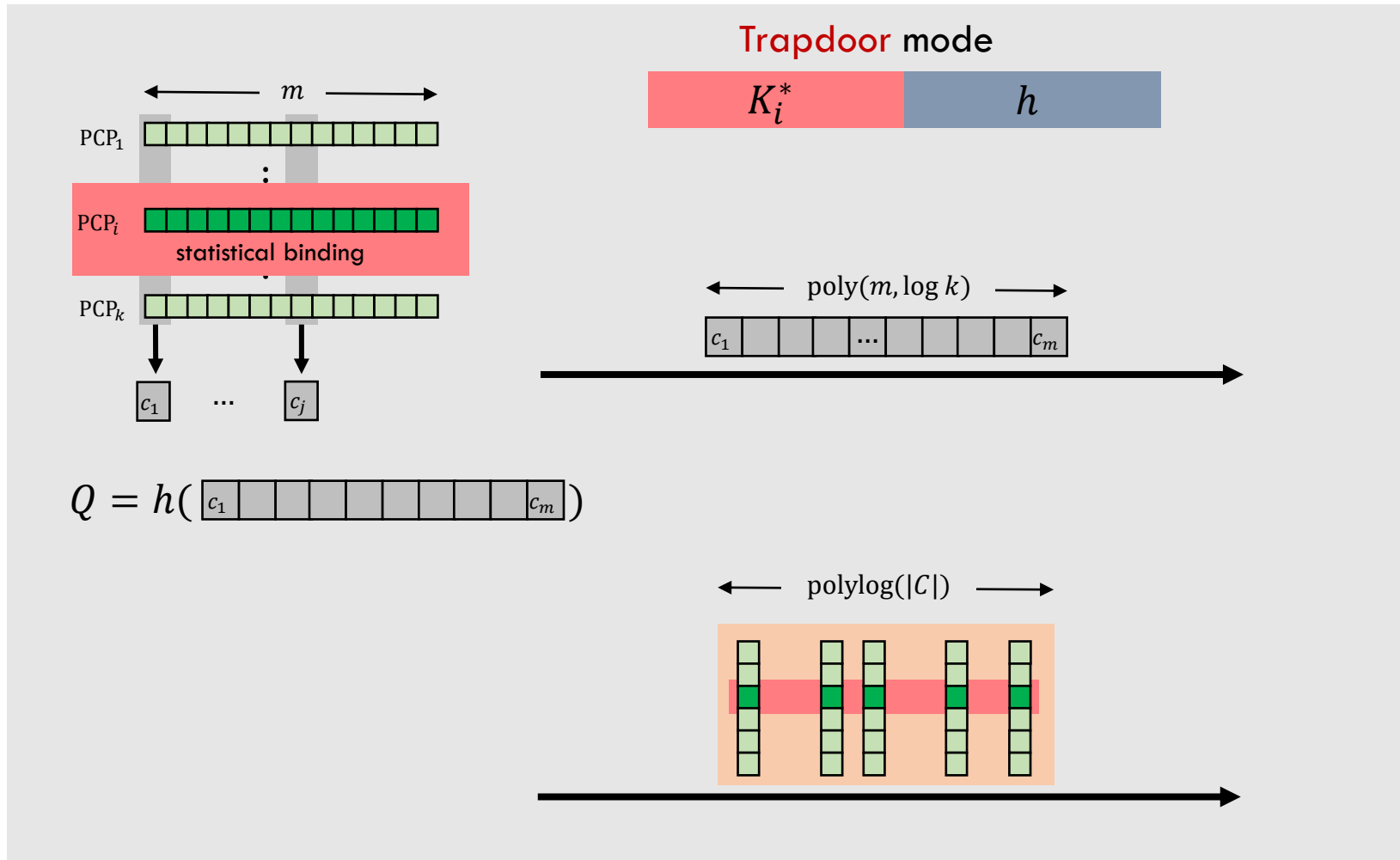


- Verify:
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Dual Mode Argument for Batch Index

$$L_C = \{i \mid \exists w \text{ s.t. } C(i, w) = 1\}$$

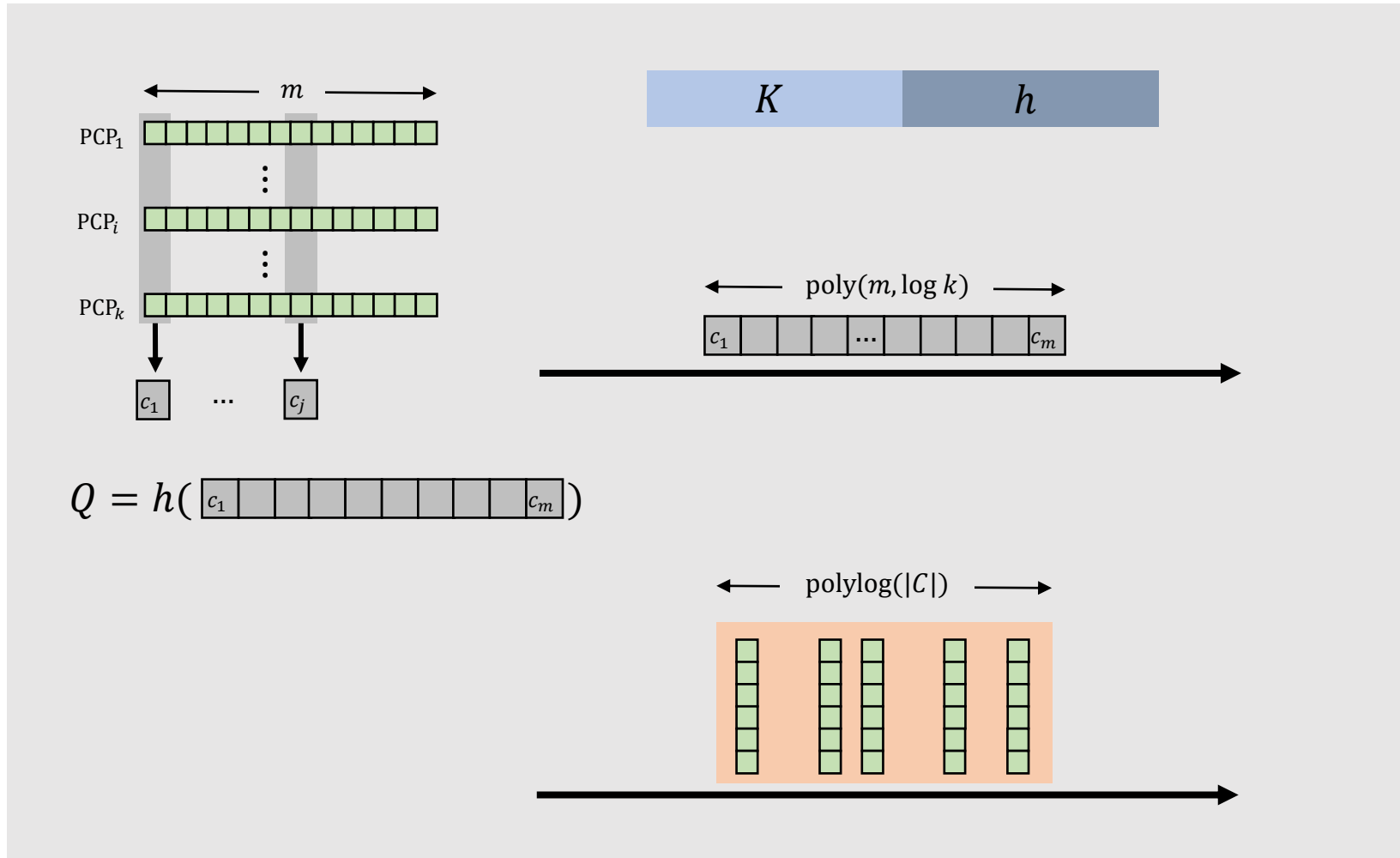
$$\forall i \in [k], i \in L_C$$



[Holmgren-Lombardi-Rothblum'21]
Assuming **LWE**, the transformation is sound.

- Verify:
1. Commitment openings are valid.
 2. PCP responses verify on Q

SNARG for Batch Index



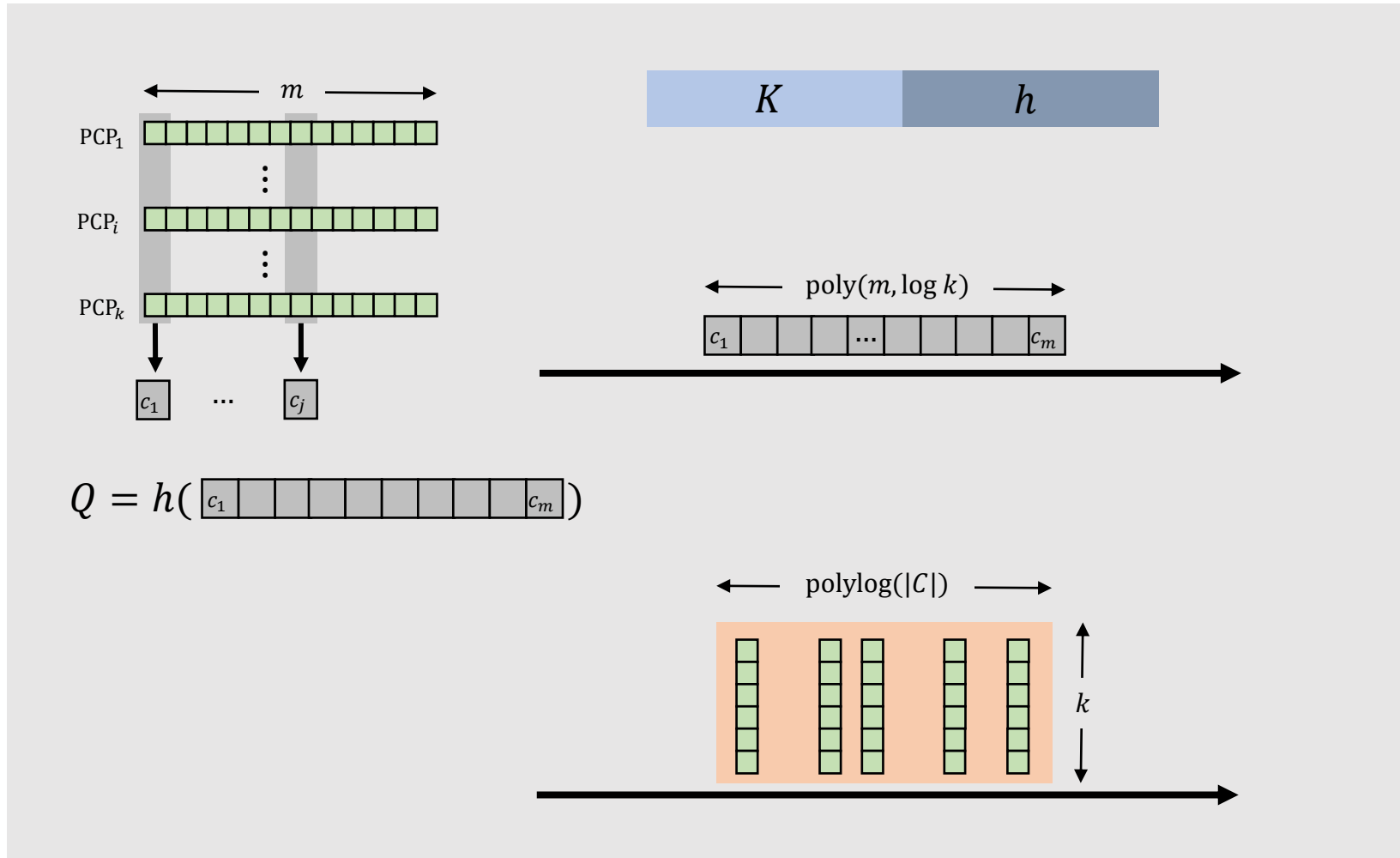
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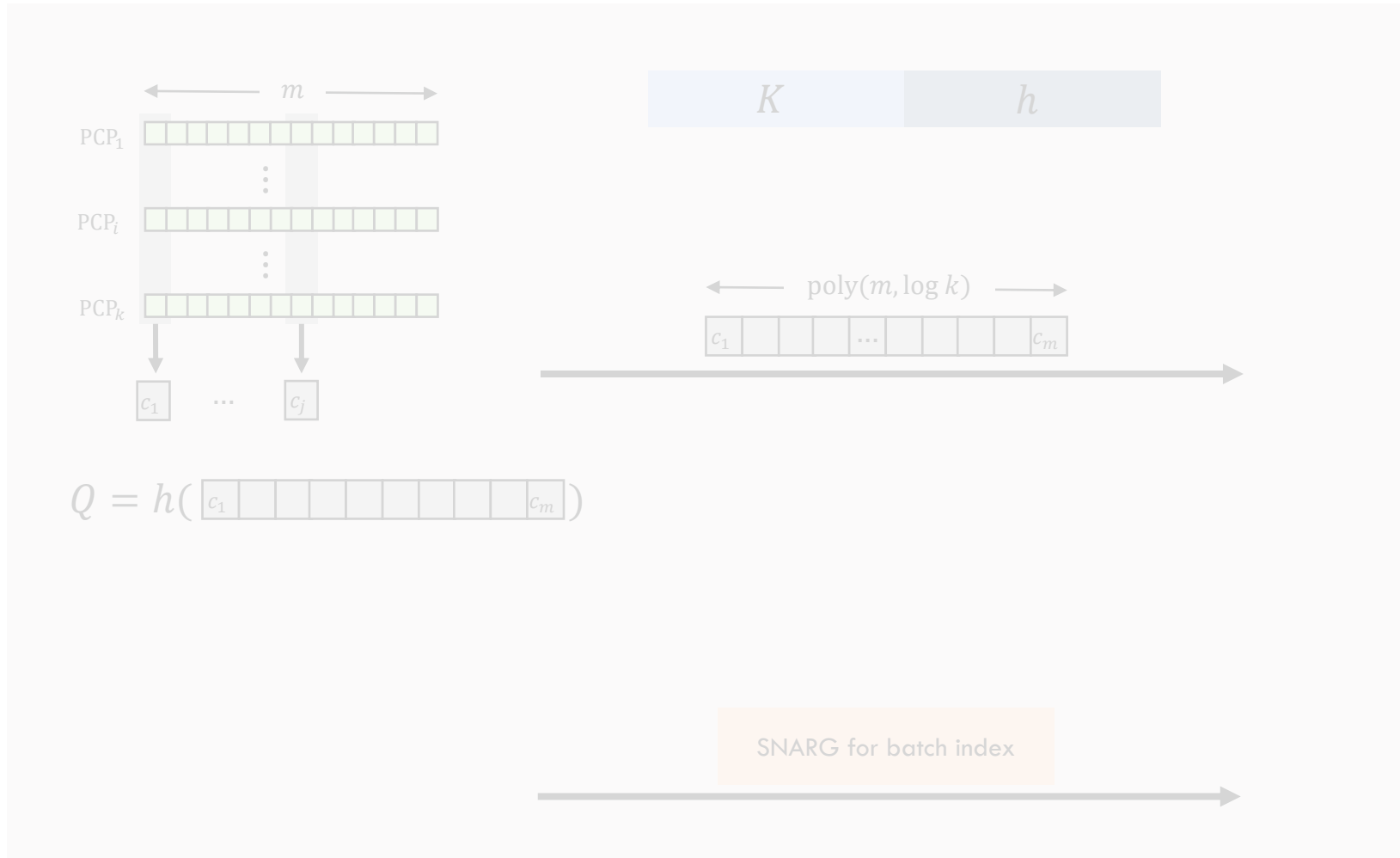
$$\forall i \in [k], i \in L_C$$

$$|\text{Verifier}| = \Omega(k)$$

Verify:

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SNARG for Batch Index



$$L_C = \{i \mid \exists w \text{ s.t. } C(i, w) = 1\}$$

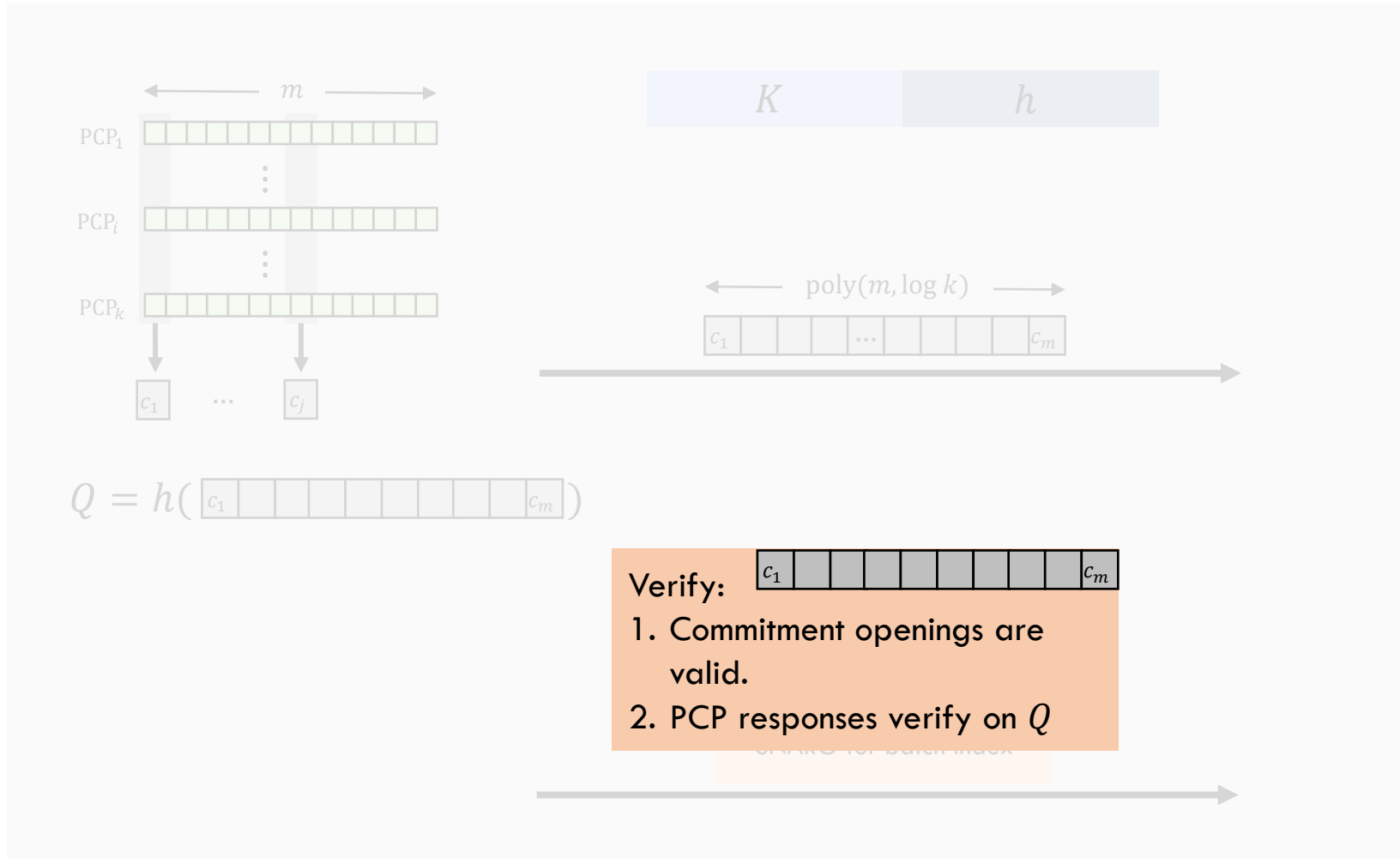
$$\forall i \in [k], i \in L_C$$

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SNARG for Batch Index

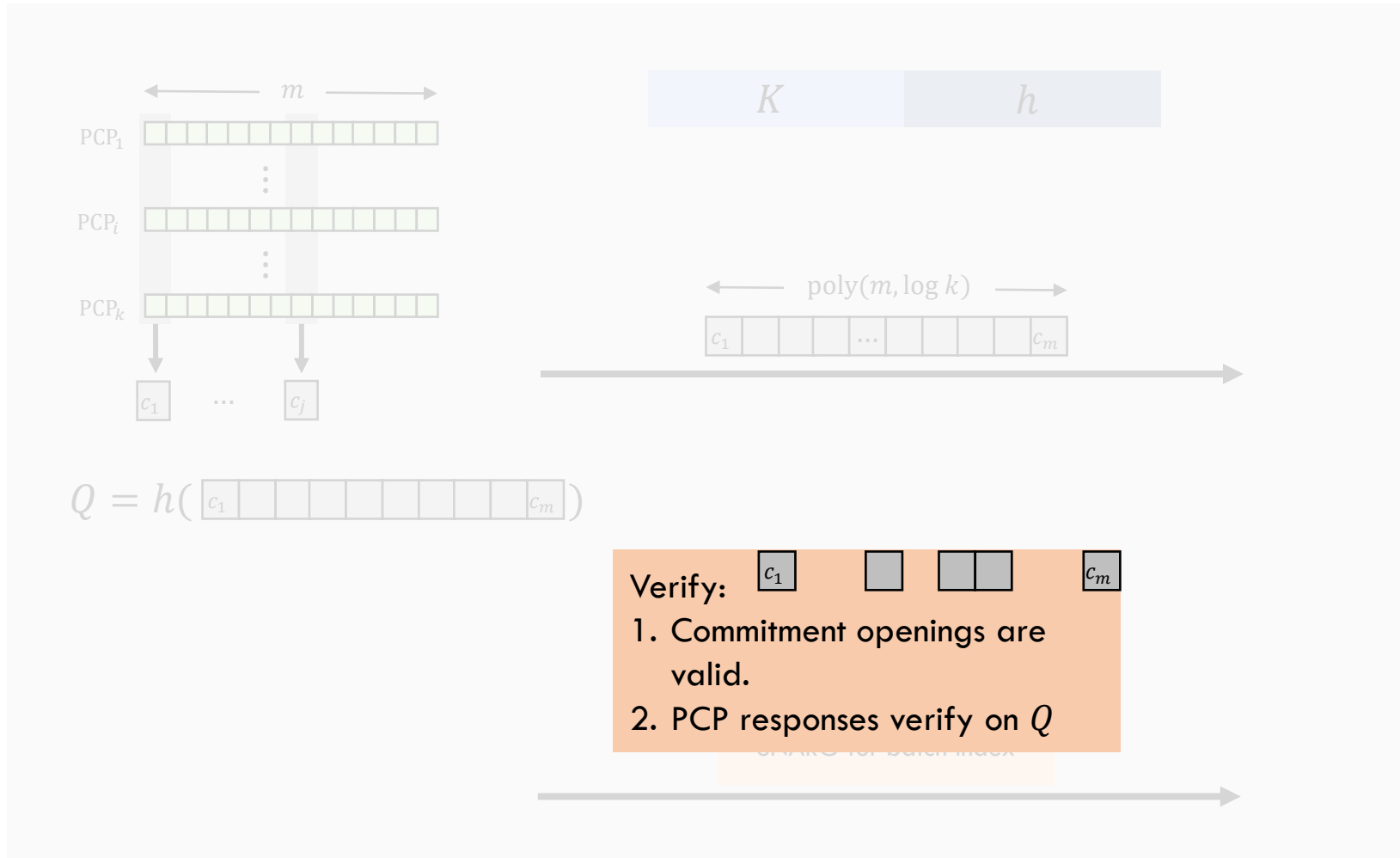


$$L_C = \{i \mid \exists w \text{ s.t. } C(i, w) = 1\}$$

$$\forall i \in [k], i \in L_C$$

$$|\text{Person}| = \Omega(k)$$

SNARG for Batch Index

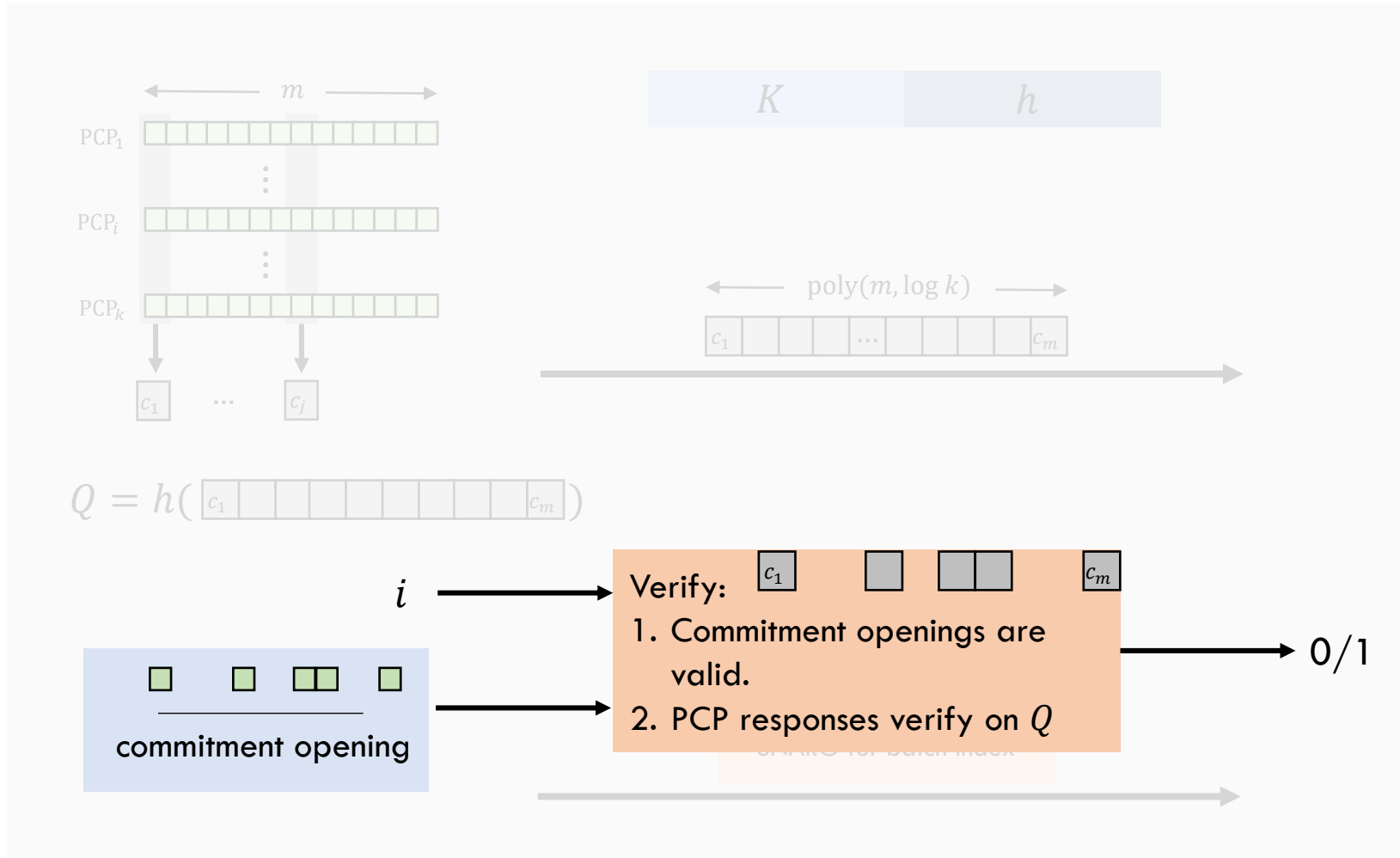


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$$|\text{Person}| = \Omega(k)$$

SNARG for Batch Index

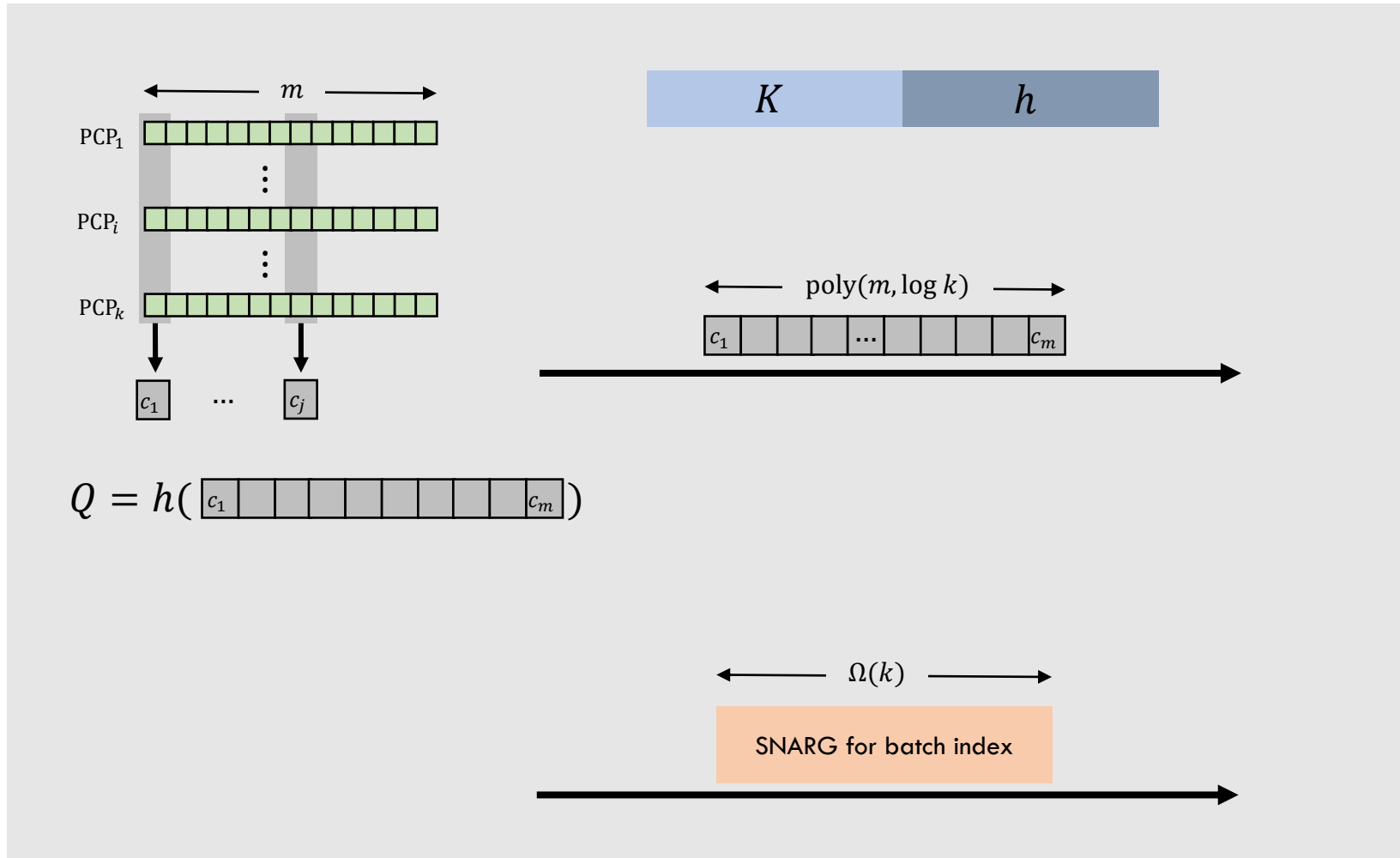


$$L_C = \{i \mid \exists w \text{ s.t. } C(i, w) = 1\}$$

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SNARG for Batch Index



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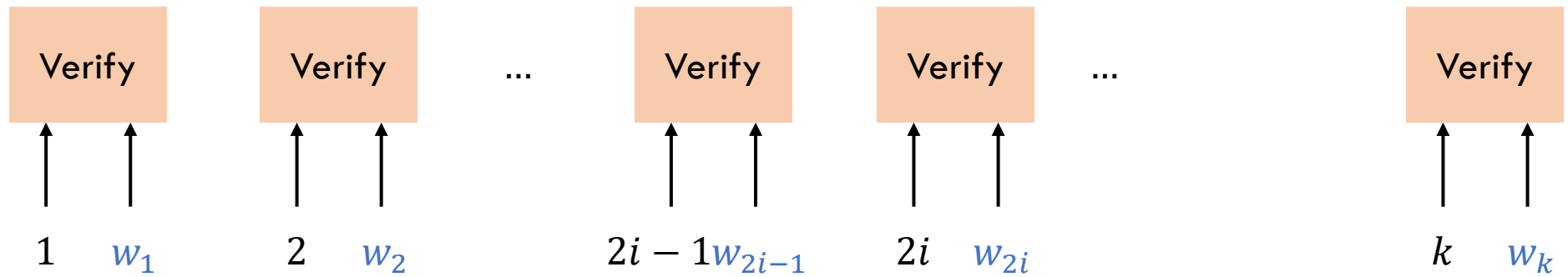
$$| \text{Person} | = \Omega(k)$$

Verify:

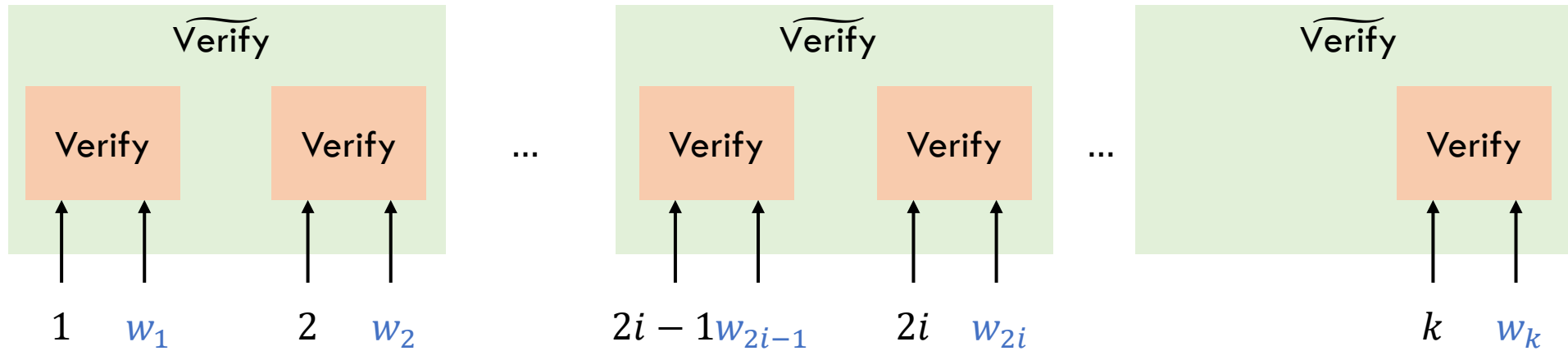


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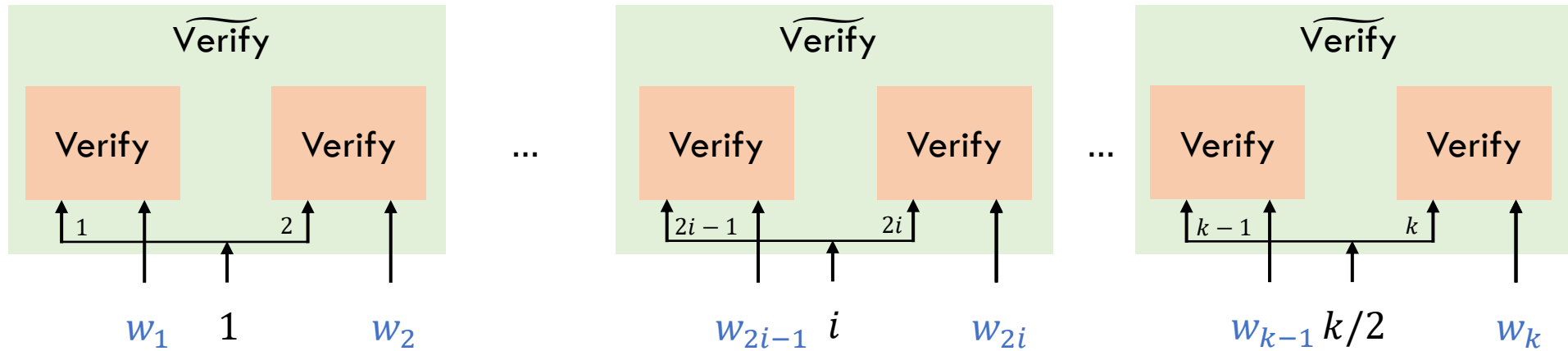
Grouping Instances for Recursion



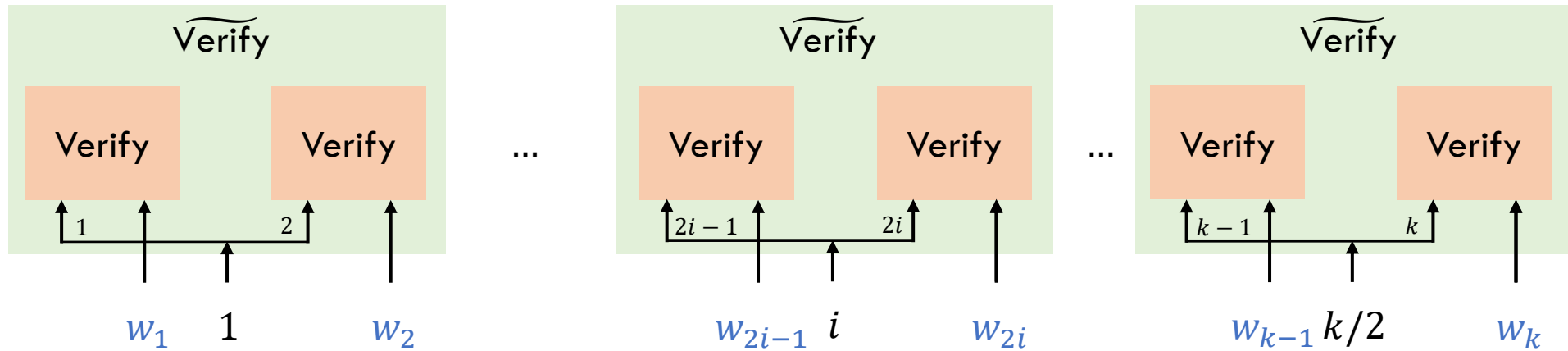
Grouping Instances for Recursion



Grouping Instances for Recursion



Grouping Instances for Recursion



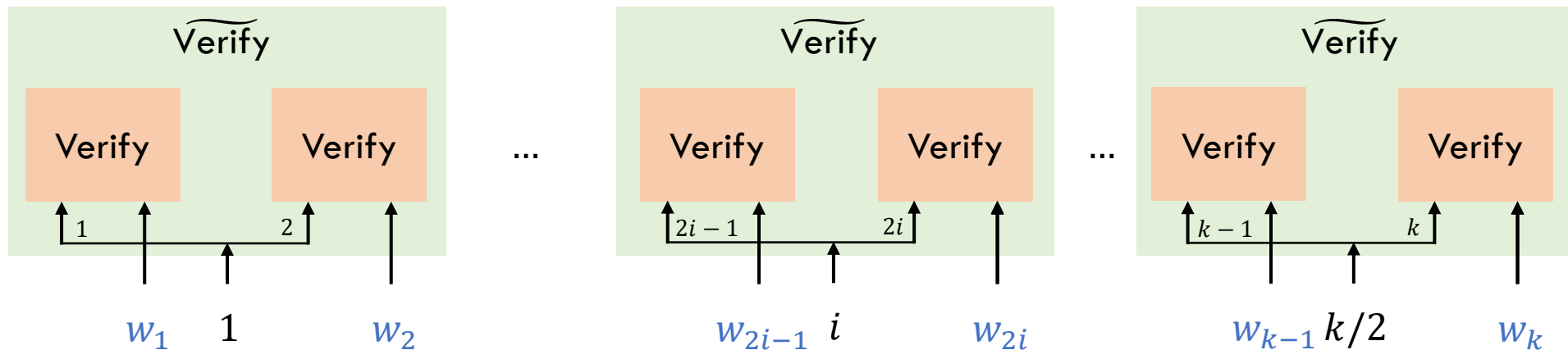
$k/2$ instances for circuit $\widetilde{\text{Verify}}$

Grouping Instances for Recursion

Recurse $\log(k)$ times

Further Challenges: Prevent exponential growth in $\widetilde{\text{Verify}}$

- Tool: PCP with Fast Online verification



$k/2$ instances for circuit $\widetilde{\text{Verify}}$

Recap

LWE

SNARGs for Batch NP

SNARGs for polynomial
time computation.

Dual Mode Interactive Batch Arguments + Fiat-Shamir
instantiation via CIH

No Signaling SSB Commitments

Thank you. Questions?

Arka Rai Choudhuri

arkarc@berkeley.edu

ia.cr/2021/808